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ÖNGERİLME ETKİSİNDEKİ TABAKALI YARIM DÜZLEM İÇİN LAMB PROBLEMİNİN İNCELENMESİ

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İÇİNDEKİLER

SİMGE LİSTESİ

λ, μ	Lamé değişmezleri
ρ	Kütle yoğunluğu
E	Elastisite modülü
ν	Poisson oranı
u_i	Yerdeğiştirme vektörünün bileşenleri
σ_{ij}	Gerilme tansörünün bileşenleri
P_0	Tekil yük
$\delta(x_i)$	Dirac-Delta fonksiyonu
ω	Yükün frekansı
∇_i	Gradyan operatörü
ε_{ij}	Şekil değiştirme tansörünün bileşenleri
ε_{ijk}	Permütasyon sembolü
t	Zaman
x_i	Dik kartezyen koordinat sistemi.
φ, ψ_i	Yerdeğiştirme potansiyelleri
c_L, c_T, c_R	Dalga hızları
Θ, Ω	Tanımlanan fonksiyonlar
k_L, k_T, k_R	Dalga yavaşlıkları
$f_F(k, x_2, \ldots, x_k)$	$f(x_1, \ldots, x_k)$ fonksiyonunun x_1'e göre üstel Fourier dönüşmüşü
δ_{ij}	Ortogonal koordinatlarda birim tansör
ζ	Karmaşık değişken
α, β	Bilinmeyen katsayılar
η_1, η_2	Öngerilmelere ilişkin katsayılar
σ_{0ij}	Öngerilme tansörü
ϑ	Tanımlanan açı
Ω	Diagramların çiziminde kullanılan hız boyutunda bir parametre
n_i	Birim dış normal

ŞEKİL LİSTESİ

ONSOZ

Tezi hazırlamam sırasında değerli görüş ve önerilerini esirgemeyerek büyük yardımları olan ve beraber çalıştığımız dört sene içinde verdiği bilgi ve kazandırdığı yaklaşımlarla kendimi yetiştirmemde büyük katkısı olan tez danışmanım sayın Prof. Dr. Surkay D. Akbarov 'a içtenlikle teşekkürlerimi sunarım.

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ÖZET

Bu çalışmada öngerilme etkisindeki tabakalı bir yarım düzlemde, yüzeyde normal doğrultuda ve zamana göre harmonik bir tekil yükün etkimesi durumunda oluşan gerilme dağılımına öngerilmelerin etkisi incelenmiştir. Lineer olmayan Cauchy hareket denkleminden yola çıkılarak elde edilen yönetici denklemler, öngerilmelerin şiddetinin öngerilme durumunun üzerine eklenen hareketten dolayı oluşan gerilmelerin şiddetinden yeterince büyük olduğu kabulüne dayanılarak uygun şekilde lineerleştirilmiştir. Stasyoner durum gözönüne alındığı için problemin bağımlı değişkenlerinin genlikleri zamana göre harmonik yapıda olmakta ve bu durum da parça türevli denklem sistemindeki bağımsız değişken sayısını bir azaltarak ikiye indirmektedir. Bu denklem sistemi yerdeğiştirme potansiyelleri kullanılarak ayrıldıktan (dekuple edildikten) sonra üstel Fourier dönüşümü uygulanırsa, bu potansiyelleri içeren ve analitik çözümü elde edilebilen iki tane sıradan türevli denklem elde edilir. Asıl bilinmeyenler olan gerilmeler ve yerdeğiştirmeler ise ters Fourier dönüşümünün sayısal olarak yapılması ile elde edilmektedir. Formülasyonu verilen model lineer olmayan yaklaşım kullanılarak elde edildiğinden burada elde edilen sonuçları, öngerilmelerin ve yükün ayrı ayrı etkimesi durumunda klasik teoriden elde edilen çözümlerin süperpozisyonu yardımı ile elde etmek mümkün değildir. Tabaka kalınlığının, frekansın, tabaka ile yarım düzlemin elastisite modüllerinin oranının değişimlerinin çeşitli öngerilme kombinasyonlarının varlığı durumunda gerilme dağılımlarına olan etkileri araştırılmış ve pratikteki bazı mühendislik problemlerin çözümüne ışık tutacak sonuçlara yer verilmiştir.

ABSTRACT

The effects of initial stresses within an elastic stratified half-plane on the stress fields superimposed by a harmonic point-perturbation source acting on the free surface in normal direction, have been investigated. The governing equations of this phenomenon, derived through non-linear Cauchy equations of motion, have been linearized regarding to the assumption that the initial stresses are much greater than the induced harmonic stress fields in amplitude. As it is the stationary case being interested in here, the amplitudes of the dependent variables of the problem become harmonic, decreasing the number of the independent variables from 3 to 2 in the governing system of partial differential equations. When displacement potentials are employed, decoupling of this system into two discrete partial differential equations is possible, and applying the exponential Fourier transformation results in two ordinary differential equations, the analytical solutions of which can be obtained straightforwardly. Then the unknown fields are obtained through inverse Fourier transforms numerically. Although the mathematical model used is linear, having been derived through a non-linear approach, it is not possible to obtain the same results by considering these two states, when the initial stresses and the surface perturbation act individually, via the principle of superposition. The effects of the variations in the layer thickness, the frequency and the ratio of the modulus of elasticities, when there are different combinations of initial stresses in sign, to the secondary stress field have been investigated and some results have been given, supposed to enlighten some specific problems of engineering.

1. GİRİŞ ve ÖNCEKİ ÇALIŞMALAR.

Günümüz teknolojisinin hemen tüm dallarında sürekli ortamlar mekaniğinin dinamik problemleri ile karşılaşıldığından bu alandaki bilimsel incelemelerin sayısı katlanarak artmaya devam etmektedir. Bu incelemelerin büyük bir kısmını oluşturan lineer problemlerin sistematik sunum ve incelemeleri Ewing vd. (1957), Achenbach (1973), Eringen ve Şuhubi (1974a;1974b) vd kaynaklarda yer almaktadır. Ancak tabiatta gerçekleşen dinamik olayların büyük bir kısmının lineer olmaması, sürekli ortamların lineer olmayan dinamik problemlerinin araştırılmasına önem kazandırmaktadır ve bu problemlerin önemli bir kısmı Green ve Atkins (1960), Eringen (1964), Trusdell (1966)vd'de sistematik olarak incelenmiştir.

Lineer olmayan problemlerden teori ve uygulama bakımlarından incelenmesi önemli olan konulardan birisi öngerilmeli ortamların dinamiğidir. Teknolojik gereksinmelerden dolayı yapı elemanlarında, tektonik veya sismik etkilerden dolayı arzı oluşturan tabakalarda, imalat işlemleri sırasında kompozit malzemelerde ve biomekanikte öngerilmelerin oluştuğu durumlarda, bunların etkisinin gözönüne alınmasının gerekeceği açıktır. Böyle durumlarda genellikle, incelenmesi çok zor hatta bazen imkansız olan lineer olmayan başlangıç ve sınır değer problemleriyle karşılaşılır. Ancak öngerilmelerin ortamda dinamik etkiden dolayı oluşan gerilmelerden değerce çok büyük olması durumunda, bu olaylar sözkonusu lineer olmayan denklemlerin lineerleştirilmesi ile de incelenebilir. Böyle durumlarda lineerleştirilen denklemlerde öngerilmeler sıfır alınırsa uygun klasik hareket denklemleri elde edilmektedir.

Öngerilmeli sürekli ortamların dinamiğine ait yapılmış olan araştırmaların kısa bir özetini vermek gerekirse, öncelikle konuya ait genel bilgiler Green vd. (1952), Bergman ve Shahbender (1958), Green (1963), Smitt (1963), Biot (1965), Fine ve Shield (1966), Tang (1967), Mott (1971), Surhendu (1972), Wilson (1972), Wattson (1973), Chakraborty ve De (1982), Guz (1999) v.b. kaynaklarda verilmiştir. Konu ile ilgili araştırmalar üç grupta toplanabilir; **i.**Yüzey dalgalarının yayılımına öngerilmelerin etkisi, **ii.**Malzeme özelliklerinin öngerilmeli ortamlarda dalga yayılımına etkisi ve **iii.**Yapı elemanlarındaki öngerilmelerin dalga yayılımına etkisi. İlk gruptaki incelemelere örnek olarak; Hayes ve Rivlin (1961), Wilson (1973a), Wagh (1974), Dey ve Addy (1977), Bhattacharya ve Sengupta (1978), Chadwick ve Jarwis (1979a; 1979b; 1979c), Chattopadhyay ve Kar (1981), Chattopadhyay vd. (1982), Chakraborty ve Dey (1980, 1982), Guz (1988) vd. gösterilebilir. Bu gruba ait araştırmaların büyük bir kısmının özeti ise Bagno ve Guz (1997) 'de yer almaktadır. İkinci grupta toplanabilen çalışmalara örnek olarak Hayes ve Rivlin (1961), Toupin ve Bernstein

(1961), Crecraft (1962), Kostrov ve Nikitin (1969), Dey (1971), Belward (1972), Kurashige (1972,1979), Weitsmann (1973), Rogers vd. (1976), Sawyers ve Rivlin (1978), Pan (1979), Reismann ve Pawlik (1979), verilebilir. Üçüncü gruba da Ramakanth (1964), Şuhubi (1965), Chen ve Wright (1966), Demiray ve Şuhubi (1970), Wilson (1973b,1977), Bradford ve Dong (1975), Belward (1976), Bhattacharya (1977), Kladinoga ve Bogdanov (2000) örnek verilebilir. Diğer taraftan öngerilmeli ortamlarda dalga yayılımına ait araştırmaların büyük çoğunluğunu içeren sistematik bir inceleme Guz (1986a; 1986b) de yer almaktadır.

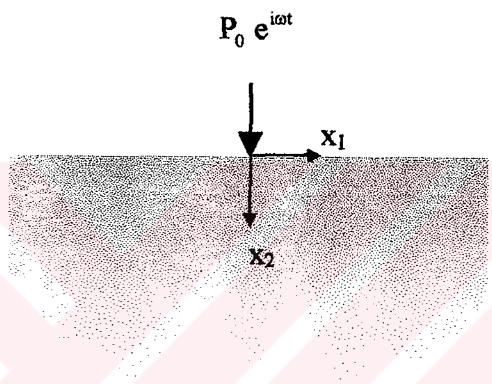
Yukarıdaki incelemelerin hepsinde öngerilmelerin, dalga yayılım hızının onun uzunluğuna bağımlılığına (dispersiyon) olan etkisi veya öngerilmelerin Rayleighh, Love ve Stoneley gibi yüzey dalgalarının yayılma hızına etkileri incelenmiştir. Öngerilmelerin ortamda dinamik dış etkiden dolayı oluşan gerilme ve şekil değiştirme yayılımlarına olan etkisini inceleyen araştırmalar ise çok az sayıda olup, örnek olarak Payton (1969,1971,1972), Koshman (1981a; 1981b) verilebilir. Sırasıyla bu iki yazara ait çalışmalarda öngerilmelerin yarım düzlemde ve düzlemde dalga yayılımına etkileri araştırılmıştır. Az sayıda çalışma yapılmış olmakla beraber bu konunun teori ve uygulama açısından çok yararlı ve gerekli olduğu, tabakalı bir yarım düzlemde oluşan dinamik gerilme yayılımına öngerilmelerin etkisinin bilinmesinin, kompozit malzemeler mekaniği, jeomekanik ve maden yataklarının mekaniği açılarından önemi düşünüldüğünde, açıkça görülmektedir. Dolayısıyla bu tezde incelelen, homogen öngerilme etkisindeki tabakalı yarım düzlemde, serbest yüzeyde etkiyen periyodik (harmonik) tekil dış kuvvetten dolayı oluşan gerilme yayılımına bu öngerilmelerin etkisinin incelenmesi, teori ve uygulama bakımından önem kazanan güncel bir konudur ve burada yapılan çalışmalar bu konuya ilişkin ilk incelemelerdir. Kaldı ki literatürde, tezde incelenen problemlerin öngerilmesiz (klasik) durumlarına ait sayısal incelemeler de çok kısıtlı veya olmayıp bu nedenle uygun klasik durumlar için de –özel birer durum olarak– uygulamalar yapılmıştır. Bu söylenilenlerden fazla ayrıntıya girmeden tezdeki araştırmanın amaçları şöyle özetlenilebilir.

1. Öngerilmesiz yarım düzlem için klasik Lamb probleminin formülasyonu, çözüm yöntemlerinin açıklanması ve literatürde bulunamayan sayısal sonuçların elde edilmesi.
2. Uygun lineerleştirilmiş hareket denklemleri ve sınır koşullarının elde edilerek öngerilmeli ve tabakalı yarım düzlem için Lamb probleminin formülasyonu,
3. Üstel Fourier dönüşümü yardımı ile uygun analitik elde edilmesi ve sayısal sonuçların ters Fourier dönüşümü ile bulunması için gerekli bilgisayar programının hazırlanması
4. Öngerilmelerin gerilme yayılımlarına olan etkilerini gösteren çok sayıda incelemenin yapılarak sonuçların yorumlanması.

2. KLASİK LAMB PROBLEMİNİN ÇÖZÜM YÖNTEMLERİ

Lamb probleminin çözümü hakkında literatürde genellikle yalnız yüzey yerdeğiştirmelerinin dağılımı için yaklaşık asimptodik ifadelerin verilmesi ile yetinilmekte, bütün yarım düzlemde oluşan gerilme ve yerdeğiştirme dağılımlarının frekansa ve yüzeye olan uzaklığa bağımlı olan değişimlerini ayrıntılı olarak ortaya koyan grafiklere ise rastlanılmamaktadır. Bu bölümde literatürdeki bu eksiği dolduracak kapsamda bir incelemenin yapılması amaçlanmıştır. Eşitliklerde indis notasyonu kullanılmış olup toplama uylaşımı (tekrarlanan indisler üzerinde toplama) geçerlidir.

2.1 Problemin Tanımı ve Çözüm Yöntemi.



Şekil 2.1 Lamb problemi.

Şekil 2.1'de gösterilen Lamb probleminin yönetici denklemi ve sınır koşulları,

$$
\begin{aligned}
&(\lambda+\mu)\, u_{j,ji} + \mu\, u_{i,jj} = \rho\, \ddot{u}_i \\
&\sigma_{ij} = \lambda\,\theta\,\delta_{ij} + 2\mu\,\varepsilon_{ij}\ , \ \theta = u_{k,k} \qquad i, j, k = 1,2 \\
&\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})
\end{aligned}
\tag{2.1}
$$

$$
\sigma_{12}\big|_{x_2=0} = 0,\ \ \sigma_{22}\big|_{x_2=0} = -P_0\, \delta(x_1)\, e^{i\omega t} \tag{2.2}
$$

$$
u_i, \sigma_{ij}\big|_{x_2\to\infty} = 0 \qquad i, j = 1,2 \tag{2.3}
$$

şeklindedir. Burada bilinen notasyon kullanılmıştır. Kütle kuvvetlerinin olmadığı ve düzlem şekil değiştirme durumunun varlığı kabul edilmiştir. Stasyoner durum gözönüne alındığı için,

$$
\{u_i(x_k,t), \sigma_{ij}(x_k,t), \varepsilon_{ij}(x_k,t)\} = \{\bar{u}_i(x_k), \bar{\sigma}_{ij}(x_k), \bar{\varepsilon}_{ij}(x_k)\} e^{i\omega t} \qquad i, j, k = 1,2 \tag{2.4}
$$

yazılabilir. (2.4); (2.1)-(2.3) de kullanılırsa genlikler için yönetici denklem ve sınır koşulları,

$$\begin{aligned}&(\lambda+2\mu)\,\overline{u}_{1,11}+\mu\,\overline{u}_{1,22}+\rho\,\omega^2\,\overline{u}_1+(\lambda+\mu)\,\overline{u}_{2,12}=0\\&(\lambda+\mu)\,\overline{u}_{1,12}+\mu\,\overline{u}_{2,11}+(\lambda+2\mu)\,\overline{u}_{2,22}+\rho\,\omega^2\,\overline{u}_2=0\end{aligned} \tag{2.5}$$

$$\overline{\sigma}_{12}\Big|_{x_2=0}=0,\quad \overline{\sigma}_{22}\Big|_{x_2=0}=-P_0\,\delta(x_1) \tag{2.6}$$

$$\overline{u}_i,\overline{\sigma}_{ij}\Big|_{x_2\to\infty}=0\qquad i,j=1,2 \tag{2.7}$$

şeklinde elde edilir. Yerdeğiştirme vektörünü $\overline{\varphi}$ ve $\overline{\psi}_i$ potansiyelleri cinsinden veren,

$$\overline{u}_i=\overline{\varphi}_{,i}+\varepsilon_{ijk}\overline{\psi}_{k,j} \tag{2.8}$$

eşitliği uyarınca $\overline{u}_3=0$ sıfır olduğu için,

$$\begin{aligned}\overline{u}_1&=\overline{\varphi}_{,1}+\overline{\psi}_{3,2}\\\overline{u}_2&=\overline{\varphi}_{,2}-\overline{\psi}_{3,1}\end{aligned} \tag{2.9}$$

yazılırsa (2.5)'e göre,

$$\begin{aligned}&(\lambda+2\mu)\,\overline{\varphi}_{,ii}+\rho\,\omega^2\,\overline{\varphi}=0\\&\mu\,\overline{\psi}_{3,ii}+\rho\,\omega^2\,\overline{\psi}_3=0\end{aligned} \tag{2.10}$$

eşitlikleri bulunur. Lineer-elastik, homogen ve izotrop ortamda enine ve boyuna dalga hızları,

$$c_L=\left(\frac{\lambda+2\mu}{\rho}\right)^{1/2},\quad c_T=\left(\frac{\mu}{\rho}\right)^{1/2} \tag{2.11}$$

ve yavaşlıkları (slowness),

$$k_L=\frac{\omega}{c_L},\quad k_T=\frac{\omega}{c_T} \tag{2.12}$$

olup buna göre (2.10),

$$\begin{aligned}&\overline{\varphi}_{,ii}+k_L^2\,\overline{\varphi}=0\\&\overline{\psi}_{3,ii}+k_T^2\,\overline{\psi}_3=0\end{aligned} \tag{2.13}$$

şeklinde yazılabilir. (2.13)'e ve (2.6)-(2.7) ile verilen sınır koşullarına,

$$f_F(k,x_2) = \int_{-\infty}^{\infty} f(x_1,x_2)\, e^{ikx_1}\, dx_1 \tag{2.14}$$

üstel Fourier dönüşümü uygulanırsa,

$$\begin{aligned} &\overline{\varphi}_{F,22} - (k^2 - k_L^2)\,\overline{\varphi}_F = 0 \\ &\overline{\psi}_{3F,22} - (k^2 - k_T^2)\,\overline{\psi}_{3F} = 0 \end{aligned} \tag{2.15}$$

$$\overline{\sigma}_{12F}\big|_{x_2=0} = 0, \quad \overline{\sigma}_{22F}\big|_{x_2=0} = -P_0 \tag{2.16}$$

$$\overline{u}_{iF}, \overline{\sigma}_{ijF}\big|_{x_2\to\infty} = 0 \quad i,j = 1,2 \tag{2.17}$$

bulunur. (2.15) 'in çözümleri,

$$\overline{\varphi}_F = \alpha(k)\, e^{rx_2}, \overline{\psi}_{3F} = \beta(k)\, e^{tx_2} \tag{2.18}$$

yapısında aranır ve,

$$\upsilon = \left(k^2 - k_L^2\right)^{1/2}, \upsilon' = \left(k^2 - k_T^2\right)^{1/2} \tag{2.19}$$

kısaltmaları kullanılırsa,

$$r_{1,2} = \mp\upsilon\, , t_{1,2} = \mp\upsilon' \tag{2.20}$$

bulunur. (2.17) ve Şekil 2.1'e göre $r<0$ ve $t<0$ olmalıdır. Bu durumda,

$$\begin{aligned} &\overline{\varphi}_F = \alpha(k)\, e^{-\upsilon x_2} \\ &\overline{\psi}_{3F} = \beta(k)\, e^{-\upsilon' x_2} \end{aligned} \tag{2.21}$$

yazılır. Lineer elastik, homojen ve izotrop cisime ait bünye denklemleri,

$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij}\ , \ \theta = u_{k,k} \tag{2.22}$$

ve lineer kinematik bağıntılar,

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2.23}$$

olup (2.9) uyarınca gerilme tansörünün bağımsız bileşenleri yerdeğiştirme potansiyellerine,

$$
\begin{aligned}
\overline{\sigma}_{11F} &= (\lambda+2\mu)(-k^2\,\overline{\varphi}_F - i\,k\,\overline{\psi}_{3F,2}) + \lambda(\overline{\varphi}_{F,22} + i\,k\,\overline{\psi}_{3F,2}) \\
\overline{\sigma}_{22F} &= (\lambda+2\mu)(\overline{\varphi}_{F,22} + i\,k\,\overline{\psi}_{3F,2}) + \lambda(-k^2\,\overline{\varphi}_F - i\,k\,\overline{\psi}_{3F,2}) \\
\overline{\sigma}_{12F} &= \mu(-2\,i\,k\,\overline{\varphi}_{F,2} + \overline{\psi}_{3F,22} + k^2\,\overline{\psi}_{3F})
\end{aligned}
\tag{2.24}
$$

eşitlikleri ile bağlanabilir ve ayrıca yine (2.9)'a göre,

$$
\begin{aligned}
\overline{u}_{1F} &= -i\,k\,\overline{\varphi}_F + \overline{\psi}_{3F,2} \\
\overline{u}_{2F} &= \overline{\varphi}_{F,2} + i\,k\,\overline{\psi}_{3F}
\end{aligned}
\tag{2.25}
$$

eşitlikleri yazılabilir ve (2.21) yardımı ile,

$$
\begin{aligned}
\overline{u}_{1F} &= -i\,k\,\alpha(k)\,e^{-\upsilon x_2} - \upsilon'\,\beta(k)\,e^{-\upsilon' x_2} \\
\overline{u}_{2F} &= -\upsilon\,\alpha(k)\,e^{-\upsilon x_2} + i\,k\,\beta(k)\,e^{-\upsilon' x_2}
\end{aligned}
\tag{2.26}
$$

ve,

$$
\begin{aligned}
\overline{\sigma}_{11F} &= \left[-(\lambda+2\mu)\,k^2 + \lambda\upsilon^2\right]\alpha(k)\,e^{-\upsilon x_2} + 2\,i\,\mu\,k\,\upsilon'\,\beta(k)\,e^{-\upsilon' x_2} \\
\overline{\sigma}_{22F} &= \left[(\lambda+2\mu)\,\upsilon^2 - \lambda\,k^2\right]\alpha(k)\,e^{-\upsilon x_2} - 2\,i\,\mu\,k\,\upsilon'\,\beta(k)\,e^{-\upsilon' x_2} \\
\overline{\sigma}_{12F} &= 2\,i\,\mu\,k\,\upsilon\,\alpha(k)\,e^{-\upsilon x_2} + \mu\,(2k^2 - k_T^2)\,\beta(k)\,e^{-\upsilon' x_2}
\end{aligned}
\tag{2.27}
$$

bulunur. $\alpha(k)$ ve $\beta(k)$'nın belirlenmesi için (2.16) ile verilen sınır şartları kullanılırsa,

$$
\begin{bmatrix} 2k^2 - k_T^2 & -2\,i\,k\,\upsilon' \\ 2\,i\,k\,\upsilon & 2k^2 - k_T^2 \end{bmatrix}
\begin{Bmatrix} \alpha(k) \\ \beta(k) \end{Bmatrix}
=
\begin{Bmatrix} -P_0/\mu \\ 0 \end{Bmatrix}
\tag{2.28}
$$

lineer cebirsel denklem sistemi elde edilir ve,

$$
\Delta(k) = (2k^2 - k_T^2)^2 - 4k^2\upsilon\,\upsilon'
\tag{2.29}
$$

denilirse çözüm,

$$
\overline{\varphi}_F = -\frac{P_0}{\mu}\frac{2k^2 - k_T^2}{\Delta(k)}e^{-\upsilon x_2}
\tag{2.30}
$$

$$
\overline{\psi}_{3F} = i\,\frac{P_0}{\mu}\frac{2k\,\upsilon}{\Delta(k)}e^{-\upsilon' x_2}
\tag{2.31}
$$

olarak elde edilir ve (2.26) ve (2.27) yardımı ile,

$$\bar{u}_{1F} = i\,\frac{P_0}{\mu}\,\frac{(2k^2 - k_T^2)\,e^{-\upsilon x_2} - 2\upsilon\,\upsilon'\,e^{-\upsilon' x_2}}{\Delta(k)}\,k \tag{2.32}$$

$$\bar{u}_{2F} = \frac{P_0}{\mu}\,\frac{(2k^2 - k_T^2)\,e^{-\upsilon x_2} - 2k^2\,e^{-\upsilon' x_2}}{\Delta(k)}\,\upsilon \tag{2.33}$$

ve,

$$\bar{\sigma}_{11F} = \frac{P_0}{\mu}\,\frac{\left[(\lambda+2\mu)k^2 - \lambda\,\upsilon^2\right]\left(2k^2 - k_T^2\right)\,e^{-\upsilon x_2} - 4\mu\,k^2\,\upsilon\,\upsilon'\,e^{-\upsilon' x_2}}{\Delta(k)} \tag{2.34}$$

$$\bar{\sigma}_{22F} = \frac{P_0}{\mu}\,\frac{\left[-(\lambda+2\mu)\upsilon^2 + \lambda\,k^2\right]\left(2k^2 - k_T^2\right)\,e^{-\upsilon x_2} + 4\mu\,k^2\,\upsilon\,\upsilon'\,e^{-\upsilon' x_2}}{\Delta(k)} \tag{2.35}$$

$$\bar{\sigma}_{12F} = i\,P_0\,\frac{2k\left(2k^2 - k_T^2\right)\,\upsilon\,\left(-e^{-\upsilon x_2} + e^{-\upsilon' x_2}\right)}{\Delta(k)} \tag{2.36}$$

bulunur. (2.14) 'e ait ters dönüşüm,

$$f(x_1, x_2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f_F(k, x_2)\,e^{-ikx_1}\,dk \tag{2.37}$$

olup buna göre (2.32)–(2.36) 'dan,

$$\bar{u}_1 = i\,\frac{P_0}{2\pi\mu}\int_{-\infty}^{\infty}\frac{(2k^2 - k_T^2)\,e^{-\upsilon x_2} - 2\upsilon\,\upsilon'\,e^{-\upsilon' x_2}}{\Delta(k)}\,k\,e^{-ikx_1}\,dk \tag{2.38}$$

$$\bar{u}_2 = \frac{P_0}{2\pi\mu}\int_{-\infty}^{\infty}\frac{(2k^2 - k_T^2)\,e^{-\upsilon x_2} - 2k^2\,e^{-\upsilon' x_2}}{\Delta(k)}\,\upsilon\,e^{-ikx_1}\,dk \tag{2.39}$$

ve,

$$\bar{\sigma}_{11} = \frac{P_0}{2\pi\mu}\int_{-\infty}^{\infty}\frac{\left[(\lambda+2\mu)k^2 - \lambda\,\upsilon^2\right]\left(2k^2 - k_T^2\right)\,e^{-\upsilon x_2} - 4\mu\,k^2\,\upsilon\,\upsilon'\,e^{-\upsilon' x_2}}{\Delta(k)}\,e^{-ikx_1}\,dk \tag{2.40}$$

$$\bar{\sigma}_{22} = \frac{P_0}{2\pi\mu}\int_{-\infty}^{\infty}\frac{\left[-(\lambda+2\mu)\upsilon^2 + \lambda\,k^2\right]\left(2k^2 - k_T^2\right)\,e^{-\upsilon x_2} + 4\mu\,k^2\,\upsilon\,\upsilon'\,e^{-\upsilon' x_2}}{\Delta(k)}\,e^{-ikx_1}\,dk \tag{2.41}$$

$$\bar{\sigma}_{12} = i\,\frac{P_0}{2\pi}\int_{-\infty}^{\infty}\frac{2k\left(2k^2 - k_T^2\right)\,\upsilon\,\left(-e^{-\upsilon x_2} + e^{-\upsilon' x_2}\right)}{\Delta(k)}\,e^{-ikx_1}\,dk \tag{2.42}$$

bulunur. Bu integraller sınırlarının sonsuzda olması ve tekil noktalarının bulunması nedeniyle her iki bakımdan da has (proper) değildir ve analitik olarak hesaplanamazlar. Tekillik,

$$k_R = \frac{\omega}{c_R} \tag{2.43}$$

olmak üzere $\pm k_R$ noktalarındadır. c_R elastik bir cismin sınırında oluşan Rayleigh dalgasının yayılım hızı olup (2.29) ile verilen ifadenin köküdür ve Poisson oranına bağlı olmakla birlikte yaklaşık olarak 0,92 c_T 'ye eşittir. Bu tekillik nedeni ile integrallerin ancak Cauchy anlamında değeri vardır. Esasen (2.28) ile verilen denklem sistemi homogen olarak düşünülürse, homogen sınır koşullarının varlığı durumunda oluşan özdeğer probleminin çözümüne karşı gelir. İntegral sınırlarının sonsuzda olması, yerdeğiştirme ve gerilme fonksiyonlarının Fourier dönüşümünü haiz olma şartlarını sağlamaları nedeniyle sayısal işlemlerde sorun yaratmaz.

2.2 Yüzey Yerdeğiştirmelerinin Dağılımının İki Farklı Yoldan Elde Edilmesi.

(2.38) ve (2.39) eşitliklerinde $x_2=0$ konulursa yüzeydeki yerdeğiştirmeler,

$$\bar{u}_{01} = i\frac{P_0}{2\pi\mu}\int_{-\infty}^{\infty}\frac{2k^2 - k_T^2 - 2\upsilon\upsilon'}{\Delta(k)}\,k\,e^{-ikx_1}\,dk \tag{2.44}$$

$$\bar{u}_{02} = -\frac{P_0}{2\pi\mu}\int_{-\infty}^{\infty}k_T^2\,\frac{\upsilon}{\Delta(k)}\,e^{-ikx_1}\,dk \tag{2.45}$$

olarak bulunur. Burada,

$$\Phi(k) = \frac{2k^2 - k_T^2 - 2\upsilon\upsilon'}{\Delta(k)}\,k\,e^{-ikx_1} \tag{2.46}$$

$$\Psi(k) = k_T^2\,\frac{\upsilon}{\Delta(k)}\,e^{-ikx_1} \tag{2.47}$$

kısaltmaları yapılırsa, (2.44) ve (2.45) eşitlikleri,

$$\bar{u}_{01} = i\frac{P_0}{2\pi\mu}\int_{-\infty}^{\infty}\Phi(k)\,dk \tag{2.48}$$

$$\bar{u}_{02} = -\frac{P_0}{2\pi\mu}\int_{-\infty}^{\infty}\Psi(k)\,dk \tag{2.49}$$

olarak yazılabilir. Bu integraller analitik olarak hesaplanamadıklarından sayısal olarak hesaplanıp, karmaşık düzlemde hesaplanacak değerleri ile karşılaştırılarak kontrol edilecektir.

2.2.1 Ters Fourier dönüşümünün direkt sayısal integrasyon ile yapılması.

(2.46) ve (2.47) daki υ ve υ' terimleri nedeni ile integrallerin üç aralıkta hesaplanacaktır.

i. $0<k<k_L$ durumu.

Bu durumda,

$$\upsilon = (k^2 - k_L^2)^{1/2} = i\,(k_L^2 - k^2)^{1/2} = i\,\hat{\upsilon} \tag{2.50}$$

$$\upsilon' = (k^2 - k_T^2)^{1/2} = i\,(k_T^2 - k^2)^{1/2} = i\,\hat{\upsilon}' \tag{2.51}$$

eşitlikleri kullanılır. Burada

$$\hat{\upsilon} = (k_L^2 - k^2)^{1/2} \tag{2.52}$$

$$\hat{\upsilon}' = (k_T^2 - k^2)^{1/2} \tag{2.53}$$

olmaktadır. Buna göre integrandlar bu aralıkta,

$${}^{1}\Phi(k) = \frac{2k^2 - k_T^2 + 2\hat{\upsilon}\,\hat{\upsilon}'}{(2k^2 - k_T^2)^2 + 4k^2\,\hat{\upsilon}\,\hat{\upsilon}'}\,k\,e^{-ikx_1} \tag{2.54}$$

$${}^{1}\Psi(k) = k_T^2\,\frac{i\,\hat{\upsilon}}{(2k^2 - k_T^2)^2 + 4k^2\,\hat{\upsilon}\,\hat{\upsilon}'}\,e^{-ikx_1} \tag{2.55}$$

yapısını kazanır ve (2.48) ve (2.49) yardımı ile,

$${}^{1}\overline{u}_{01} = \frac{P_0}{2\pi\mu}\int_{-k_L}^{k_L} \frac{2k^2 - k_T^2 + 2\hat{\upsilon}\,\hat{\upsilon}'}{(2k^2 - k_T^2)^2 + 4k^2\,\hat{\upsilon}\,\hat{\upsilon}'}\,k\sin(kx_1)\,dk \tag{2.56}$$

$${}^{1}\overline{u}_{02} = -\frac{P_0}{2\pi\mu}\int_{-k_L}^{k_L} k_T^2\,\frac{\hat{\upsilon}}{(2k^2 - k_T^2)^2 + 4k^2\,\hat{\upsilon}\,\hat{\upsilon}'}\sin(kx_1)\,dk \tag{2.57}$$

bulunur. Tek ve çift fonksiyonların orijine göre simetrik aralıklardaki belirli integrallerine ait özelliklerden,

$$ {}^{1}\overline{u}_{01}=\frac{P_0}{\pi\mu}\int_0^{k_L}\frac{2k^2-k_T^2+2\hat{\upsilon}\,\hat{\upsilon}'}{(2k^2-k_T^2)^2+4k^2\,\hat{\upsilon}\,\hat{\upsilon}'}k\sin(kx_1)\,dk \tag{2.58} $$

$$ {}^{1}\overline{u}_{02}=0 \tag{2.59} $$

bulunur.

ii. $k_L<k<k_T$ durumu.

Bu durumda,

$$ \upsilon=(k^2-k_L^2)^{1/2} \tag{2.60} $$

$$ \upsilon'=(k_T^2-k^2)^{1/2}=i\,\hat{\upsilon}' \tag{2.61} $$

eşitlikleri kullanılır. Buna göre integrandlar bu aralıkta,

$$ {}^{2}\Phi(k)=\frac{2k^2-k_T^2-2\upsilon\,i\,\hat{\upsilon}'}{(2k^2-k_T^2)^2-4k^2\,\upsilon\,i\,\hat{\upsilon}'}k\,e^{-ikx_1} \tag{2.62} $$

$$ {}^{2}\Psi(k)=k_T^2\frac{\upsilon}{(2k^2-k_T^2)^2-4k^2\,\upsilon\,i\,\hat{\upsilon}'}e^{-ikx_1} \tag{2.63} $$

yapısındadır. Bu eşitliklerin pay ve paydaları, paydanın karmaşık eşleniği ile çarpılırsa,

$$ {}^{2}\Phi(k)=\frac{(2k^2-k_T^2)^2-2i(2k^2-k_T^2)\upsilon\hat{\upsilon}'+4ik^2+8k^2\upsilon^2\hat{\upsilon}'^2}{(2k^2-k_T^2)^4+16k^4\upsilon^2\hat{\upsilon}'^2}(2k^2-k_T^2)\,k\,e^{-ikx_1} \tag{2.64} $$

$$ {}^{2}\Psi(k)=k_T^2\frac{(2k^2-k_T^2)^2\,\upsilon+4ik^2\,\upsilon^2\,\hat{\upsilon}'}{(2k^2-k_T^2)^4+16k^4\,\upsilon^2\,\hat{\upsilon}'^2}e^{-ikx_1} \tag{2.65} $$

bulunur ve (2.48) ve (2.49) yardımı ile,

$$ \begin{aligned} {}^{2}\overline{u}_{01}=\frac{P_0}{2\pi\mu}\left(\int_{-k_T}^{-k_L}+\int_{k_L}^{k_T}\right)&\left\{\frac{\left[(2k^2-k_T^2)^2+8k^2\,\upsilon\hat{\upsilon}'\right]\sin(kx_1)}{(2k^2-k_T^2)^4+16k^4\,\upsilon^2\,\hat{\upsilon}'^2}\right.\\ &\left.+\frac{\left[2(2k^2-k_T^2)\upsilon\hat{\upsilon}'-4k^2\,\upsilon\hat{\upsilon}'\right]\cos(kx_1)}{(2k^2-k_T^2)^4+16k^4\,\upsilon^2\,\hat{\upsilon}'^2}\right\}(2k^2-k_T^2)\,k\,dk \end{aligned} \tag{2.66} $$

$$ {}^{2}\overline{u}_{02}=-\frac{P_0}{2\pi\mu}\left\{\int_{-k_T}^{-k_L}+\int_{k_L}^{k_T}\right\}k_T^2\frac{(2k^2-k_T^2)^2\upsilon\cos(kx_1)+4k^2\,\upsilon^2\,\hat{\upsilon}'\sin(kx_1)}{(2k^2-k_T^2)^4+16k^4\,\upsilon^2\,\hat{\upsilon}'^2}\,dk \tag{2.67} $$

ve sonuç olarak,

$$^2\bar{u}_{01}=\frac{P_0}{\pi\mu}\int_{k_L}^{k_T}\frac{(2k^2-k_T^2)^3+8k^2\upsilon^2\hat{\upsilon}'^2}{(2k^2-k_T^2)^4+16k^4\upsilon^2\hat{\upsilon}'^2}k\sin(kx_1)\,dk \tag{2.68}$$

$$^2\bar{u}_{02}=-\frac{P_0}{\pi\mu}\int_{k_L}^{k_T}k_T^2\frac{(2k^2-k_T^2)^2\,\upsilon}{(2k^2-k_T^2)^4+16k^4\upsilon^2\hat{\upsilon}'^2}\cos(kx_1)\,dk \tag{2.69}$$

bulunur.

iii. $k_T<k$ durumu.

Bu durumda ilk verilen bağıntılarda değişiklik yapma gereği yoktur.

$$^3\Phi(k)=\frac{2k^2-k_T^2-2\upsilon\upsilon'}{(2k^2-k_T^2)^2-4k^2\upsilon\upsilon'}k\,e^{-ikx_1} \tag{2.70}$$

$$^3\Psi(k)=k_T^2\frac{\upsilon}{(2k^2-k_T^2)^2-4k^2\upsilon\upsilon'}e^{-ikx_1} \tag{2.71}$$

olup buradan (2.48)-(2.49) yardımı ile,

$$^3\bar{u}_{01}=\frac{P_0}{\pi\mu}\int_{k_T}^{\infty}\frac{2k^2-k_T^2-2\upsilon\upsilon'}{(2k^2-k_T^2)^2-4k^2\upsilon\upsilon'}k\sin(kx_1)\,dk \tag{2.72}$$

$$^3\bar{u}_{02}=-\frac{P_0}{\pi\mu}\int_{k_T}^{\infty}k_T^2\frac{\upsilon}{(2k^2-k_T^2)^2-4k^2\upsilon\upsilon'}\cos(kx_1)\,dk \tag{2.73}$$

bulunur. Sonuç olarak yüzey yerdeğiştirmeleri,

$$\bar{u}_{0i}={}^1\bar{u}_{0i}+{}^2\bar{u}_{0i}+{}^2\bar{u}_{0i} \tag{2.74}$$

ile hesaplanır.

2.2.2 Ters Fourier dönüşümünün karmaşık fonksiyonlar yardımı ile yapılması.

(2.46) ve (2.47) ile verilen ifadeler $\zeta=k+i\tau$ karmaşık değişkeni cinsinden,

$$\Phi(\zeta) = \frac{2\zeta^2 - k_T^2 - 2\upsilon\,\upsilon'}{\Delta(\zeta)}\zeta\, e^{-i\zeta x_1} \tag{2.75}$$

$$\Psi(\zeta) = k_T^2 \frac{\upsilon}{\Delta(\zeta)} e^{-i\zeta x_1} \tag{2.76}$$

şeklinde yazılırsa (2.48) ve (2.49) ile verilen integraller karmaşık düzlemde rezidü yöntemi ile hesaplanabilir. Bu durumda,

$$\upsilon(\zeta) = \left(\zeta^2 - k_L^2\right)^{1/2} \tag{2.77}$$

$$\upsilon'(\zeta) = \left(\zeta^2 - k_T^2\right)^{1/2} \tag{2.78}$$

$$\Delta(\zeta) = (2\zeta^2 - k_T^2)^2 - 4\zeta^2\,\upsilon\,\upsilon' \tag{2.79}$$

olur ve buna göre $\Phi(\zeta)$ ve $\Psi(\zeta)$ fonksiyonlarının $(\pm k_R;0)$ da kutuplarının, $(\pm k_L;0)$, $(\pm k_T;0)$ da ise dallanma noktalarının var olduğu görülür. Bu nedenle Cauchy teoreminin uygulanabilmesi için fonksiyon uygun kesimlerle holomorf hale getirilmelidir. Frekansın $\omega=p-i\sigma$ şeklinde karmaşık kabul edilmesi kutup ve dallanma noktalarını gerçel eksen üzerinden alarak eğimi Im ω/Re ω olan bir doğru üzerine taşır. (2.77) ve (2.78) uyarınca dört Riemann yaprağı olup uygun yaprak, φ ve ψ_3 potansiyellerinin $x_2\rightarrow\infty$ durumunda sonsuz değer almamaları için $\mathrm{Re}\upsilon\geq 0$ ve $\mathrm{Re}\upsilon'\geq 0$ şartlarına göre belirlenir. Buna göre kesim çizgileri $\mathrm{Re}\,\upsilon = 0, \mathrm{Re}\,\upsilon' = 0$ olmalıdır. $\upsilon^2 = \zeta^2 - k_L^2$ olduğuna göre $\mathrm{Re}\,\upsilon = 0$ olması için,

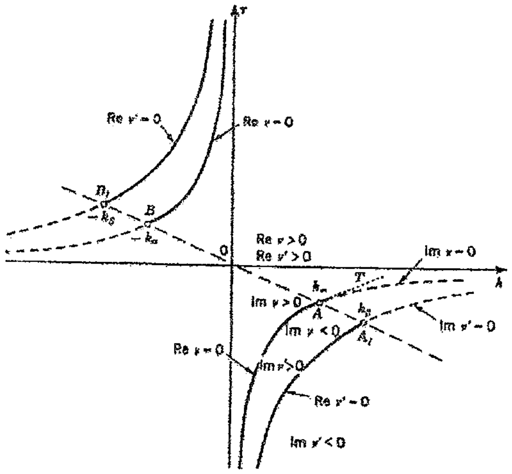
$$\upsilon^2 = k^2 - \tau^2 + 2\,i\,k\,\tau - \frac{p^2 - \sigma^2 - 2\,i\,p\,\sigma}{c_L^2} < 0,\ \upsilon^2 \in R \tag{2.80}$$

olmalıdır. Buna göre,

$$k\,\tau = -\frac{p\,\sigma}{c_L^2},\ k^2 - \tau^2 < \frac{p^2 - \sigma^2}{c_L^2} \tag{2.81}$$

şartları sağlanmalıdır. Bunlardan ilki kesim çizgilerinin hiperboller olması gerektiğini, ikincisi de hiperbollerin hangi parçalarının kesim olarak kullanılabileceğini gösterir. Şekil 2.2 uyarınca hiperbollerin dolu çizilen parçaları üzerinde $\mathrm{Re}\,\upsilon = 0$ ve $\mathrm{Re}\,\upsilon' = 0$, kesikli çizilen parçaları üzerinde ise $\mathrm{Im}\,\upsilon = 0$ veya $\mathrm{Im}\,\upsilon' = 0$ olmaktadır. Çünkü bu kısımlarda,

$$k\,\tau = -\frac{p\,\sigma}{c_L^2},\ k^2 - \tau^2 > \frac{p^2 - \sigma^2}{c_L^2} \tag{2.82}$$



Şekil 2.2 Re ω > 0 için dallanma noktaları ve kesim çizgileri.

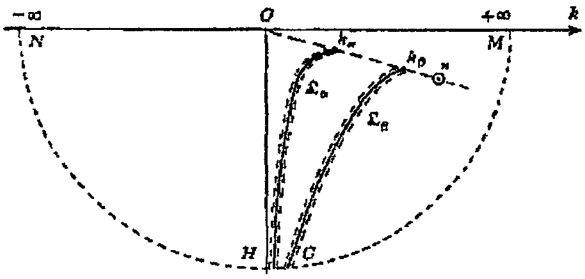
olduğundan υ^2 veya υ'^2 gerçel ve pozitiftir. Hiperbolün örneğin Ak parçası üzerinde $\mathrm{Im}\,\upsilon = 0$ olduğu için sağ yarım düzlemde $\mathrm{Im}\,\upsilon$ ancak Ak üzerinden geçerken işaret değiştirir (Şekil2.2)

$$\zeta = k_L + \delta\, e^{i\theta}\, , - \pi < \theta < \pi \tag{2.83}$$

dönüşümü yapılırsa,

$$\upsilon(\zeta) = +\left(\zeta^2 - k_L^2\right)^{1/2} \cong \left(2\, k_L\, \delta\right)^{1/2} e^{i\frac{\theta}{2}} = \left(2\, k_L\, \delta\right)^{1/2} \left(\cos\theta/2 + i \sin\theta/2\right) \tag{2.84}$$

bulunur, ve $\mathrm{Im}\,\upsilon$'nün ilgili kesimin sol tarafında pozitif sağ tarafında ise negatif olduğu görülür. Aynı yaklaşım $\mathrm{Im}\,\upsilon'$ için de yapılabilir. (θ açısı k_L noktasındaki teğetinden ölçülecektir. Değişim aralığının $-\pi<\theta<\pi$ olarak alınması hiperbolün dolu çizilen kısmını kesmemesi içindir) $\mathrm{Re}\,\upsilon$ ve $\mathrm{Re}\,\upsilon'$ ise bütün düzlemde kesim çizgilerini kesmeyen uygun yörüngelerin hiç birinde işaret değiştirmez ve pozitiftir. Burada rezidü teoremini uygulamak için önce ω karmaşık alınıp Şekil 2.3'teki gibi bir çevre seçilecek, sonra ω nın gerçel olması limit durumu gözönüne alınarak aranılan sonuçlar elde edilecektir.



Şekil 2.3 İntegrasyon çevresi.

Örnek olarak,

$$\oint \Phi(\zeta)\,d\zeta = \left(\int_M^N + \int_N^H + \int_{L_\alpha} + \int_{L_\beta} + \int_G^M \right) \Phi(\zeta)\,d\zeta = 2\pi i \sum \mathrm{Res} \tag{2.85}$$

integrali gözönüne alındığında Şekil 2.3'e göre öncelikle NH ve GM sonsuz yayları üzerindeki integraller, integrandda yer alan exp(-iζx_1) terimi nedeniyle sıfıra eşit olur çünkü,

$$\tau \to -\infty \;\therefore\; \exp(-i\zeta x_1) = \exp[-i\,(k+i\tau)\,x_1] = 0 \tag{2.86}$$

olur. Bu durumda aranılan integraller,

$$\int_{-\infty}^{\infty} \Phi(k)\,dk = -2\pi i\,\mathrm{Res}\big|_{\zeta=k_R} + \left(\int_{L_\alpha} + \int_{L_\beta} \right) \Phi(\zeta)\,d\zeta \tag{2.87}$$

$$\int_{-\infty}^{\infty} \Psi(k)\,dk = -2\pi i\,\mathrm{Res}\big|_{\zeta=k_R} + \left(\int_{L_\alpha} + \int_{L_\beta} \right) \Psi(\zeta)\,d\zeta \tag{2.88}$$

eşitlikleri ile elde edilebilir. Buna göre,

$$\bar{u}_{01} = i\frac{P_0}{2\pi\mu} \int_{-\infty}^{\infty} \Phi(k)\,dk = i\frac{P_0}{2\pi\mu} \lim_{\sigma\to 0} \left[-2\pi i\,\mathrm{Res}\big|_{\zeta=k_R} + \left(\int_{L_\alpha} + \int_{L_\beta} \right) \Phi(\zeta)\,d\zeta \right] \tag{2.89}$$

$$\bar{u}_{02} = -\frac{P_0}{2\pi\mu} k_T^2 \int_{-\infty}^{\infty} \psi(k)\,dk = -\frac{P_0}{2\pi\mu} k_T^2 \lim_{\sigma\to 0} \left[-2\pi i\,\mathrm{Res}\big|_{\zeta=k_R} + \left(\int_{L_\alpha} + \int_{L_\beta} \right) \Psi(\zeta)\,d\zeta \right] \tag{2.90}$$

ve,

$$\bar{u}_{01} = i\frac{P_0}{2\pi\mu} \left[\lim_{\sigma\to 0} \left(-2\pi i\,\mathrm{Res}\big|_{\zeta=k_R} \right) + I_{\alpha\varphi} + I_{\beta\varphi} \right] \tag{2.91}$$

$$\bar{u}_{02} = -\frac{P_0}{2\pi\mu} k_T^2 \left[\lim_{\sigma\to 0} \left(-2\pi i\,\mathrm{Res}\big|_{\zeta=k_R} \right) + I_{\alpha\psi} + I_{\beta\psi} \right] \tag{2.92}$$

yazılabilir. Kapalı bölgedeki tek kutup olan $\zeta = k_R$; $\Delta(\zeta)$ 'nın birinci basamaktan sıfır noktası, ve integrandlar $M(\zeta)/N(\zeta)$ yapısında olduğu için rezidüler $M(\zeta)/N'(\zeta)$ yapısında olup,

$$\lim_{\sigma\to 0} \mathrm{Res}\,\Phi(\zeta)\big|_{\zeta=k_R} = \frac{k_R \left[2k_R^2 - k_T^2 - 2\left(k_R^2 - k_L^2\right)^{1/2} \left(k_R^2 - k_T^2\right)^{1/2} \right]}{\Delta'(k_R)} e^{-ik_R x_1} \tag{2.93}$$

$$\lim_{\sigma\to 0} \operatorname{Res} \Psi(\zeta)\big|_{\zeta=k_R} = \frac{\left(k_R^2 - k_L^2\right)^{1/2}}{\Delta'(k_R)} e^{-ik_R x_1} \tag{2.94}$$

şeklindedir. Burada,

$$\Delta'(k_R) = 8k_R(2k_R^2 - k_T^2) - \left[8k_R(k_R^2 - k_L^2)^{1/2} + \frac{4k_R^3}{(k_R^2 - k_L^2)^{1/2}}\right](k_R^2 - k_T^2)^{1/2} - 4k_R^3 \frac{(k_R^2 - k_L^2)^{1/2}}{(k_R^2 - k_T^2)^{1/2}} \tag{2.95}$$

$$H = -\frac{k_r\left[2k_r^2 - k_T^2 - 2(k_R^2 - k_L^2)^{1/2}(k_R^2 - k_T^2)^{1/2}\right]}{\Delta'(k_R)} \tag{2.96}$$

$$K = -k_T^2 \frac{(k_R^2 - k_L^2)^{1/2}}{\Delta'(k_R)} \tag{2.96}$$

tanımlamaları yapılmıştır. Buna göre,

$$\lim_{\sigma\to 0} \operatorname{Res}\left[\Phi(\zeta)\right]\big|_{\zeta=k_R} = -H e^{-ik_R x_1} \tag{2.97}$$

$$\lim_{\sigma\to 0} \operatorname{Res}\left[\Psi(\zeta)\right]\big|_{\zeta=k_R} = -\frac{K}{k_T^2} e^{-ik_R x_1} \tag{2.98}$$

yazılabilir. Buna göre (2.91) ve (2.92)'den yararlanılarak,

$$\bar{u}_{01} = -\frac{P_0}{\mu} H e^{-ik_R x_1} + i\frac{P_0}{2\pi\mu}(I_{\alpha\varphi} + I_{\beta\varphi}) \tag{2.99}$$

$$\bar{u}_{02} = -i\frac{P_0}{\mu} K e^{-ik_R x_1} - \frac{P_0}{2\pi\mu} k_T^2 (I_{\alpha\psi} + I_{\beta\psi}) \tag{2.100}$$

yazılabilir. Şimdi kesim çizgileri üzerinde tanımlanan integraller,

$\upsilon_I = \operatorname{Im}\upsilon > 0$, Birinci çeyrek düzlemde

$\upsilon_I' = \operatorname{Im}\upsilon' > 0$, Birinci çeyrek düzlemde

$\upsilon_{II}' = \operatorname{Im}\upsilon' < 0$, Dördüncü çeyrek düzlemde

olmak üzere Şekil 2.4 yardımı ile,

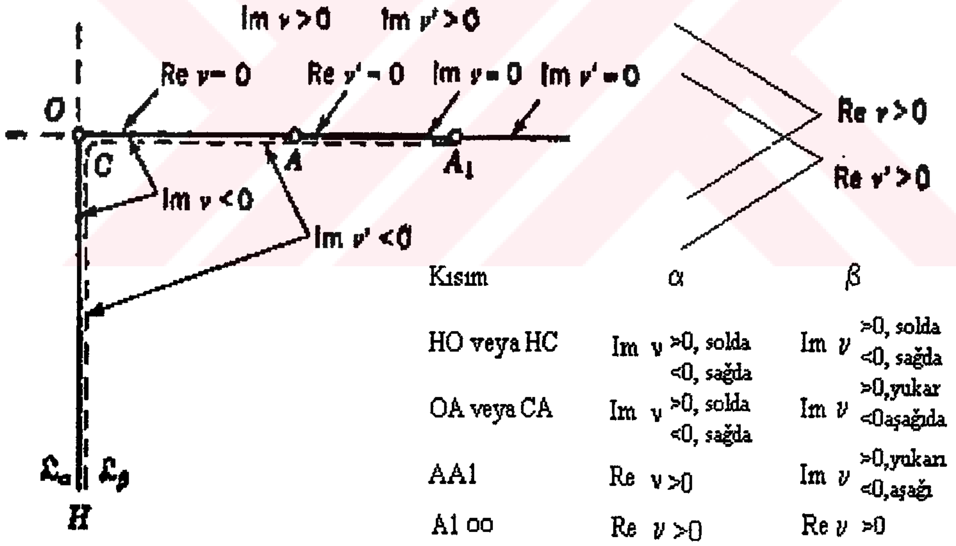
$$\begin{aligned} I_{\alpha\varphi} = &\int_{-i\infty}^{0} \left[\frac{2\zeta^2 - k_T^2 - 2\upsilon_I \upsilon_I'}{\left(2\zeta^2 - k_T^2\right)^2 - 4\zeta^2 \upsilon_I \upsilon_I'} - \frac{2\zeta^2 - k_T^2 + 2\upsilon_I \upsilon_I'}{\left(2\zeta^2 - k_T^2\right)^2 + 4\zeta^2 \upsilon_I \upsilon_I'}\right] \zeta e^{-i\zeta x_1} d\zeta \\ &+ \int_{0}^{k_L} \left[\frac{2k^2 - k_T^2 - 2\upsilon_I \upsilon_I'}{\left(2k^2 - k_T^2\right)^2 - 4k^2 \upsilon_I \upsilon_I'} - \frac{2k^2 - k_T^2 + 2\upsilon_I \upsilon_I'}{\left(2k^2 - k_T^2\right)^2 + 4\zeta^2 \upsilon_I \upsilon_I'}\right] k e^{-ikx_1} d\zeta \end{aligned} \tag{2.101}$$

$$
\begin{aligned}
I_{\beta\varphi} = &\int_{-i\infty}^{0}\left[\frac{2\zeta^2-k_T^2+2\upsilon_I\,\upsilon_I'}{\left(2\zeta^2-k_T^2\right)^2+4\zeta^2\,\upsilon_I\,\upsilon_I'}-\frac{2\zeta^2-k_T^2-2\upsilon_I\,\upsilon_I'}{\left(2\zeta^2-k_T^2\right)^2-4\zeta^2\,\upsilon_I\,\upsilon_I'}\right]\zeta\, e^{-i\zeta x_1}\,d\zeta \\
&+\int_{0}^{k_L}\left[\frac{2k^2-k_T^2+2\upsilon_I\,\upsilon_I'}{\left(2k^2-k_T^2\right)^2+4k^2\,\upsilon_I\,\upsilon_I'}-\frac{2k^2-k_T^2-2\upsilon_I\,\upsilon_I'}{\left(2k^2-k_T^2\right)^2-4k^2\,\upsilon_I\,\upsilon_I'}\right]k\, e^{-ikx_1}\,d\zeta \\
&+\int_{k_L}^{k_T}\left[\frac{2k^2-k_T^2-2\upsilon\,\upsilon_I'}{\left(2k^2-k_T^2\right)^2-4k^2\,\upsilon\,\upsilon_I'}-\frac{2k^2-k_T^2-2\upsilon_I\,\upsilon_{II}'}{\left(2k^2-k_T^2\right)^2-4k^2\,\upsilon\,\upsilon_{II}'}\right]k\, e^{-ikx_1}\,d\zeta
\end{aligned}
\tag{2.102}
$$

şeklinde yazılabilir. OA_1 kenarında $\upsilon_{II}'=-\upsilon_I'$ olduğu için sonuç olarak,

$$
\begin{aligned}
I_{\alpha\varphi}+I_{\beta\varphi} &= \int_{k_L}^{k_T}\left[\frac{2k^2-k_T^2-2\upsilon\,\upsilon_I'}{\left(2k^2-k_T^2\right)^2-4k^2\,\upsilon\,\upsilon_I'}-\frac{2k^2-k_T^2+2\upsilon_I\,\upsilon_I'}{\left(2k^2-k_T^2\right)^2+4\zeta^2\,\upsilon\,\upsilon_I'}\right]k\, e^{-ikx_1}\,dk \\
&= 4k_T^2\int_{0}^{k_L}\frac{(2k^2-k_T^2)\,\upsilon\,\upsilon_I'}{\left(2k^2-k_T^2\right)^4-16k^4\,\upsilon^2\,\upsilon_I'^2}\,k\, e^{-ikx_1}\,dk
\end{aligned}
\tag{2.103}
$$

bulunur.



Kısım	α	β
HO veya HC	Im ν >0, solda <0, sağda	Im ν >0, solda <0, sağda
OA veya CA	Im ν >0, solda <0, sağda	Im ν >0,yukar <0aşağıda
AA1	Re ν >0	Im ν >0,yukarı <0,aşağı
A1 ∞	Re ν >0	Re ν >0

Şekil 2.4 Im ω=0 durumundaki kesim çizgilerine göre Im υ ve Im υ′ 'nün işaretleri

Buna göre,

$$
\overline{u}_{01} = -\frac{P_0}{\mu}H\cos(k_R x_1)+\frac{2P_0k_T^2}{\pi\mu}\int_{k_L}^{k_T}\frac{(2k^2-k_T^2)\,\upsilon\,\upsilon_I'}{\left(2k^2-k_T^2\right)^4-16k^4\,\upsilon^2\,\upsilon_I'^2}\,k\sin(kx_1)\,d\zeta
\tag{2.104}
$$

yazılabilir. Benzer şekilde,

$$
\begin{aligned}
I_{\alpha\psi} = & \int_{-i\infty}^{0} \left[\frac{\upsilon_I}{\left(2\zeta^2 - k_T^2\right)^2 - 4\zeta^2\, \upsilon_I\, \upsilon_I'} + \frac{\upsilon_I}{\left(2\zeta^2 - k_T^2\right)^2 + 4\zeta^2\, \upsilon_I\, \upsilon_I'} \right] e^{-i\zeta x_1}\, d\zeta \\
& + \int_{0}^{k_L} \left[\frac{\upsilon_I}{\left(2k^2 - k_T^2\right)^2 - 4k^2\, \upsilon_I\, \upsilon_I'} + \frac{\upsilon_I}{\left(2k^2 - k_T^2\right)^2 + 4\zeta^2\, \upsilon_I\, \upsilon_I'} \right] e^{-ikx_1}\, d\zeta
\end{aligned}
\tag{2.105}
$$

$$
\begin{aligned}
I_{\beta\varphi} = & \int_{-i\infty}^{0} \left[\frac{-\upsilon_I}{\left(2\zeta^2 - k_T^2\right)^2 + 4\zeta^2\, \upsilon_I\, \upsilon_I'} - \frac{\upsilon_I}{\left(2\zeta^2 - k_T^2\right)^2 - 4\zeta^2\, \upsilon_I\, \upsilon_I'} \right] e^{-i\zeta x_1}\, d\zeta \\
& + \int_{0}^{k_L} \left[\frac{-\upsilon_I}{\left(2k^2 - k_T^2\right)^2 + 4k^2\, \upsilon_I\, \upsilon_I'} - \frac{\upsilon_I}{\left(2k^2 - k_T^2\right)^2 - 4k^2\, \upsilon_I\, \upsilon_I'} \right] e^{-ikx_1}\, d\zeta \\
& + \int_{k_L}^{k_T} \left[\frac{\upsilon}{\left(2k^2 - k_T^2\right)^2 - 4k^2\, \upsilon\, \upsilon_I'} - \frac{\upsilon}{\left(2k^2 - k_T^2\right)^2 + 4k^2\, \upsilon\, \upsilon_I'} \right] e^{-ikx_1}\, d\zeta
\end{aligned}
\tag{2.106}
$$

yazılırsa,

$$
\begin{aligned}
I_{\alpha\psi} + I_{\beta\psi} = & \; 2 \int_{-i\infty}^{0} \frac{\upsilon_I}{\left(2\zeta^2 - k_T^2\right)^2 - 4\zeta^2\, \upsilon_I\, \upsilon_I'}\, e^{-i\zeta x_1}\, d\zeta \\
& + 2 \int_{0}^{k_L} \frac{\upsilon_I}{\left(2k^2 - k_T^2\right)^2 - 4k^2\, \upsilon_I\, \upsilon_I'}\, e^{-ikx_1}\, dk \\
& + 8 \int_{0}^{k_L} \frac{k^2\, \upsilon^2\, \upsilon_I'}{\left(2k^2 - k_T^2\right)^4 - 16k^4\, \upsilon^2\, \upsilon_I'^2}\, e^{-ikx_1}\, dk
\end{aligned}
\tag{2.107}
$$

ve sonuç olarak,

$$
\begin{aligned}
I_{\alpha\psi} + I_{\beta\psi} = & \; 2i \int_{\infty}^{0} \frac{-\left(\tau^2 + k_L^2\right)^{1/2}}{\left(2\tau^2 + k_T^2\right)^2 + 4\tau^2 \left(\tau^2 + k_L^2\right)^{1/2} \left(\tau^2 + k_T^2\right)^{1/2}}\, e^{-\tau x_1}\, d\tau \\
& + 2 \int_{0}^{k_L} \frac{\upsilon_I}{\left(2k^2 - k_T^2\right)^2 - 4k^2\, \upsilon_I\, \upsilon_I'}\, e^{-ikx_1}\, dk \\
& + 8 \int_{0}^{k_L} \frac{k^2\, \upsilon^2\, \upsilon_I'}{\left(2k^2 - k_T^2\right)^4 - 16k^4\, \upsilon^2\, \upsilon_I'^2}\, e^{-ikx_1}\, dk
\end{aligned}
\tag{2.108}
$$

bulunur. Buna göre,

$$
\begin{aligned}
\bar{u}_{02} = & -\frac{P_0}{\mu} K \sin(k_R x_1) - \frac{P_0}{\pi\mu} k_T^2 \left(\int_{0}^{k_L} \frac{\upsilon_I}{\left(2k^2 - k_T^2\right)^2 - 4k^2\, \upsilon\, \upsilon_I'} \cos(kx_1)\, dk \right. \\
& \left. + 4 \int_{k_L}^{k_T} \frac{k^2\, \upsilon^2\, \upsilon_I'}{\left(2k^2 - k_T^2\right)^4 - 16k^4\, \upsilon^2\, \upsilon_I'^2} \cos(kx_1)\, dk \right)
\end{aligned}
\tag{2.109}
$$

yazılabilir. (2.104) ve (2.109) bağıntılarında yer alan integraler için yaklaşık asimptodik açınımlar elde etmek mümkündür, ancak bunlar $x_1>>1$ için geçerli olacaktır. Bu nedenle bu eşitliklerdeki integraller sayısal olarak hesaplanır. Bu kısımda verilen incelemelerin bulunabileceği örneğin Achenbach (1973) gibi bir çok kaynakta yüzey yerdeğiştirmelerine ait asimptodik ifadelerin verilmesi dışında sayısal sonuç bulunmamaktadır. Bu nedenle şimdi x_2'nin keyfi bir değeri için gerilme ve yerdeğiştirmeleri veren ifadeler elde edilecektir.

2.3. Yarım Düzlemdeki Gerilme ve Yerdeğiştirmelerin Dağılımı.

Bu kısımda öncelikle uygun statik duruma ait problemin analitik çözümü elde edilmiş, sonra dinamik duruma (Lamb problemi) ait kaynaklarda bulunamayan ve x_2'nin keyfi değeri için geçerli olan gerilme ve yerdeğiştirme bağıntıları çıkartılarak sayısal uygulamalar yapılmıştır.

2.3.1. Uygun statik durumun incelenmesi.

$\omega=0$ alınarak (2.1) ile verilen yönetici denklemler ile (2.2) ve (2.3) ile verilen sınır koşullarına (2.14) ile tanımlanan üstel Fourier dönüşümü uygulanırsa,

$$\begin{aligned} &-(\lambda+2\mu)k^2\,\bar{u}_{1F}+\mu\,\bar{u}_{1F,22}-i\,k\,(\lambda+\mu)\,\bar{u}_{2F,2}=0 \\ &-i\,k\,(\lambda+\mu)\,\bar{u}_{1F,2}-\mu\,k^2\,\bar{u}_{2F}+(\lambda+2\mu)\,\bar{u}_{2F,22}=0 \end{aligned} \tag{2.110}$$

$$\sigma_{12F}\big|_{x_2=0}=0,\quad \sigma_{22F}\big|_{x_2=0}=-P_0\,\delta(x_1) \tag{2.111}$$

$$u_{iF},\sigma_{ijF}\big|_{x_j\to\infty}=0\quad ,\, i,j=1,2 \tag{2.112}$$

bulunur. (2.110) eşitliklerinden ilki x_2'ye göre bir kere türetilip ikincisinde kullanılırsa,

$$u_{1F,2222}-2k^2\,u_{1F,22}+k^4\,u_{1F}=0 \tag{2.113}$$

bulunur. Bu homogen diferansiyel denklemin çözümü,

$$u_{1F}=a\,e^{-kx_2}+b\,x_2\,e^{-kx_2}+c\,e^{kx_2}+d\,x_2\,e^{kx_2} \tag{2.114}$$

olup (2.112) uyarınca,

$$u_{1F}=a\,e^{-|k|x_2}+b\,x_2\,e^{-|k|x_2}=\begin{cases} a\,e^{-kx_2}+b\,x_2\,e^{-kx_2}, k>0 \\ a\,e^{kx_2}+b\,x_2\,e^{kx_2}, k<0 \end{cases} \tag{2.115}$$

yazılır. (2.115); (2.110) eşitliklerinin ikincisinde kullanılırsa,

$$\beta^2 = \frac{\lambda+\mu}{\lambda+2\mu} \;,\; 1-\beta^2 = \frac{\mu}{\lambda+2\mu} \;,\; A_1 = i\,k\,\beta^2\left(b - a|k|\right) \;,\; A_2 = -i\,k\,\beta^2\, b\,|k| \tag{2.116}$$

olmak üzere,

$$u_{2F,22} - (1-\beta^2)\,k^2\,u_{2F} = A_1(k)\,e^{-|k|x_2} + A_2(k)\,x_2\,e^{-|k|x_2} \tag{2.117}$$

eşitliği elde edilir. Bunun genel çözümü, homogen çözüm; (2.113) ile gözönüne alındığından sıfır olduğu için özel çözüme eşit olup, bu da örneğin süperpozisyon ilkesi ile,

$$u_{2F} = -i\left(\frac{\lambda+3\mu}{\lambda+\mu}\,b + a\,|k| + b\,|k|\,x_2\right)\frac{e^{-|k|x_2}}{k} \tag{2.118}$$

olarak bulunur. a ve b 'nin belirlenmesi için (2.22) uyarınca,

$$\begin{aligned} \sigma_{22F} &= (\lambda+2\mu)\,u_{2F,2} - i\,k\,\lambda\,u_{1F} \\ \sigma_{12F} &= \mu\,(u_{1F,2} - i\,k\,u_{2F}) \end{aligned} \tag{2.119}$$

yazılıp (2.111) koşulları kullanılırsa (2.115) ve (2.118) yardımı ile,

$$a = -i\,\frac{P_0}{2\,k\,(\lambda+\mu)} \;,\; b = i\,\frac{P_0}{2\mu}\frac{|k|}{k} \tag{2.120}$$

bulunur. Buna göre,

$$u_{1F} = i\,\frac{P_0}{2\mu}\left(-\frac{\mu}{\lambda+\mu}\frac{e^{-|k|x_2}}{k} + \frac{|k|}{k}\,x_2\,e^{-|k|x_2}\right) \tag{2.121}$$

$$u_{2F} = \frac{P_0}{2\mu}\left(\frac{\lambda+2\mu}{\lambda+\mu}\frac{|k|}{k^2}\,e^{-|k|x_2} + x_2\,e^{-|k|x_2}\right) \tag{2.122}$$

eşitlikleri elde edilir ve (2.37) uyarınca,

$$u_1 = i\,\frac{P_0}{4\pi\mu}\int_{-\infty}^{\infty}\left(-\frac{\mu}{\lambda+\mu}\frac{e^{-|k|x_2}}{k} + \frac{|k|}{k}\,x_2\,e^{-|k|x_2}\right)e^{-ikx_1}\,dk \tag{2.123}$$

$$u_2 = \frac{P_0}{4\pi\mu} \int_{-\infty}^{\infty} \left(\frac{\lambda+2\mu}{\lambda+\mu} \frac{|k|}{k^2} e^{-|k|x_2} + x_2\, e^{-|k|x_2} \right) e^{-ikx_1}\, dk \tag{2.124}$$

ve bunların düzenlenmesi ile,

$$\begin{aligned} u_1 = {} & \frac{P_0}{8\pi\mu} \int_{-\varepsilon}^{\varepsilon} \left(-\frac{\mu}{\lambda+\mu} \frac{e^{-|k|x_2}}{k} + \frac{|k|}{k} x_2\, e^{-|k|x_2} \right) \sin(kx_1)\, dk \\ & + \frac{P_0}{2\pi\mu} \int_{\varepsilon}^{\infty} \left(-\frac{\mu}{\lambda+\mu} \frac{e^{-kx_2}}{k} + x_2\, e^{-kx_2} \right) \sin(kx_1)\, dk \end{aligned} \tag{2.125}$$

$$\begin{aligned} u_2 = {} & \frac{P_0}{8\pi\mu} \int_{-\varepsilon}^{\varepsilon} \left(\frac{\lambda+2\mu}{\lambda+\mu} \frac{|k|}{k^2} e^{-|k|x_2} + x_2\, e^{-|k|x_2} \right) \cos(kx_1)\, dk \\ & + \frac{P_0}{2\pi\mu} \int_{0}^{\infty} \left(\frac{\lambda+2\mu}{\lambda+\mu} \frac{1}{k} e^{-kx_2} + x_2\, e^{-kx_2} \right) \cos(kx_1)\, dk \end{aligned} \tag{2.126}$$

bulunur. Burada ε çok küçük bir sayı olup $x_1=0$ noktasındaki tekilliğin Cauchy anlamında etkisini hesaplamak için alınan komşuluğun sınırını tanımlar. (2.14), (2.22) ve (2.23) eşitlikleri birlikte kullanılırsa,

$$\sigma_{11F} = -i\,k\,(\lambda+2\mu)\,u_{1F} + \lambda\, u_{2F,2} \tag{2.127}$$

$$\sigma_{11F} = (\lambda+2\mu)\,u_{2F,2} - i\,k\,\lambda\, u_{1F} \tag{2.128}$$

$$\sigma_{12F} = \mu\left(u_{1F,2} - i\,k\,u_{2F}\right) \tag{2.129}$$

bulunur. Burada (2.121) ve (2.122) eşitlikleri kullanılırsa,

$$\sigma_{11F} = P_0 \left(-e^{-|k|x_2} + |k|\, x_2\, e^{-|k|x_2} \right) \tag{2.130}$$

$$\sigma_{22F} = -P_0 \left(e^{-|k|x_2} + |k|\, x_2\, e^{-|k|x_2} \right) \tag{2.131}$$

$$\sigma_{12F} = -i\,P_0\,k\,x_2\, e^{-|k|x_2} \tag{2.132}$$

bulunur ve (2.37) uyarınca,

$$\sigma_{11F} = \frac{P_0}{\pi} \int_0^{\infty} \left(-e^{-kx_2} + k\,x_2\, e^{-kx_2} \right) \cos(kx_1)\, dk \tag{2.133}$$

$$\sigma_{22F} = -\frac{P_0}{\pi} \int_0^{\infty} \left(e^{-kx_2} + k\,x_2\, e^{-kx_2} \right) \cos(kx_1)\, dk \tag{2.134}$$

$$\sigma_{12F} = -\frac{P_0}{\pi} x_2 \int_0^{\infty} e^{-kx_2}\, k \sin(kx_1)\, dk \tag{2.135}$$

yazılabilir. Burada,

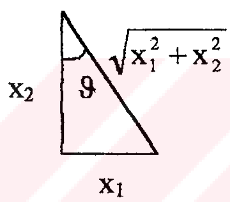
$$I_1 = \int_0^\infty e^{-kx_2} \cos(kx_1)\, dk = \frac{x_2}{x_1^2 + x_2^2} \tag{2.136}$$

$$I_2 = \int_0^\infty k\, e^{-kx_2} \cos(kx_1)\, dk = \frac{x_2^2 - x_1^2}{\left(x_1^2 + x_2^2\right)^2} \tag{2.137}$$

$$I_3 = \int_0^\infty e^{-kx_2} \sin(kx_1)\, dk = \frac{x_1}{x_1^2 + x_2^2} \tag{2.138}$$

$$I_4 = \int_0^\infty k\, e^{-kx_2} \sin(kx_1)\, dk = \frac{2x_1 x_2}{\left(x_1^2 + x_2^2\right)^2} \tag{2.139}$$

eşitlikleri kullanılırsa gerekli düzenlemeler yapılarak Şekil 2.5 yardımıyla sonuç olarak,



Şekil 2.5 ϑ açısının tanımlanması.

$$\sigma_{11} = -\frac{2\,P_0}{\pi} \frac{\sin^2 \vartheta \cos^2 \vartheta}{x_2} \tag{2.140}$$

$$\sigma_{22} = -\frac{2\,P_0}{\pi} \frac{\cos^4 \vartheta}{x_2} \tag{2.141}$$

$$\sigma_{12} = -\frac{2\,P_0}{\pi} \frac{\sin \vartheta \cos^3 \vartheta}{x_2} \tag{2.142}$$

bulunur.

2.3.2. Dinamik durumun incelenmesi.

(2.38)-(2.42) eşitlikleri

$$\Theta(k) = \frac{(2k^2 - k_T^2)\, e^{-\upsilon x_2} - 2\upsilon\, \upsilon'\, e^{-\upsilon' x_2}}{\Delta(k)}\, k\, e^{-ikx_1} \tag{2.143}$$

$$\Omega(k)=\frac{(2k^2-k_T^2)\,e^{-\upsilon x_2}-2k^2\,e^{-\upsilon' x_2}}{\Delta(k)}\,\upsilon\,e^{-ikx_1} \tag{2.144}$$

$$A(k)=\frac{\left[(\lambda+2\mu)k^2-\lambda\,\upsilon^2\right]\left(2k^2-k_T^2\right)e^{-\upsilon x_2}-4\mu\,k^2\,\upsilon\,\upsilon'\,e^{-\upsilon' x_2}}{\Delta(k)}\,e^{-ikx_1} \tag{2.145}$$

$$B(k)=\frac{\left[-(\lambda+2\mu)\upsilon^2+\lambda\,k^2\right]\left(2k^2-k_T^2\right)e^{-\upsilon x_2}+4\mu\,k^2\,\upsilon\,\upsilon'\,e^{-\upsilon' x_2}}{\Delta(k)}\,e^{-ikx_1} \tag{2.146}$$

$$C(k)=\frac{2k\left(2k^2-k_T^2\right)\upsilon\,(-e^{-\upsilon x_2}+e^{-\upsilon' x_2})}{\Delta(k)}\,e^{-ikx_1} \tag{2.147}$$

kısaltmaları kullanılarak şu yapıda yazılabilir;

$$\overline{u}_1=i\frac{P_0}{2\pi\mu}\int_{-\infty}^{\infty}\Theta(k)\,dk \tag{2.148}$$

$$\overline{u}_2=\frac{P_0}{2\pi\mu}\int_{-\infty}^{\infty}\Omega(k)\,dk \tag{2.149}$$

$$\overline{\sigma}_{11}=\frac{P_0}{2\pi\mu}\int_{-\infty}^{\infty}A(k)\,dk \tag{2.150}$$

$$\overline{\sigma}_{22}=\frac{P_0}{2\pi\mu}\int_{-\infty}^{\infty}B(k)\,dk \tag{2.151}$$

$$\overline{\sigma}_{12}=i\frac{P_0}{2\pi}\int_{-\infty}^{\infty}C(k)\,dk \tag{2.152}$$

Üç bölgedeki ${}^m\Theta(k)$. ${}^m\Omega(k)$, ${}^mA(k)$, ${}^mB(k)$ ve ${}^mC(k)$,m=1,3 fonksiyonları şöyle hesaplanır.

i. $0<k<k_L$ bölgesi.

Bu durumda,

$$\upsilon=(k^2-k_L^2)^{1/2}=i\,(k_L^2-k^2)^{1/2}=i\,\hat{\upsilon} \tag{2.153}$$

$$\upsilon'=(k^2-k_T^2)^{1/2}=i\,(k_T^2-k^2)^{1/2}=i\,\hat{\upsilon}' \tag{2.154}$$

eşitlikleri kullanılır ve integrandlar bu aralıkta,

$${}^1\Theta(k)=\frac{(2k^2-k_T^2)\,e^{-i\hat{\upsilon}x_2}+2\,\hat{\upsilon}\,\hat{\upsilon}'\,e^{-i\hat{\upsilon}'x_2}}{(2k^2-k_T^2)^2+4k^2\,\hat{\upsilon}\,\hat{\upsilon}'}\,k\,e^{-ikx_1} \tag{2.155}$$

$$^{1}\Omega(k) = i\,\frac{(2k^2 - k_T^2)\,e^{-i\hat{\upsilon}x_2} - 2k^2\,e^{-i\hat{\upsilon}'x_2}}{(2k^2 - k_T^2)^2 + 4k^2\,\hat{\upsilon}\,\hat{\upsilon}'}\,\hat{\upsilon}\,e^{-ikx_1} \tag{2.156}$$

$$^{1}A(k) = \frac{\left[(\lambda + 2\mu)k^2 + \lambda\,\hat{\upsilon}^2\right]\left(2k^2 - k_T^2\right)e^{-i\hat{\upsilon}x_2} + 4\mu\,k^2\,\hat{\upsilon}\,\hat{\upsilon}'\,e^{-i\hat{\upsilon}'x_2}}{(2k^2 - k_T^2)^2 + 4k^2\,\hat{\upsilon}\,\hat{\upsilon}'}\,e^{-ikx_1} \tag{2.157}$$

$$^{1}B(k) = \frac{\left[(\lambda + 2\mu)\hat{\upsilon}^2 + \lambda\,k^2\right]\left(2k^2 - k_T^2\right)e^{-i\hat{\upsilon}x_2} - 4\mu\,k^2\,\hat{\upsilon}\,\hat{\upsilon}'\,e^{-i\hat{\upsilon}'x_2}}{(2k^2 - k_T^2)^2 + 4k^2\,\hat{\upsilon}\,\hat{\upsilon}'}\,e^{-ikx_1} \tag{2.158}$$

$$^{1}C(k) = i\,\frac{2k\left(2k^2 - k_T^2\right)\hat{\upsilon}\,(-e^{-i\hat{\upsilon}x_2} + e^{-i\hat{\upsilon}'x_2})}{(2k^2 - k_T^2)^2 + 4k^2\,\hat{\upsilon}\,\hat{\upsilon}'}\,e^{-ikx_1} \tag{2.159}$$

yapısını kazanır. (2.148)-(2.152) uyarınca (2.155) ve (2.159)'nın sanal, (2.156)-(2.158) 'in de gerçel kısımları kullanılacak ve tek fonksiyonlar etki etmeyeceği için gözönüne alınmayacaktır. Buna göre sonuç olarak,

$$^{1}\overline{u}_1 = \frac{P_0}{\pi\mu}\int_0^{k_L}\frac{\left(2k^2 - k_T^2\right)\cos(\hat{\upsilon}x_2) + 2\overline{\upsilon}\,\overline{\upsilon}'\cos(\hat{\upsilon}'x_2)}{(2k^2 - k_T^2)^2 + 4k^2\,\hat{\upsilon}\,\hat{\upsilon}'}\,k\,\sin\left(kx_1\right)dk \tag{2.160}$$

$$^{1}\overline{u}_2 = \frac{P_0}{\pi\mu}\int_0^{k_L}\frac{\left(2k^2 - k_T^2\right)\sin(\hat{\upsilon}x_2) - 2k^2\sin(\hat{\upsilon}'x_2)}{(2k^2 - k_T^2)^2 + 4k^2\,\hat{\upsilon}\,\hat{\upsilon}'}\,\hat{\upsilon}\,\cos\left(kx_1\right)dk \tag{2.161}$$

$$^{1}\overline{\sigma}_{11} = \frac{P_0}{\pi\mu}\int_0^{k_L}\frac{\left[(\lambda + 2\mu)k^2 + \lambda\,\hat{\upsilon}^2\right](2k^2 - k_T^2)\cos(\hat{\upsilon}x_2) + 4\mu\,k^2\,\hat{\upsilon}\,\hat{\upsilon}'\cos(\hat{\upsilon}'x_2)}{(2k^2 - k_T^2)^2 + 4k^2\,\hat{\upsilon}\,\hat{\upsilon}'}\cos\left(kx_1\right)dk \tag{2.162}$$

$$^{1}\overline{\sigma}_{22} = \frac{P_0}{\pi\mu}\int_0^{k_L}\frac{\left[(\lambda + 2\mu)\hat{\upsilon}^2 + \lambda\,k^2\right](2k^2 - k_T^2)\cos(\hat{\upsilon}x_2) - 4\mu\,k^2\,\hat{\upsilon}\,\hat{\upsilon}'\cos(\overline{\upsilon}'x_2)}{(2k^2 - k_T^2)^2 + 4k^2\,\hat{\upsilon}\,\hat{\upsilon}'}\cos\left(kx_1\right)dk \tag{2.163}$$

$$^{1}\overline{\sigma}_{12} = \frac{P_0}{\pi}\int_0^{k_L}\frac{2k\,(2k^2 - k_T^2)\,\hat{\upsilon}\left[\sin(\hat{\upsilon}'x_2) - \sin(\hat{\upsilon}x_2)\right]}{(2k^2 - k_T^2)^2 + 4k^2\,\hat{\upsilon}\,\hat{\upsilon}'}\sin\left(kx_1\right)dk \tag{2.164}$$

bulunur.

b. $k_L<k<k_T$ bölgesi.

Bu durumda,

$$\upsilon = (k^2 - k_L^2)^{1/2} \tag{2.165}$$

$$\upsilon' = (k_T^2 - k^2)^{1/2} = i\,\hat{\upsilon}' \tag{2.166}$$

eşitlikleri kullanılır ve buna göre,

$$^{2}\Theta(k)=\frac{(2k^2-k_T^2)\,e^{-\upsilon x_2}-2\upsilon\, i\,\hat{\upsilon}'\,e^{-i\hat{\upsilon}'x_2}}{(2k^2-k_T^2)^2-4k^2\,\upsilon\, i\,\hat{\upsilon}'}\,k\,e^{-ikx_1} \tag{2.167}$$

$$^{2}\Omega(k)=\frac{(2k^2-k_T^2)\,e^{-\upsilon x_2}-2k^2\,e^{-i\hat{\upsilon}'x_2}}{(2k^2-k_T^2)^2-4k^2\,\upsilon\, i\,\hat{\upsilon}'}\,\upsilon\,e^{-ikx_1} \tag{2.168}$$

$$^{2}A(k)=\frac{\left[(\lambda+2\mu)k^2-\lambda\,\upsilon^2\right]\left(2k^2-k_T^2\right)e^{-\upsilon x_2}-4\mu\,k^2\,\upsilon\, i\,\hat{\upsilon}'\,e^{-i\hat{\upsilon}'x_2}}{(2k^2-k_T^2)^2-4k^2\,\upsilon\, i\,\hat{\upsilon}'}\,e^{-ikx_1} \tag{2.169}$$

$$^{2}B(k)=\frac{\left[-(\lambda+2\mu)\upsilon^2+\lambda k^2\right]\left(2k^2-k_T^2\right)e^{-\upsilon x_2}+4\mu\,k^2\,\upsilon\, i\,\hat{\upsilon}'\,e^{-i\hat{\upsilon}'x_2}}{(2k^2-k_T^2)^2-4k^2\,\upsilon\, i\,\hat{\upsilon}'}\,e^{-ikx_1} \tag{2.170}$$

$$^{2}C(k)=\frac{2k\left(2k^2-k_T^2\right)\upsilon\left(-e^{-\upsilon x_2}+e^{-i\hat{\upsilon}'x_2}\right)}{(2k^2-k_T^2)^2-4k^2\,\upsilon\, i\,\hat{\upsilon}'}\,e^{-ikx_1} \tag{2.171}$$

bulunur. (2.167)-(2.171) eşitliklerinin pay ve paydaları, paydanın karmaşık eşleniği ile çarpılıp düzenlendikten sonra (2.148)-(2.152) uyarınca uygun sanal veya gerçel kısımları alınır ve bunların içinden tek fonksiyon olan kısımları atılırsa sonuç olarak,

$$\begin{aligned}^{2}\bar{u}_1=\frac{P_0}{\pi\mu}\int_{k_L}^{k_T}\Bigg\{&\frac{\left(2k^2-k_T^2\right)^2\left[\left(2k^2-k_T^2\right)e^{-\upsilon x_2}-2\upsilon\,\hat{\upsilon}'\sin(\hat{\upsilon}'x_2)\right]}{(2k^2-k_T^2)^4+16k^4\,\upsilon^2\,\hat{\upsilon}'^2}\\&+\frac{8k^2\,\upsilon^2\,\hat{\upsilon}'^2\cos(\hat{\upsilon}'x_2)}{(2k^2-k_T^2)^4+16k^4\,\upsilon^2\,\hat{\upsilon}'^2}\Bigg\}\,k\sin\left(kx_1\right)dk\end{aligned} \tag{2.172}$$

$$\begin{aligned}^{2}\bar{u}_2=\frac{P_0}{\pi\mu}\int_{k_L}^{k_T}\Bigg\{&\frac{\left(2k^2-k_T^2\right)^2\left[\left(2k^2-k_T^2\right)e^{-\upsilon x_2}-2k^2\cos(\hat{\upsilon}'x_2)\right]}{(2k^2-k_T^2)^4+16k^4\,\upsilon^2\,\hat{\upsilon}'^2}\\&-\frac{8k^4\,\upsilon\,\hat{\upsilon}'\sin(\hat{\upsilon}'x_2)}{(2k^2-k_T^2)^4+16k^4\,\upsilon^2\,\hat{\upsilon}'^2}\Bigg\}\,\upsilon\cos\left(kx_1\right)dk\end{aligned} \tag{2.173}$$

$$\begin{aligned}^{2}\bar{\sigma}_{11}=\frac{P_0}{\pi\mu}\int_{k_L}^{k_T}\Bigg\{&\frac{(2k^2-k_T^2)^2\left\{\left[(\lambda+2\mu)k^2-\lambda\,\upsilon^2\right](2k^2-k_T^2)\,e^{-\upsilon x_2}-4\mu\,k^2\,\upsilon\,\hat{\upsilon}'\sin(\hat{\upsilon}'x_2)\right\}}{(2k^2-k_T^2)^4+16k^4\,\upsilon^2\,\hat{\upsilon}'^2}\\&+\frac{16\mu\,k^4\,\upsilon^2\,\hat{\upsilon}'^2\cos(\hat{\upsilon}'x_2)}{(2k^2-k_T^2)^4+16k^4\,\upsilon^2\,\hat{\upsilon}'^2}\Bigg\}\cos\left(kx_1\right)dk\end{aligned} \tag{2.174}$$

$$\begin{aligned}^{2}\bar{\sigma}_{22}=\frac{P_0}{\pi\mu}\int_{k_L}^{k_T}\Bigg\{&\frac{(2k^2-k_T^2)^2\left\{\left[-(\lambda+2\mu)\upsilon^2+\lambda\,k^2\right](2k^2-k_T^2)\,e^{-\upsilon x_2}+4\mu\,k^2\,\upsilon\,\hat{\upsilon}'\sin(\hat{\upsilon}'x_2)\right\}}{(2k^2-k_T^2)^4+16k^4\,\upsilon^2\,\hat{\upsilon}'^2}\\&-\frac{16\mu\,k^4\,\upsilon^2\,\hat{\upsilon}'^2\cos(\hat{\upsilon}'x_2)}{(2k^2-k_T^2)^4+16k^4\,\upsilon^2\,\hat{\upsilon}'^2}\Bigg\}\cos\left(kx_1\right)dk\end{aligned} \tag{2.175}$$

$$^{2}\bar{\sigma}_{12}=\frac{P_0}{\pi}\int_{k_L}^{k_T}\frac{(2k^2-k_T^2)^2\left[-e^{-\upsilon x_2}+\cos(\hat{\upsilon}'x_2)\right]+4k^2\,\upsilon\,\hat{\upsilon}'\sin(\hat{\upsilon}'x_2)}{(2k^2-k_T^2)^4+16k^4\,\upsilon^2\,\hat{\upsilon}'^2}\,2(2k^2-k_T^2)\,\upsilon\,k\sin\left(kx_1\right)dk \tag{2.176}$$

bulunur.

c. $k_T<k$ bölgesi.

Bu durumda (2.143)-(2.147) eşitlikleri kullanılır ve (2.148)-(2.152) yardımı ile,

$$^3\bar{u}_1=\frac{P_0}{\pi\mu}\int_{k_T}^{\infty}\frac{\left(2k^2-k_T^2\right)e^{-\upsilon x_2}-2\upsilon\,\upsilon' e^{-\upsilon' x_2}}{(2k^2-k_T^2)^2-4k^2\,\upsilon\,\upsilon'}\,k\sin\left(kx_1\right)dk \tag{2.177}$$

$$^3\bar{u}_2=\frac{P_0}{\pi\mu}\int_{k_T}^{\infty}\frac{\left(2k^2-k_T^2\right)e^{-\upsilon x_2}-2k^2\,e^{-\upsilon' x_2}}{(2k^2-k_T^2)^2-4k^2\,\upsilon\,\upsilon'}\,\upsilon\cos\left(kx_1\right)dk \tag{2.178}$$

$$^3\bar{\sigma}_{11}=\frac{P_0}{\pi\mu}\int_{k_T}^{\infty}\frac{\left[(\lambda+2\mu)k^2-\lambda\upsilon^2\right](2k^2-k_T^2)\,e^{-\upsilon x_2}-4\mu k^2\,\upsilon\,\upsilon' e^{-\upsilon' x_2}}{(2k^2-k_T^2)^2-4k^2\,\upsilon\,\upsilon'}\cos\left(kx_1\right)dk \tag{2.179}$$

$$^3\bar{\sigma}_{22}=\frac{P_0}{\pi\mu}\int_{k_T}^{\infty}\frac{\left[-(\lambda+2\mu)\upsilon^2+\lambda k^2\right](2k^2-k_T^2)\,e^{-\upsilon x_2}+4\mu k^2\,\upsilon\,\upsilon' e^{-\upsilon' x_2}}{(2k^2-k_T^2)^2-4k^2\,\upsilon\,\upsilon'}\cos\left(kx_1\right)dk \tag{2.180}$$

$$^3\bar{\sigma}_{12}=\frac{P_0}{\pi}\int_{k_T}^{\infty}\frac{-e^{-\upsilon x_2}+e^{-\upsilon' x_2}}{(2k^2-k_T^2)^2-4k^2\,\upsilon\,\upsilon'}\,2\,(2k^2-k_T^2)\,\upsilon\,k\sin\left(kx_1\right)dk \tag{2.181}$$

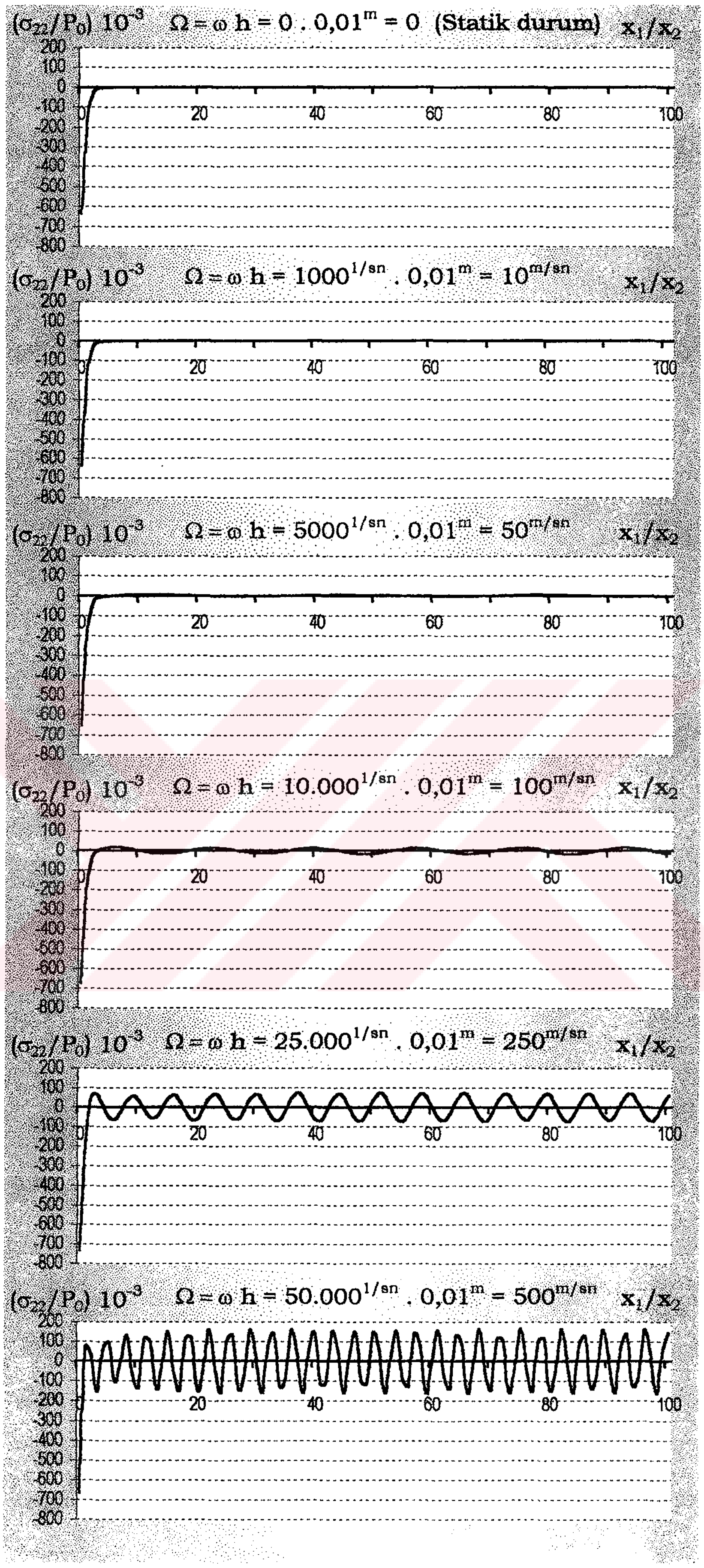
bulunur. Sonuç olarak gerilme ve yerdeğiştirmeler şu eşitlikler yardımı ile bulunabilir;

$$\bar{u}_i={}^1\bar{u}_i+{}^2\bar{u}_i+{}^2\bar{u}_i \tag{2.182}$$

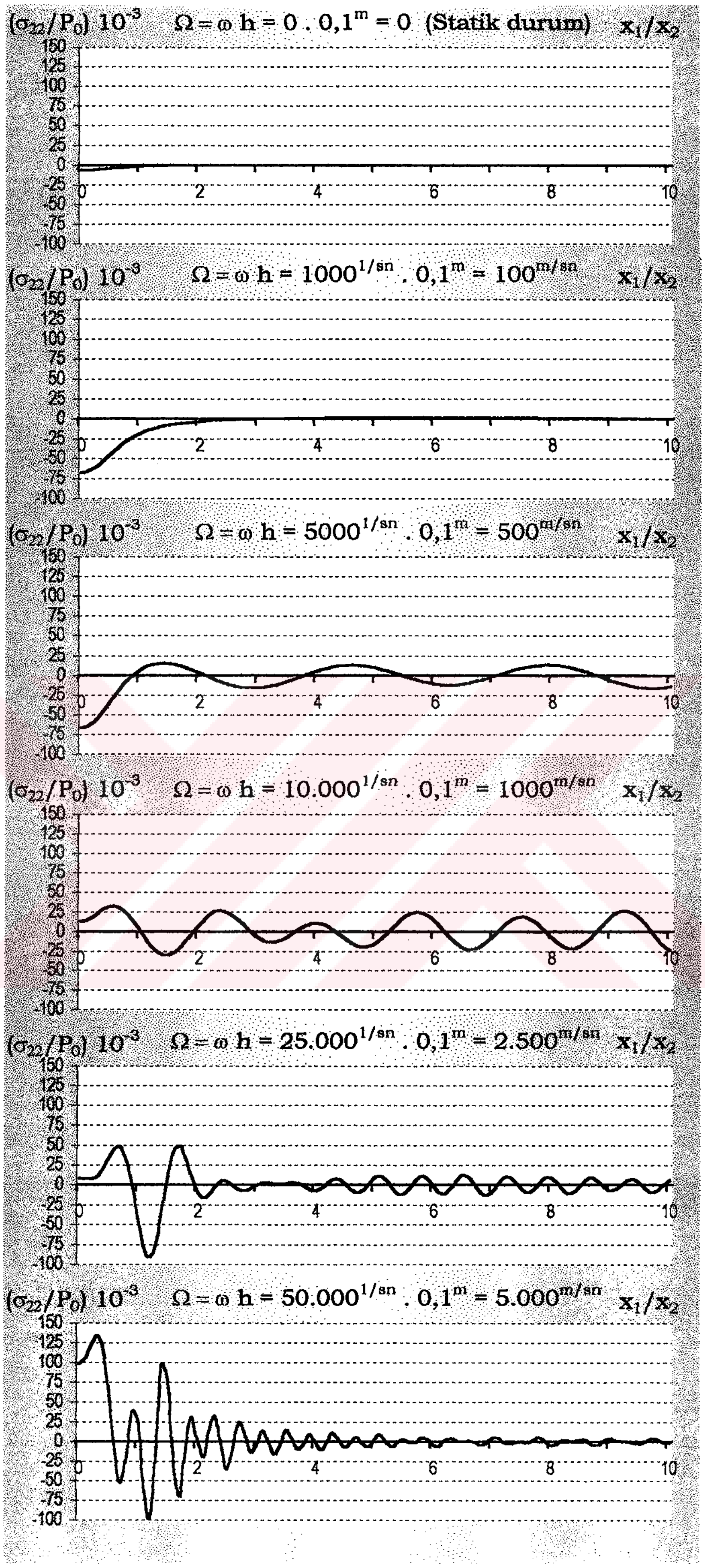
$$\bar{\sigma}_{ij}={}^1\bar{\sigma}_{ij}+{}^2\bar{\sigma}_{ij}+{}^3\bar{\sigma}_{ij} \tag{2.183}$$

2.4. Homogen Yarım Düzlem Durumuna Ait Sayısal İncelemeler.

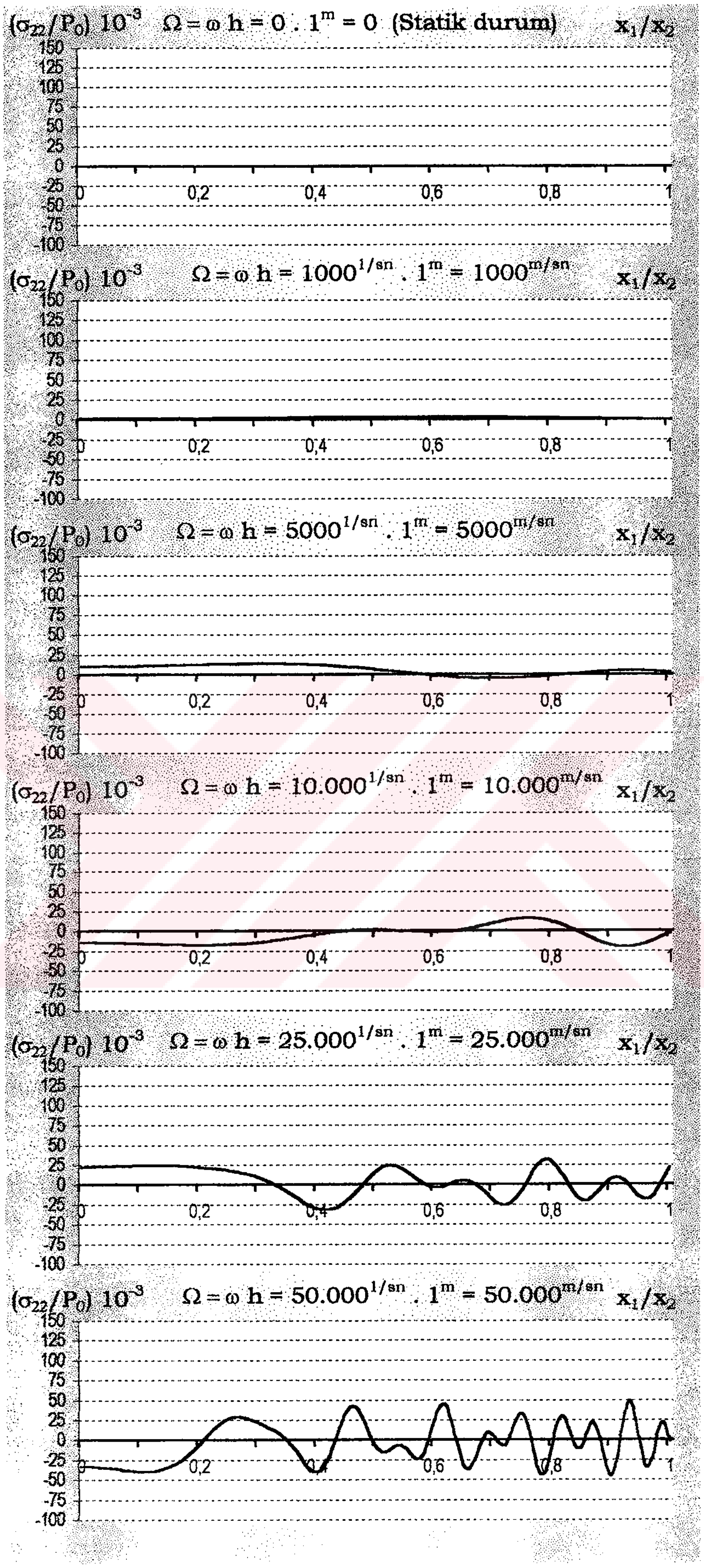
Bu kısımda homogen yarım düzlemde uygun statik ve dinamik durumlarda oluşan $\bar{\sigma}_{22}$ büyüklüklerinin frekansa ve yüzeyden olan uzaklığa bağlı yayılımlarını gösteren boyutsuz grafikler verilmiştir. Sayısal işlemlerde $E=70^{\ \mathrm{GPa}}$, $\nu=0.25$, $\rho=3.10^{3\ \mathrm{kg/m^3}}$ alınmıştır. Bu durumda enine ve boyuna dalga hızları; $c_L=5291{,}503^{\ \mathrm{m/sn}}$, $c_T=3055{,}051^{\ \mathrm{m/sn}}$ olmaktadır. Frekansın sıfırdan başlayarak çeşitli değerleri için $\bar{\sigma}_{22}$'nin yüzeyden değişik derinliklerde alınan yatay çizgiler üzerindeki yayılımları boyutsuz grafiklerle verilmiştir. Bu şekilde keyfi bir $\sigma_{22}(x_1,x_2)e^{i\omega t}$ yayılımı $\Omega=\omega\,x_2$ eşitliğini sağlayan Ω sayısına karşı gelen grafikten elde edilebilir. Şekil 2.6-2.9' dan görüldüğü gibi homogen yarım düzlem durumunda serbest yüzeyde uygulanan tekil harmonik yük nedeni ile Rayleigh yüzey dalgaları oluşmaktadır. Bu dalgalardan dolayı oluşan gerilme yayılımlarının genliklerinin ise x_2'nin artmasıyla hızla azaldığı izlenmektedir. Bu diagramların yorumlanması sonuçlar bölümünde yapılmıştır.



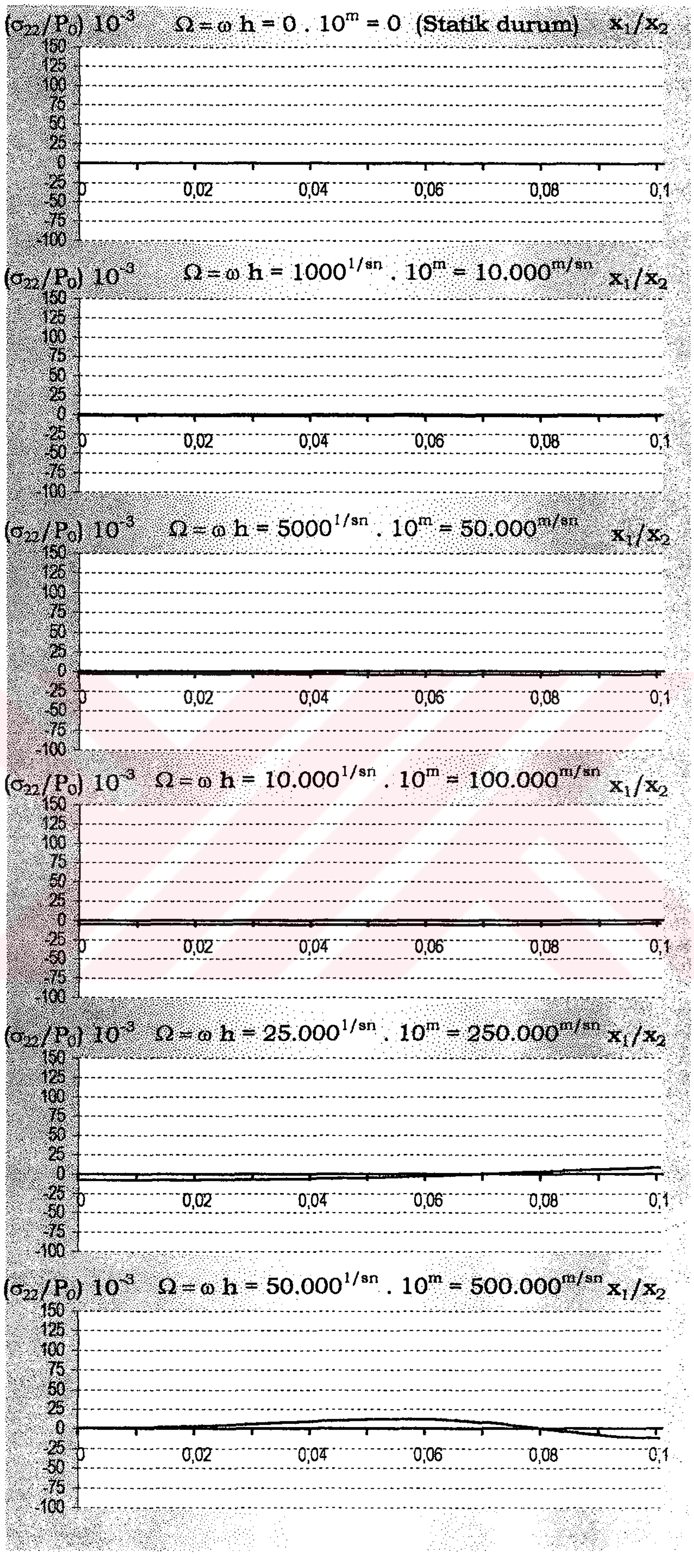
Şekil 2.6 Homogen yarım düzlemde x_2=0.01^m için σ_{22} yayılımı



Şekil 2.7 Homogen yarım düzlemde $x_2=0.1^m$ için σ_{22} yayılımı



Şekil 2.8 Homogen yarım düzlemde $x_2=1^m$ için σ_{22} yayılımı



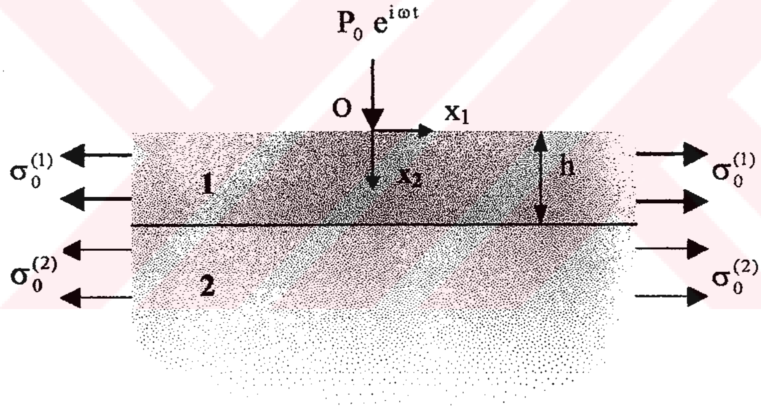
Şekil 2.9 Homogen yarım düzlemde $x_2=10^m$ için σ_{22} yayılımı

3. ÖNGERİLMELİ ve TABAKALI YARIM DÜZLEM İÇİN LAMB PROBLEMİ.

Bu bölümde üzerinde öngerilmeli tabakası bulunan öngerilmeli yarım düzlemin Lamb problemi incelenmiştir. Öncelikle uygun lineer olmayan hareket denklemleri ve sınır koşullarından lineerleştirilmiş hareket denklemleri ve sınır koşulları elde edilmiş, daha sonra öngerilme etkisindeki tabakalı yarım düzlem için Lamb probleminin formülasyonu verilerek bu problem üstel Fourier dönüşümü yardımı ile incelenmiştir. Çok kapsamlı olan matematik işlemler ve bilgisayar programları yapılarak bir çok özel durumda gerilme yayılımına ait sayısal uygulamalar yapılmış ve elde edilen sonuçlar yardımı ile yüzeyde etkiyen tekil harmonik yükten dolayı oluşan gerilme yayılımına öngerilmelerin etkisi incelenmiştir.

3.1. Lineerleştirilmiş Hareket Denklemlerinin ve Sınır Koşullarının Elde Edilmesi.

İleride aranılan büyüklüklerin bağlı olduğu koordinatların Lagrange koordinatları olduğu kabul edilecektir.



Şekil 3.1 Öngerilmeli ve tabakalı yarım düzlem için Lamb problemi

Lineer elastik izotrop bir cisimde geometrik non-lineer alan denklemleri ve sınır koşulları,

$$
\begin{aligned}
&\left[\sigma_{jn}\left(\delta_i^n + u_{i,n}\right)\right]_{,j} = \rho\, \ddot{u}_i \\
&\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{n,i}u_{n,j}) \\
&\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij} \quad \theta = \varepsilon_{kk} = u_{k,k} \\
&(u_i)_{S_I} = f_i\,, \ \left[\sigma_{jn}(\delta_{in} + u_{i,n})\right]_{S_{II}} n_j = p_i \quad (S_I \cup S_{II} = S)
\end{aligned}
\qquad (3.1)
$$

şeklinde yazılır (Guz, 1999). Burada,

$$\sigma'_{ij}=\sigma_{ij}+\sigma_{0ij}\ ,\ \varepsilon'_{ij}=\varepsilon_{ij}+\varepsilon_{0ij}\ ,\ u'_{ij}=u_{ij}+u_{0ij} \tag{3.2}$$

toplamları yazılarak,

$$\sigma_{ij}\langle\langle\sigma_{0ij}\ ,\ \varepsilon_{ij}\langle\langle\varepsilon_{0ij}\ ,\ u_i\langle\langle u_{0i}\ ,f_i\langle\langle f_{0i}\ ,p_i\langle\langle p_{0i} \tag{3.3}$$

eşitsizliklerinin sağlandığı kabul edilecektir. Ayrıca (3.1) alan denklemlerinin $\sigma_{0ij}\ ,\varepsilon_{0ij}\ ,u_{0ij}$ büyüklükleri için de sağlandığı kabul edilecektir. Burada σ_{ij} yüzeydeki pertürbasyondan dolayı oluşan gerilme tansörünü, σ_{0ij} ise öngerilme tansörünü göstermektedir. Öngerilme tansörü ve ortam homogen olarak kabul edildiği için,

$$\forall x_k \in V \therefore\ \rho^{(m)}(x_k)=\rho^{(m)}\ \wedge\ \sigma_{0ij}^{(m)}(x_k)=\sigma_{0ij}^{(m)} \tag{3.4}$$

olmaktadır. (3.2) eşitliği ile tanımlanan durum için (3.1) eşitlikleri gözönüne alınırsa,

$$\begin{gathered}\left[\left(\sigma_{jn}+\sigma_{0jn}\right)\left(\delta_i^n+u_{i,n}+u_{0i,n}\right)\right]_{,j}=\rho\left(\ddot{u}_i+\ddot{u}_{0i}\right)\\ \left(u_i+u_{0i}\right)_{S_I}=f_i+f_{0i}\ ,\ \left[\left(\sigma_{jn}+\sigma_{0jn}\right)\left(\delta_i^n+u_{i,n}+u_{0i,n}\right)\right]_{S_{II}}n_j=p_{0i}+p_i\end{gathered} \tag{3.5}$$

bulunur. (3.5) ile verilen eşitliklerin ilki yukarıdaki kabuller çerçevesinde düzenlenirse,

$$\left[\sigma_{0jn}\left(\delta_i^n+u_{0i,n}\right)\right]_{,j}+\left(\sigma_{jn}\,\delta_i^n+\sigma_{jn}\,u_{i,n}+\sigma_{jn}\,u_{0i,n}+\sigma_{0jn}\,u_{i,n}\right)_{,j}=\rho\ \left(\ddot{u}_i+\ddot{u}_{0i}\right) \tag{3.6}$$

eşitliği, ve $\left[\sigma_{0jn}\left(\delta_i^n+u_{0i,n}\right)\right]_{,j}=\rho\,\ddot{u}_{0i}$ olduğuna göre (3.6)'dan,

$$\left(\sigma_{jn}\,\delta_i^n+\sigma_{jn}\,u_{i,n}+\sigma_{jn}\,u_{0i,n}+\sigma_{0jn}\,u_{i,n}\right)_{,j}=\rho\ \ddot{u}_i \tag{3.7}$$

eşitliği elde edilir. Burada (3.3) uyarınca bir mertebe küçük terimler ihmal edilirse,

$$\left[\sigma_{jn}\left(\delta_i^n+u_{0i,n}\right)+\sigma_{0jn}\,u_{i,n}\right]_{,j}=\rho\ \ddot{u}_i \tag{3.8}$$

bulunur ve incelenen durumda $u_{oi,n}<<1$ ve $\delta_{in}+u_{0i,n}\approx\delta_{in}$ olduğu için,

$$\left(\sigma_{ji}+\sigma_{0jn}\,u_{i,n}\right)_{,j}=\rho\ \ddot{u}_i \tag{3.9}$$

eşitliği elde edilir ve bu eşitlik (3.4) uyarınca,

$$\sigma_{ji,j}+\sigma_{0jn}\,u_{i,nj}=\rho\,\ddot{u}_i \tag{3.10}$$

şeklinde yazılabilir. (3.1) in üçüncü eşitliği, (3.2) ve yukarıdaki kabullere dayanılarak,

$$\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}) \tag{3.11}$$

ile verilen lineerleştirilmiş kinematik bağıntıları ve küçük pertürbasyonlar için,

$$\sigma_{ij}=\lambda\varepsilon_{kk}\delta_{ij}+2\mu\varepsilon_{ij} \tag{3.12}$$

ile verilen Hooke kanunları yazılabilir. (3.12) eşitliği (3.11) yardımı ile,

$$\sigma_{ij}=\lambda\,u_{k,k}\,\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right) \tag{3.13}$$

şeklinde yazılır ve (3.10) ile verilen hareket denklemi de,

$$\sigma_{ij,i}+\sigma_{0in}\,u_{j,in}=\rho\,\ddot{u}_j \tag{3.14}$$

şeklinde yazılırsa (3.13); (3.14) 'de kullanılarak,

$$\lambda\,u_{k,ki}\,\delta_{ij}+\mu\left(u_{i,ji}+u_{j,ii}\right)+\sigma_{0in}\,u_{j,in}=\rho\,\ddot{u}_j \tag{3.15}$$

şeklinde yerdeğiştirmeler cinsinden yazılmış lineerleştirilmiş hareket denklemleri elde edilir. Şekil 3.1'e göre $\sigma_{011}=\sigma_0, \sigma_{022}=\sigma_{033}=0, i\neq j \therefore \sigma_{0ij}=0$ olduğuna göre (3.15) eşitliği,

$$\lambda\,u_{k,kj}+\mu\left(u_{i,ji}+u_{j,ii}\right)+\sigma_0\,u_{j,11}=\rho\,\ddot{u}_j\ ,\ (\sigma_{i2}|_{x_2=\text{sabit}}=p_i) \tag{3.16}$$

şeklinde veya açık olarak,

$$\begin{aligned}(\lambda+2\mu+\sigma_0)\,u_{1,11}+\mu\,u_{1,22}+(\lambda+\mu)\,u_{2,12}&=\rho\,\ddot{u}_1\\(\lambda+\mu)\,u_{1,12}+(\mu+\sigma_0)\,u_{2,11}+(\lambda+2\mu)\,u_{2,22}&=\rho\,\ddot{u}_2\end{aligned} \tag{3.17}$$

şeklinde yazılabilir. Bundan sonraki incelemeler bu eşitlikler kullanılarak yapılacaktır. Ayrıca yukarıda verilen işlemlerde öngerilmelerin klasik lineer elastik teori çerçevesinde kaldığı kabul edilmiştir.

3.2. Problemin Formülasyonu ve Çözüm Yöntemi.

Şekil 3.1 ve (3.17) uyarınca problemin yönetici denklemi ile sınır ve süreklilik koşulları,

$$\begin{aligned}&\left(\lambda^{(m)}+2\mu^{(m)}+\sigma_0^{(m)}\right)u_{1,11}^{(m)}+\mu^{(m)}u_{1,22}^{(m)}+\left(\lambda^{(m)}+\mu^{(m)}\right)u_{2,12}^{(m)}=\rho^{(m)}\ddot{u}_1^{(m)}\\&\left(\lambda^{(m)}+\mu^{(m)}\right)u_{1,12}^{(m)}+\left(\mu^{(m)}+\sigma_0^{(m)}\right)u_{2,11}^{(m)}+\left(\lambda^{(m)}+2\mu^{(m)}\right)u_{2,22}^{(m)}=\rho^{(m)}\ddot{u}_2^{(m)}\end{aligned}\quad m=1,2 \tag{3.18}$$

$$\sigma_{12}^{(1)}\Big|_{x_2=0}=0,\quad \sigma_{22}^{(1)}\Big|_{x_2=0}=-P_0\,\delta(x_1)\,e^{i\omega t} \tag{3.19}$$

$$u_i^{(2)},\sigma_{ij}^{(2)}\ \Big|_{x_2\to\infty}=0\quad i,j=1,2 \tag{3.20}$$

$$u_i^{(1)}\Big|_{x_2=h}=u_i^{(2)}\Big|_{x_2=h}\quad i=1,2 \tag{3.21}$$

$$\sigma_{2i}^{(1)}\Big|_{x_2=h}=\sigma_{2i}^{(2)}\Big|_{x_2=h}\quad i=1,2 \tag{3.22}$$

şeklinde olur. Burada kütle kuvvetlerinin var olmadığı ve düzlem şekil değiştirme durumunun sözkonusu olduğu kabul edilmiştir ($\varepsilon_{33}=0$). Ayrıca stasyoner durum gözönüne alındığı için,

$$\left\{u_i^{(m)}(x_i,t),\sigma_{ij}^{(m)}(x_i,t),\varepsilon_{ij}^{(m)}(x_i,t)\right\}=\left\{\bar{u}_i^{(m)}(x_i),\bar{\sigma}_{ij}^{(m)}(x_i),\bar{\varepsilon}_{ij}^{(m)}(x_i)\right\}e^{i\omega t}\quad i,j,m=1,2 \tag{3.23}$$

yazılabilir. (3.23) eşitlikleri (3.18)-(3.22) 'de gözönüne alınırsa genlikler için aşağıdaki yönetici denklem, sınır ve süreklilik koşulları elde edilir.

$$\begin{aligned}&\left(\lambda^{(m)}+2\mu^{(m)}+\sigma_0^{(m)}\right)\bar{u}_{1,11}^{(m)}+\mu^{(m)}\bar{u}_{1,22}^{(m)}+\left(\lambda^{(m)}+\mu^{(m)}\right)\bar{u}_{2,12}^{(m)}+\rho^{(m)}\omega^2\bar{u}_1^{(m)}=0\\&\left(\lambda^{(m)}+\mu^{(m)}\right)\bar{u}_{1,12}^{(m)}+\left(\mu^{(m)}+\sigma_0^{(m)}\right)\bar{u}_{2,11}^{(m)}+\left(\lambda^{(m)}+2\mu^{(m)}\right)\bar{u}_{2,22}^{(m)}+\rho^{(m)}\omega^2\bar{u}_2^{(m)}=0\end{aligned}\quad m=1,2 \tag{3.24}$$

$$\bar{\sigma}_{12}^{(1)}\Big|_{x_2=0}=0,\quad \bar{\sigma}_{22}^{(1)}\Big|_{x_2=0}=-P_0\,\delta(x_1) \tag{3.25}$$

$$\bar{u}_i^{(2)},\bar{\sigma}_{ij}^{(2)}\ \Big|_{x_2\to\infty}=0\quad i,j=1,2 \tag{3.26}$$

$$\bar{u}_i^{(1)}\Big|_{x_2=h}=\bar{u}_i^{(2)}\Big|_{x_2=h}\quad i=1,2 \tag{3.27}$$

$$\bar{\sigma}_{2i}^{(1)}\Big|_{x_2=h}=\bar{\sigma}_{2i}^{(2)}\Big|_{x_2=h}\quad i=1,2 \tag{3.28}$$

Düzlem şekil değiştirme durumunda $\bar{u}_i^{(m)}$ bileşenleri $\bar{\varphi}$ ve $\bar{\vec{\psi}}$ potansiyellerine,

$$\begin{aligned}\bar{u}_1^{(m)}&=\bar{\varphi}_{,1}^{(m)}+\bar{\psi}_{3,2}^{(m)}\\ \bar{u}_2^{(m)}&=\bar{\varphi}_{,2}^{(m)}-\bar{\psi}_{3,1}^{(m)}\end{aligned}\quad m=1,2 \tag{3.29}$$

ile bağlanabilir. $\bar{\psi}_3$; $\bar{\vec{\psi}}$ 'nin x_3 doğrultusundaki bileşenidir. (3.29); (3.24)'te kullanılırsa,

$$\left(\lambda^{(m)}+2\mu^{(m)}+\sigma_0^{(m)}\right)\left(\overline{\varphi}_{,111}^{(m)}+\overline{\psi}_{3,211}^{(m)}\right)+\mu^{(m)}\left(\overline{\varphi}_{,122}^{(m)}+\overline{\psi}_{3,222}^{(m)}\right)$$
$$+\left(\lambda^{(m)}+\mu^{(m)}\right)\left(\overline{\varphi}_{,221}^{(m)}-\overline{\psi}_{3,112}^{(m)}\right)+\rho^{(m)}\,\omega^2\left(\overline{\varphi}_{,1}^{(m)}+\overline{\psi}_{3,2}^{(m)}\right)=0 \quad m=1,2 \tag{3.30}$$

$$\left(\lambda^{(m)}+\mu^{(m)}\right)\left(\overline{\varphi}_{,112}^{(m)}+\overline{\psi}_{3,221}^{(m)}\right)+\left(\mu^{(m)}+\sigma_0^{(m)}\right)\left(\overline{\varphi}_{,211}^{(m)}-\overline{\psi}_{3,111}^{(m)}\right)$$
$$+\left(\lambda^{(m)}+2\mu^{(m)}\right)\left(\overline{\varphi}_{,222}^{(m)}-\overline{\psi}_{3,112}^{(m)}\right)+\rho^{(m)}\,\omega^2\left(\overline{\varphi}_{,2}^{(m)}-\overline{\psi}_{3,1}^{(m)}\right)=0 \quad m=1,2 \tag{3.31}$$

bulunur. Bu eşitlikleri ayırmak için (3.30) x_1'e, (3.31) x_2'ye göre türetilip birlikte kullanılırsa,

$$\left(\lambda^{(m)}+2\mu^{(m)}\right)\left(\overline{\varphi}_{,1111}^{(m)}+2\overline{\varphi}_{,1122}^{(m)}+\overline{\varphi}_{,2222}^{(m)}\right)+\sigma_0^{(m)}\left(\overline{\varphi}_{,1111}^{(m)}+\overline{\varphi}_{,1122}^{(m)}\right)+\rho^{(m)}\,\omega^2\left(\overline{\varphi}_{,11}^{(m)}+\overline{\varphi}_{,22}^{(m)}\right)=0 \quad m=1,2 \tag{3.32}$$

ve (3.30); x_2'ye, (3.31) de x_1'e göre türetilerek birarada kullanılırsa,

$$\mu^{(m)}\left(\overline{\psi}_{3,1111}^{(m)}+2\overline{\psi}_{3,1122}^{(m)}+\overline{\psi}_{3,2222}^{(m)}\right)+\sigma_0^{(m)}\left(\overline{\psi}_{3,1111}^{(m)}+\overline{\psi}_{3,1122}^{(m)}\right)+\rho^{(m)}\,\omega^2\left(\overline{\psi}_{3,11}^{(m)}+\overline{\psi}_{3,22}^{(m)}\right)=0 \quad m=1,2 \tag{3.33}$$

eşitlikleri elde edilir. Bunlar harmonik ve biharmonik operatörler kullanılarak,

$$\left(\lambda^{(m)}+2\mu^{(m)}\right)\nabla^4\overline{\varphi}^{(m)}+\sigma_0^{(m)}\,\nabla^2\overline{\varphi}_{,11}^{(m)}+\rho^{(m)}\,\omega^2\,\nabla^2\overline{\varphi}^{(m)}=0 \quad m=1,2 \tag{3.34}$$

$$\mu^{(m)}\,\nabla^4\overline{\psi}_3^{(m)}+\sigma_0^{(m)}\,\nabla^2\overline{\psi}_{3,11}^{(m)}+\rho^{(m)}\,\omega^2\,\nabla^2\overline{\psi}_3^{(m)}=0 \quad m=1,2 \tag{3.35}$$

ve sonuç olarak,

$$\left(\lambda^{(m)}+2\mu^{(m)}\right)\nabla^2\overline{\varphi}^{(m)}+\sigma_0^{(m)}\,\overline{\varphi}_{,11}^{(m)}+\rho^{(m)}\,\omega^2\,\overline{\varphi}^{(m)}=0 \quad m=1,2 \tag{3.36}$$

$$\mu^{(m)}\,\nabla^2\overline{\psi}_3^{(m)}+\sigma_0^{(m)}\,\overline{\psi}_{3,11}^{(m)}+\rho^{(m)}\,\omega^2\,\overline{\psi}_3^{(m)}=0 \quad m=1,2 \tag{3.37}$$

şeklinde yazılabilir. (3.36) ve (3.37) eşitliklerine,

$$f_F(k,x_2)=\int_{-\infty}^{\infty} f(x_1,x_2)\,e^{ikx_1}\,dx_1 \tag{3.38}$$

üstel Fourier dönüşümü uygulanır ve,

$$c_L^{(m)}=\left(\frac{\lambda^{(m)}+2\mu^{(m)}}{\rho^{(m)}}\right)^{1/2}, \quad c_T^{(m)}=\left(\frac{\mu^{(m)}}{\rho^{(m)}}\right)^{1/2} \quad m=1,2 \tag{3.39}$$

ile tanımlanan dalga hızları ve,

$$k_L^{(m)} = \frac{\omega}{c_L^{(m)}}, \quad k_T^{(m)} = \frac{\omega}{c_T^{(m)}} \qquad m=1,2 \tag{3.40}$$

ile tanımlanan dalga yavaşlıkları kullanılırsa,

$$\overline{\varphi}_{F,22}^{(m)} - \left[k^2 \left(1 + \frac{\sigma_0^{(m)}}{\lambda^{(m)} + 2\mu^{(m)}} \right) - k_L^{(m)2} \right] \overline{\varphi}_F^{(m)} = 0 \qquad m=1,2 \tag{3.41}$$

$$\overline{\psi}_{3F,22}^{(m)} - \left[k^2 \left(1 + \frac{\sigma_0^{(m)}}{\mu^{(m)}} \right) - k_T^{(m)2} \right] \overline{\psi}_{3F}^{(m)} = 0 \qquad m=1,2 \tag{3.42}$$

eşitlikleri elde edilir. Burada,

$$\eta_1^{(m)} = \frac{\sigma_0^{(m)}}{\lambda^{(m)} + 2\mu^{(m)}} \qquad m=1,2 \tag{3.43}$$

$$\eta_2^{(m)} = \frac{\sigma_0^{(m)}}{\mu^{(m)}} \qquad m=1,2 \tag{3.44}$$

kısaltmaları tanımlanırsa (3.41) ve (3.42) eşitlikleri,

$$\overline{\varphi}_{F,22}^{(m)} - \left[k^2 \left(1 + \eta_1^{(m)} \right) - k_L^{(m)\,2} \right] \overline{\varphi}_F^{(m)} = 0 \qquad m=1,2 \tag{3.45}$$

$$\overline{\psi}_{3F,22}^{(m)} - \left[k^2 \left(1 + \eta_2^{(m)} \right) - k_T^{(m)2} \right] \overline{\psi}_{3F}^{(m)} = 0 \qquad m=1,2 \tag{3.46}$$

şeklini alır. (3.25)-(3.28) ile verilen sınır ve süreklilik koşullarının Fourier dönüşümleri ise,

$$\overline{\sigma}_{12F}\Big|_{x_2=0} = 0, \quad \overline{\sigma}_{22F}\Big|_{x_2=0} = -P_0 \tag{3.47}$$

$$\overline{u}_{iF}, \overline{\sigma}_{ijF}\Big|_{x_2\to\infty} = 0 \quad i,j = 1,2 \tag{3.48}$$

$$\overline{u}_{iF}^{(1)}\Big|_{x_2=h} = \overline{u}_{iF}^{(2)}\Big|_{x_2=h} \quad i=1,2 \tag{3.49}$$

$$\overline{\sigma}_{2iF}^{(1)}\Big|_{x_2=h} = \overline{\sigma}_{2iF}^{(2)}\Big|_{x_2=h} \quad i=1,2 \tag{3.50}$$

şeklindedir. (3.45) ve (3.46)'nin çözümleri,

$$\overline{\varphi}_F^{(m)} = \alpha^{(m)}(k)\, e^{r x_2}, \overline{\psi}_{3F}^{(m)} = \beta^{(m)}(k)\, e^{t x_2} \qquad m=1,2 \tag{3.51}$$

yapısında aranır ve,

$$\upsilon^{(m)} = \left[k^2\left(1+\eta_1^{(m)}\right)-k_L^{(m)2}\right]^{1/2} \qquad m=1,2 \tag{3.52}$$

$$\upsilon'^{(m)} = \left[k^2\left(1+\eta_2^{(m)}\right)-k_T^{(m)2}\right]^{1/2} \qquad m=1,2 \tag{3.53}$$

kısaltmaları yapılırsa,

$$r_{1,2}^{(m)} = \mp\upsilon^{(m)}\,,\; t_{1,2}^{(m)} = \mp\upsilon'^{(m)} \qquad m=1,2 \tag{3.54}$$

bulunur. (3.48) uyarınca $r^{(2)}<0$ ve $t^{(2)}<0$ olmalıdır. Buna göre potansiyeller,

$$\overline{\varphi}_F^{(1)} = \alpha_1(k)e^{-\upsilon^{(1)}x_2} + \alpha_2(k)e^{\upsilon^{(1)}x_2} \tag{3.55}$$

$$\overline{\psi}_{3F}^{(1)} = \beta_1(k)e^{-\upsilon'^{(1)}x_2} + \beta_2(k)e^{\upsilon'^{(1)}x_2} \tag{3.56}$$

ve,

$$\overline{\varphi}_F^{(2)} = \alpha_3(k)e^{-\upsilon^{(2)}x_2} \tag{3.57}$$

$$\overline{\psi}_{3F}^{(2)} = \beta_3(k)e^{-\upsilon'^{(2)}x_2} \tag{3.58}$$

yapısında olmalıdır. $\alpha_i(k)$ ve $\beta_i(k)$ fonksiyonların belirlenmesi için (3.47) ile verilen sınır ve (3.49)-(3.50) ile verilen süreklilik koşulları kullanılır. (3.11) ile verilen lineer kinematik bağıntılar, (3.12) ile verilen bünye denklemleri, (3.29) ve Fourier dönüşümüne ait özellikler kullanılarak, gerilme tansörünün bağımsız bileşenleri yer değiştirme potansiyellerine,

$$\overline{\sigma}_{11F}^{(m)} = \left(\lambda^{(m)}+2\mu^{(m)}\right)\left(-k^2\overline{\varphi}_F^{(m)} - ik\overline{\psi}_{3F,2}^{(m)}\right) + \lambda^{(m)}\left(\overline{\varphi}_{F,22}^{(m)} + ik\overline{\psi}_{3F,2}^{(m)}\right) \qquad m=1,2 \tag{3.59}$$

$$\overline{\sigma}_{22F}^{(m)} = \left(\lambda^{(m)}+2\mu^{(m)}\right)\left(\overline{\varphi}_{F,22}^{(m)} + ik\overline{\psi}_{3F,2}^{(m)}\right) + \lambda^{(m)}\left(-k^2\overline{\varphi}_F^{(m)} - ik\overline{\psi}_{3F,2}^{(m)}\right) \qquad m=1,2 \tag{3.60}$$

$$\overline{\sigma}_{12F}^{(m)} = \mu^{(m)}\left(-2ik\overline{\varphi}_{F,2}^{(m)} + \overline{\psi}_{3F,22}^{(m)} + k^2\overline{\psi}_{3F}^{(m)}\right) \qquad m=1,2 \tag{3.61}$$

eşitlikleri ile bağlanabilir. Ayrıca (3.29) eşitliğine Fourier dönüşümü uygulanarak,

$$\overline{u}_{1F}^{(m)} = -ik\overline{\varphi}_F^{(m)} + \overline{\psi}_{3F,2}^{(m)} \qquad m=1,2 \tag{3.62}$$

$$\overline{u}_{2F}^{(m)} = \overline{\varphi}_{F,2}^{(m)} + ik\overline{\psi}_{3F}^{(m)} \qquad m=1,2 \tag{3.63}$$

bağıntıları yazılabilir. (3.55)-(3.58) eşitlikleri, (3.59)-(3.63) 'de kullanılırsa,

$$\overline{u}_{1F}^{(1)} = -ik(\alpha_1 e^{-\upsilon^{(1)}x_2} + \alpha_2 e^{\upsilon^{(1)}x_2}) - \upsilon'^{(1)}(\beta_1 e^{-\upsilon'^{(1)}x_2} - \beta_2 e^{\upsilon'^{(1)}x_2}) \tag{3.64}$$

$$\overline{u}_{2F}^{(1)} = -\upsilon^{(1)}(\alpha_1 e^{-\upsilon^{(1)}x_2} - \alpha_2 e^{\upsilon^{(1)}x_2}) + ik(\beta_1 e^{-\upsilon'^{(1)}x_2} + \beta_2 e^{\upsilon'^{(1)}x_2}) \tag{3.65}$$

$$\overline{\sigma}_{11F}^{(1)} = \left[-(\lambda^{(1)}+2\mu^{(1)})k^2+\lambda^{(1)}\upsilon^{(1)2}\right](\alpha_1 e^{-\upsilon^{(1)}x_2}+\alpha_2 e^{\upsilon^{(1)}x_2})+2i\mu^{(1)}k\upsilon'^{(1)}(\beta_1 e^{-\upsilon'^{(1)}x_2}-\beta_2 e^{\upsilon'^{(1)}x_2}) \tag{3.66}$$

$$\overline{\sigma}_{22F}^{(1)} = \left[(\lambda^{(1)}+2\mu^{(1)})\upsilon^{(1)2}-\lambda^{(1)}k^2\right](\alpha_1 e^{-\upsilon^{(1)}x_2}+\alpha_2 e^{\upsilon^{(1)}x_2})+2i\mu^{(1)}k\upsilon'^{(1)}(-\beta_1 e^{-\upsilon'^{(1)}x_2}+\beta_2 e^{\upsilon'^{(1)}x_2}) \tag{3.67}$$

$$\overline{\sigma}_{12F}^{(1)} = 2i\mu^{(1)}k\upsilon^{(1)}(\alpha_1 e^{-\upsilon^{(1)}x_2}-\alpha_2 e^{\upsilon^{(1)}x_2})+\mu^{(1)}\left[\left(2+\eta_2^{(1)}\right)k^2-k_T^{(1)2}\right](\beta_1 e^{-\upsilon'^{(1)}x_2}+\beta_2 e^{\upsilon'^{(1)}x_2}) \tag{3.68}$$

$$\overline{u}_{1F}^{(2)} = -ik\alpha_3 e^{-\upsilon^{(2)}x_2}-\upsilon'^{(2)}\beta_3 e^{-\upsilon'^{(2)}x_2} \tag{3.69}$$

$$\overline{u}_{2F}^{(2)} = -\upsilon^{(2)}\alpha_3 e^{-\upsilon^{(2)}x_2}+ik\beta_3 e^{-\upsilon'^{(2)}x_2} \tag{3.70}$$

$$\overline{\sigma}_{11F}^{(2)} = \left[-(\lambda^{(2)}+2\mu^{(2)})k^2+\lambda^{(2)}\upsilon^{(2)2}\right]\alpha_3 e^{-\upsilon^{(2)}x_2}+2i\mu^{(2)}k\upsilon'^{(2)}\beta_3 e^{-\upsilon'^{(2)}x_2} \tag{3.71}$$

$$\overline{\sigma}_{22F}^{(2)} = \left[(\lambda^{(2)}+2\mu^{(2)})\upsilon^{(2)2}-\lambda^{(2)}k^2\right]\alpha_3 e^{-\upsilon^{(2)}x_2}-2i\mu^{(2)}k\upsilon'^{(2)}\beta_3 e^{-\upsilon'^{(2)}x_2} \tag{3.72}$$

$$\overline{\sigma}_{12F}^{(2)} = 2i\mu^{(2)}k\upsilon^{(2)}\alpha_3 e^{-\upsilon^{(2)}x_2}+\mu^{(2)}\left[\left(2+\eta_2^{(2)}\right)k^2-k_T^{(2)2}\right]\beta_3 e^{-\upsilon'^{(2)}x_2} \tag{3.73}$$

eşitlikleri elde edilir. $\alpha_i(k)$, $\beta_i(k)$ $(i=1,3)$ bilinmeyen fonksiyonlarının elde edilebilmesi için χ_i ve H_i katsayıları sırasıyla (E1.1)-(E1.10) ve (E1.11)-(E1.16) 'daki gibi tanımlanarak,

$$\overline{u}_{1F}^{(1)} = -ik(\alpha_1 H_1+\alpha_2 H_2)-\upsilon'^{(1)}(\beta_1 H_3-\beta_2 H_4) \tag{3.74}$$

$$\overline{u}_{2F}^{(1)} = -\upsilon^{(1)}(\alpha_1 H_1-\alpha_2 H_2)+ik(\beta_1 H_3+\beta_2 H_4) \tag{3.75}$$

$$\overline{\sigma}_{11F}^{(1)} = \chi_9(\alpha_1 H_1+\alpha_2 H_2)+i\chi_2(\beta_1 H_3-\beta_2 H_4) \tag{3.76}$$

$$\overline{\sigma}_{22F}^{(1)} = \chi_1(\alpha_1 H_1+\alpha_2 H_2)+i\chi_2(-\beta_1 H_3+\beta_2 H_4) \tag{3.77}$$

$$\overline{\sigma}_{12F}^{(1)} = i\chi_3(\alpha_1 H_1-\alpha_2 H_2)+\chi_4(\beta_1 H_3+\beta_2 H_4) \tag{3.78}$$

$$\overline{u}_{1F}^{(2)} = -ik\alpha_3 H_5-\upsilon'^{(2)}\beta_3 H_6 \tag{3.79}$$

$$\overline{u}_{2F}^{(2)} = -\upsilon^{(2)}\alpha_3 H_5+ik\beta_3 H_6 \tag{3.80}$$

$$\overline{\sigma}_{11F}^{(2)} = \chi_{10}\alpha_3 H_5+i\chi_6\beta_3 H_6 \tag{3.81}$$

$$\overline{\sigma}_{22F}^{(2)} = \chi_5\alpha_3 H_5-i\chi_6\beta_3 H_6 \tag{3.82}$$

$$\overline{\sigma}_{12F}^{(2)} = i\chi_7\alpha_3 H_5+\chi_8\beta_3 H_6 \tag{3.83}$$

bulunur, ve (E1.17)-(E1.22) ile tanımlanan $\overline{H}_i$ katsayıları, (3.47) ile verilen sınır ve (3.49)-(3.50) ile verilen süreklilik koşulları kullanılırsa,

$$\begin{bmatrix} \chi_1 & \chi_1 & -i\chi_2 & i\chi_2 & 0 & 0 \\ i\chi_3 & -i\chi_3 & \chi_4 & \chi_4 & 0 & 0 \\ -ik\overline{H}_1 & -ik\overline{H}_2 & -\upsilon'^{(1)}\overline{H}_3 & \upsilon'^{(1)}\overline{H}_4 & ik\overline{H}_5 & \upsilon'^{(2)}\overline{H}_6 \\ -\upsilon^{(1)}\overline{H}_1 & \upsilon^{(1)}\overline{H}_2 & ik\overline{H}_3 & ik\overline{H}_4 & \upsilon^{(2)}\overline{H}_5 & -ik\overline{H}_6 \\ \chi_1\overline{H}_1 & \chi_1\overline{H}_2 & -i\chi_2\overline{H}_3 & i\chi_2\overline{H}_4 & -\chi_5\overline{H}_5 & i\chi_6\overline{H}_6 \\ i\chi_3\overline{H}_1 & -i\chi_3\overline{H}_2 & \chi_4\overline{H}_3 & \chi_4\overline{H}_4 & -i\chi_7\overline{H}_5 & -\chi_8\overline{H}_6 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta_2 \\ \alpha_3 \\ \beta_3 \end{Bmatrix} = \begin{Bmatrix} -P_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{3.84}$$

yapısında, katsayılar matrisinin elemanları k'nın fonksiyonları olan lineer cebirsel bir denklem sistemi elde edilir. (E1.23)-(E1.36) ile tanımlanan Γ_i sayıları yardımı ile (3.84) 'e ait katsayılar matrisinin determinantı,

$$\begin{aligned} \Delta = \overline{H}_5\overline{H}_6\big[&(\chi_2\chi_3+\chi_1\chi_4)(\overline{H}_2\overline{H}_3\Gamma_1+\overline{H}_1\overline{H}_4\Gamma_4)+(\chi_2\chi_3-\chi_1\chi_4)(\overline{H}_1\overline{H}_3\Gamma_5+\overline{H}_2\overline{H}_4\Gamma_3) \\ &+2(\chi_1\chi_3\overline{H}_3\overline{H}_4\Gamma_2+\chi_2\chi_4\overline{H}_1\overline{H}_2\Gamma_6)\big] \end{aligned} \tag{3.85}$$

olarak elde edilir. (3.84) denklem sisteminin çözümü için Δ_i (i=1,6) sayıları (E1.37)-(E1.42) de verildiği üzere hesaplanarak bilinmeyen α_i, β_i katsayıları (E1.43)-(E1.48) deki gibi elde edilir. Burada $\overline{\alpha}_i$, $\overline{\beta}_i$ sayıları (E1.49)-(E1.54) 'deki gibi tanımlanırsa sonuç olarak,

$$\alpha_1 = -P_0\overline{\alpha}_1 \tag{3.86}$$

$$\alpha_2 = P_0\overline{\alpha}_2 \tag{3.87}$$

$$\beta_1 = -iP_0\overline{\beta}_1 \tag{3.88}$$

$$\beta_2 = iP_0\overline{\beta}_2 \tag{3.89}$$

$$\alpha_3 = P_0\overline{\alpha}_3 \tag{3.90}$$

$$\beta_3 = iP_0\overline{\beta}_3 \tag{3.91}$$

bulunur. (3.86)-(3.91) uyarınca (3.74)-(3.83) eşitlikleri,

$$\overline{u}^{(1)}_{1F} = iP_0\left[k(\overline{\alpha}_1H_1-\overline{\alpha}_2H_2)+\upsilon'^{(1)}(\overline{\beta}_1H_3+\overline{\beta}_2H_4)\right] \tag{3.92}$$

$$\overline{u}^{(1)}_{2F} = P_0\left[\upsilon^{(1)}(\overline{\alpha}_1H_1+\overline{\alpha}_2H_2)+k(\overline{\beta}_1H_3-\overline{\beta}_2H_4)\right] \tag{3.93}$$

$$\overline{\sigma}^{(1)}_{11F} = P_0\left[\chi_9(-\overline{\alpha}_1H_1+\overline{\alpha}_2H_2)+\chi_2(\overline{\beta}_1H_3+\overline{\beta}_2H_4)\right] \tag{3.94}$$

$$\overline{\sigma}^{(1)}_{22F} = P_0\left[\chi_1(-\overline{\alpha}_1H_1+\overline{\alpha}_2H_2)-\chi_2(\overline{\beta}_1H_3+\overline{\beta}_2H_4)\right] \tag{3.95}$$

$$\overline{\sigma}^{(1)}_{12F} = iP_0\left[-\chi_3(\overline{\alpha}_1H_1+\overline{\alpha}_2H_2)+\chi_4(-\overline{\beta}_1H_3+\overline{\beta}_2H_4)\right] \tag{3.96}$$

$$\overline{u}^{(2)}_{1F} = -iP_0\left(k\overline{\alpha}_3H_5+\upsilon'^{(2)}\overline{\beta}_3H_6\right) \tag{3.97}$$

$$\overline{u}^{(2)}_{2F} = -P_0\left(\upsilon^{(2)}\overline{\alpha}_3H_5+k\overline{\beta}_3H_6\right) \tag{3.98}$$

$$\overline{\sigma}_{11F}^{(2)}=P_0\left(\chi_{10}\overline{\alpha}_3H_5-\chi_6\overline{\beta}_3H_6\right) \tag{3.99}$$

$$\overline{\sigma}_{22F}^{(2)}=P_0\left(\chi_5\overline{\alpha}_3H_5+\chi_6\overline{\beta}_3H_6\right) \tag{3.100}$$

$$\overline{\sigma}_{12F}^{(2)}=iP_0\left(\chi_7\overline{\alpha}_3H_5+\chi_8\overline{\beta}_3H_6\right) \tag{3.101}$$

şeklinde yazılabilir. (3.52)-(3.53), (E1.1)-(E1.10) ve (E1.17)-(E1.22) eşitlikleri yardımıyla, (E1.23)-(E1.36) ile verilen Γ_1, Γ_3, Γ_4, Γ_5, Γ_9, Γ_{10}, Γ_{11}, Γ_{12} ifadelerinin k'ya göre çift, Γ_2,Γ_6, Γ_7, Γ_8, Γ_{13}, Γ_{14} ifadelerinin ise tek fonksiyonlar olduğu ve buna göre, (E1.49)-(E1.54) ile tanımlanan $\overline{\alpha}_i$ ifadelerinin çift ve $\overline{\beta}_i$ ifadelerinin de tek fonksiyonlar oldukları görülür. Bu durumda (3.93), (3.94), (3.95), (3.98), (3.99) ve (3.100) ile verilen ifadeler çift, (3.92), (3.96), (3.97) ve (3.101) ile verilen ifadeler de tek fonksiyonlar olmaktadır. $\overline{u}_i^{(m)}(x_1,x_2),\overline{\sigma}_{ij}^{(m)}(x_1,x_2)$ alanları; $\overline{u}_{iF}^{(m)}(k,x_2),\overline{\sigma}_{ijF}^{(m)}(k,x_2)$ alanlarından,

$$f(x_1,x_2)=\frac{1}{2\pi}\int_{-\infty}^{\infty}f_F(k,x_2)e^{-ikx_1}dk \tag{3.102}$$

yapısındaki ters dönüşüm ile elde edilecektir. (3.92)-(3.101) ; (3.102) 'de kullanılırsa,

$$\overline{u}_1^{(1)}=i\frac{P_0}{2\pi}\int_{-\infty}^{\infty}\left[k(\overline{\alpha}_1H_1-\overline{\alpha}_2H_2)+\upsilon'^{(1)}(\overline{\beta}_1H_3+\overline{\beta}_2H_4)\right]e^{-ikx_1}dk \tag{3.103}$$

$$\overline{u}_2^{(1)}=\frac{P_0}{2\pi}\int_{-\infty}^{\infty}\left[\upsilon^{(1)}(\overline{\alpha}_1H_1+\overline{\alpha}_2H_2)+k(\overline{\beta}_1H_3-\overline{\beta}_2H_4)\right]e^{-ikx_1}dk \tag{3.104}$$

$$\overline{\sigma}_{11}^{(1)}=\frac{P_0}{2\pi}\int_{-\infty}^{\infty}\left[\chi_9(-\overline{\alpha}_1H_1+\overline{\alpha}_2H_2)+\chi_2(\overline{\beta}_1H_3+\overline{\beta}_2H_4)\right]e^{-ikx_1}dk \tag{3.105}$$

$$\overline{\sigma}_{22}^{(1)}=\frac{P_0}{2\pi}\int_{-\infty}^{\infty}\left[\chi_1(-\overline{\alpha}_1H_1+\overline{\alpha}_2H_2)-\chi_2(\overline{\beta}_1H_3+\overline{\beta}_2H_4)\right]e^{-ikx_1}dk \tag{3.106}$$

$$\overline{\sigma}_{12}^{(1)}=i\frac{P_0}{2\pi}\int_{-\infty}^{\infty}\left[-\chi_3(\overline{\alpha}_1H_1+\overline{\alpha}_2H_2)+\chi_4(-\overline{\beta}_1H_3+\overline{\beta}_2H_4)\right]e^{-ikx_1}dk \tag{3.107}$$

$$\overline{u}_1^{(2)}=-i\frac{P_0}{2\pi}\int_{-\infty}^{\infty}\left(k\overline{\alpha}_3H_5+\upsilon'^{(2)}\overline{\beta}_3H_6\right)e^{-ikx_1}dk \tag{3.108}$$

$$\overline{u}_2^{(2)}=-\frac{P_0}{2\pi}\int_{-\infty}^{\infty}\left(\upsilon^{(2)}\overline{\alpha}_3H_5+k\overline{\beta}_3H_6\right)e^{-ikx_1}dk \tag{3.109}$$

$$\overline{\sigma}_{11}^{(2)}=\frac{P_0}{2\pi}\int_{-\infty}^{\infty}\left(\chi_{10}\overline{\alpha}_3H_5-\chi_6\overline{\beta}_3H_6\right)e^{-ikx_1}dk \tag{3.110}$$

$$\overline{\sigma}_{22}^{(2)} = \frac{P_0}{2\pi}\int_{-\infty}^{\infty}\left(\chi_5\overline{\alpha}_3 H_5 + \chi_6\overline{\beta}_3 H_6\right)e^{-ikx_1}dk \tag{3.111}$$

$$\overline{\sigma}_{12}^{(2)} = i\frac{P_0}{2\pi}\int_{-\infty}^{\infty}\left(\chi_7\overline{\alpha}_3 H_5 + \chi_8\overline{\beta}_3 H_6\right)e^{-ikx_1}dk \tag{3.112}$$

eşitlikleri bulunur. (3.103)-(3.112) eşitliklerinin sanal parçaları atıldıktan sonra integrandların hepsinin integrasyon parametresi k'ya göre çift fonksiyonlar olduğu görülür. Buna göre,

$$\overline{u}_1^{(1)} = \frac{P_0}{\pi}\int_0^{\infty}\left[k(\overline{\alpha}_1 H_1 - \overline{\alpha}_2 H_2) + \upsilon'^{(1)}(\overline{\beta}_1 H_3 + \overline{\beta}_2 H_4)\right]\sin(kx_1)dk \tag{3.113}$$

$$\overline{u}_2^{(1)} = \frac{P_0}{\pi}\int_0^{\infty}\left[\upsilon^{(1)}(\overline{\alpha}_1 H_1 + \overline{\alpha}_2 H_2) + k(\overline{\beta}_1 H_3 - \overline{\beta}_2 H_4)\right]\cos(kx_1)dk \tag{3.114}$$

$$\overline{\sigma}_{11}^{(1)} = \frac{P_0}{\pi}\int_0^{\infty}\left[\chi_9(-\overline{\alpha}_1 H_1 + \overline{\alpha}_2 H_2) + \chi_2(\overline{\beta}_1 H_3 + \overline{\beta}_2 H_4)\right]\cos(kx_1)dk \tag{3.115}$$

$$\overline{\sigma}_{22}^{(1)} = \frac{P_0}{\pi}\int_0^{\infty}\left[\chi_1(-\overline{\alpha}_1 H_1 + \overline{\alpha}_2 H_2) - \chi_2(\overline{\beta}_1 H_3 + \overline{\beta}_2 H_4)\right]\cos(kx_1)dk \tag{3.116}$$

$$\overline{\sigma}_{12}^{(1)} = \frac{P_0}{\pi}\int_0^{\infty}\left[-\chi_3(\overline{\alpha}_1 H_1 + \overline{\alpha}_2 H_2) + \chi_4(-\overline{\beta}_1 H_3 + \overline{\beta}_2 H_4)\right]\sin(kx_1)dk \tag{3.117}$$

$$\overline{u}_1^{(2)} = -\frac{P_0}{\pi}\int_0^{\infty}\left(k\overline{\alpha}_3 H_5 + \upsilon'^{(2)}\overline{\beta}_3 H_6\right)\sin(kx_1)dk \tag{3.118}$$

$$\overline{u}_2^{(2)} = -\frac{P_0}{\pi}\int_0^{\infty}\left(\upsilon^{(2)}\overline{\alpha}_3 H_5 + k\overline{\beta}_3 H_6\right)\cos(kx_1)dk \tag{3.119}$$

$$\overline{\sigma}_{11}^{(2)} = \frac{P_0}{\pi}\int_0^{\infty}\left(\chi_{10}\overline{\alpha}_3 H_5 - \chi_6\overline{\beta}_3 H_6\right)\cos(kx_1)dk \tag{3.120}$$

$$\overline{\sigma}_{22}^{(2)} = \frac{P_0}{\pi}\int_0^{\infty}\left(\chi_5\overline{\alpha}_3 H_5 + \chi_6\overline{\beta}_3 H_6\right)\cos(kx_1)dk \tag{3.121}$$

$$\overline{\sigma}_{12}^{(2)} = \frac{P_0}{\pi}\int_0^{\infty}\left(\chi_7\overline{\alpha}_3 H_5 + \chi_8\overline{\beta}_3 H_6\right)\sin(kx_1)dk \tag{3.122}$$

eşitlikleri bulunur. Görüldüğü gibi gerilme ve yerdeğiştirmelerin elde edilebilmesi için hepsi $\upsilon^{(m)}$ ve $\upsilon'^{(m)}$ m=1,2 ifadelerine bağımlı $\chi_i, \overline{\alpha}_i, \overline{\beta}_i$, ve H_i değişkenlerinden oluşan ifadelerin $\{0,\infty\}$ aralığında integrali gerekmektedir. Buna göre (3.113)-(3.122) deki integrandların k'nın her değeri için gerçel değer vermediği anlaşılır. Homogen yarım düzlem durumunda bu sorunu ortadan kaldırmak için integrasyon aralığı integrandların gerçel değer verecek yapılara dönüştürüldüğü uygun parçalara bölünmüş ve sonuç bu integrallerin toplamı olarak bulunmuştu. Tabakalı yarım düzlem durumunda ise fiziksel olarak şu durumlar oluşabilir;

a. Tabaka yarım düzlemden daha serttir ve dalga hızları yüksek olacağı için $k_L^{(1)}/\sqrt{1+\eta_1^{(1)}}$ ve $k_T^{(1)}/\sqrt{1+\eta_2^{(1)}}$ sayı çifti, $k_L^{(2)}/\sqrt{1+\eta_1^{(2)}}$ ve $k_T^{(2)}/\sqrt{1+\eta_2^{(2)}}$ sayı çiftinden daha küçüktür. Bu durumda $k_L^{(1)}/\sqrt{1+\eta_1^{(1)}} < k_T^{(1)}/\sqrt{1+\eta_2^{(1)}} < k_L^{(2)}/\sqrt{1+\eta_1^{(2)}} < k_T^{(2)}/\sqrt{1+\eta_2^{(2)}}$ olmaktadır.

b. Yarım düzlem tabakadan daha serttir ve $k_L^{(2)}/\sqrt{1+\eta_1^{(2)}}$ ve $k_T^{(2)}/\sqrt{1+\eta_2^{(2)}}$ sayı çifti, $k_L^{(1)}/\sqrt{1+\eta_1^{(1)}}$ ve $k_T^{(1)}/\sqrt{1+\eta_2^{(1)}}$ sayı çiftinden daha küçüktür. Bu durumda $k_L^{(2)}/\sqrt{1+\eta_1^{(2)}} < k_T^{(2)}/\sqrt{1+\eta_2^{(2)}} < k_L^{(1)}/\sqrt{1+\eta_1^{(1)}} < k_T^{(1)}/\sqrt{1+\eta_2^{(1)}}$ olmaktadır.

c. Tabaka, yarım düzleme göre sadece az bir miktar daha serttir ve bu durumda $k_L^{(1)}/\sqrt{1+\eta_1^{(1)}} < k_L^{(2)}/\sqrt{1+\eta_1^{(2)}} < k_T^{(1)}/\sqrt{1+\eta_2^{(1)}} < k_T^{(2)}/\sqrt{1+\eta_2^{(2)}}$ olmaktadır.

d. Yarım düzlem, tabakaya göre sadece az bir miktar daha serttir ve bu durumda $k_L^{(2)}/\sqrt{1+\eta_1^{(2)}} < k_L^{(1)}/\sqrt{1+\eta_1^{(1)}} < k_T^{(2)}/\sqrt{1+\eta_2^{(2)}} < k_T^{(1)}/\sqrt{1+\eta_2^{(1)}}$ olmaktadır.

Burada mühendislik uygulamaları açısından daha çok önem kazanan (a) ve (c) durumlarının varlığı kabul edilmiştir. Diğer iki durumda da hesap yöntemini değişmeyeceği açıktır. Homogen yarım düzlem durumunda ayrıca $k_T<\infty$ aralığının $k=k_R$ noktasında integrand sonsuz değer almakta ve integralin bu aralıkta Cauchy anlamında değeri hesaplanmaktaydı. Tabakalı yarım düzlem durumunda ise durum bu kadar basit olmamaktadır. Çünkü tekillik yaratan k_R gibi bir sayının hesaplanması homogen yarım düzlem durumunda olduğu kadar kolay değildir. Homogen yarım düzlem durumunda k_R; yükün frekansının Rayleigh dalgalarının hızına bölünmesi ile elde edilmekte idi. Rayleigh dalgalarının yayılma hızı ise gerek analitik yoldan elde edilebilen çok yaklaşık bir formül ile, gerekse bu çalışmada yapılan sayısal uygulamalarda olduğu gibi sayısal olarak istenilen yaklaşıklıkla hesaplanabilir. Tabakalı yarım düzlem durumunda tekillik doğurabilecek neden tabaka ve yarım düzlem sınırında oluşabilen Stoneley dalgalarının varlığı ile ilgilidir. Bilindiği gibi bu dalgalar iki ortamın mekanik sabitlerinin orantısının bazı değerleri için oluşabilmektedir (Guz, 1986). Bu değerler dışında Stoneley dalgaları oluşmadığından bu integrallerin hesaplanmasında tekilliklere rastlanmamaktadır. (3.103)-(3.112) eşitliklerinden yararlanarak,

$$ {}^{i}\Theta^{(1)}(k)=\left[k\,({}^{i}\overline{\alpha}_1\;{}^{i}H_1-{}^{i}\overline{\alpha}_2\;{}^{i}H_2)+{}^{i}\upsilon'^{(1)}({}^{i}\overline{\beta}_1\;{}^{i}H_3+{}^{i}\overline{\beta}_2\;{}^{i}H_4)\right]e^{-ikx_1} \tag{3.123} $$

$$ {}^{i}\Omega^{(1)}(k)=\left[{}^{i}\upsilon^{(1)}({}^{i}\overline{\alpha}_1\;{}^{i}H_1+{}^{i}\overline{\alpha}_2\;{}^{i}H_2)+k({}^{i}\overline{\beta}_1\;{}^{i}H_3-{}^{i}\overline{\beta}_2\;{}^{i}H_4)\right]e^{-ikx_1} \tag{3.124} $$

$$ {}^{i}A^{(1)}(k)=\left[{}^{i}\chi_9(-{}^{i}\overline{\alpha}_1\;{}^{i}H_1+{}^{i}\overline{\alpha}_2\;{}^{i}H_2)+{}^{i}\chi_2({}^{i}\overline{\beta}_1\;{}^{i}H_3+{}^{i}\overline{\beta}_2\;{}^{i}H_4)\right]e^{-ikx_1} \tag{3.125} $$

$$ {}^{i}B^{(1)}(k)=\left[{}^{i}\chi_1(-{}^{i}\overline{\alpha}_1\ {}^{i}H_1+{}^{i}\overline{\alpha}_2\ {}^{i}H_2)-{}^{i}\chi_2({}^{i}\overline{\beta}_1\ {}^{i}H_3+{}^{i}\overline{\beta}_2\ {}^{i}H_4)\right]e^{-ikx_1} \tag{3.126}$$

$$ {}^{i}C^{(1)}(k)=\left[-{}^{i}\chi_3({}^{i}\overline{\alpha}_1\ {}^{i}H_1+{}^{i}\overline{\alpha}_2\ {}^{i}H_2)+{}^{i}\chi_4(-{}^{i}\overline{\beta}_1\ {}^{i}H_3+{}^{i}\overline{\beta}_2\ {}^{i}H_4)\right]e^{-ikx_1} \tag{3.127}$$

$$ {}^{i}\Theta^{(2)}(k)=-\left(k\ {}^{i}\overline{\alpha}_3\ {}^{i}H_5+{}^{i}\upsilon'^{(2)}\ {}^{i}\overline{\beta}_3\ {}^{i}H_6\right)e^{-ikx_1} \tag{3.128}$$

$$ {}^{i}\Omega^{(2)}(k)=-\left({}^{i}\upsilon^{(2)}\ {}^{i}\overline{\alpha}_3\ {}^{i}H_5+k\ {}^{i}\overline{\beta}_3\ {}^{i}H_6\right)e^{-ikx_1} \tag{3.129}$$

$$ {}^{i}A^{(2)}(k)=\left({}^{i}\chi_{10}\ {}^{i}\overline{\alpha}_3\ {}^{i}H_5-{}^{i}\chi_6\ {}^{i}\overline{\beta}_3\ {}^{i}H_6\right)e^{-ikx_1} \tag{3.130}$$

$$ {}^{i}B^{(2)}(k)=\left({}^{i}\chi_5\ {}^{i}\overline{\alpha}_3\ {}^{i}H_5+{}^{i}\chi_6\ {}^{i}\overline{\beta}_3\ {}^{i}H_6\right)e^{-ikx_1} \tag{3.131}$$

$$ {}^{i}C^{(2)}(k)=\left({}^{i}\chi_7\ {}^{i}\overline{\alpha}_3\ {}^{i}H_5+{}^{i}\chi_8\ {}^{i}\overline{\beta}_3\ {}^{i}H_6\right)e^{-ikx_1} \tag{3.132}$$

kısaltmaları yapılırsa,

$$ \overline{u}_1^{(1)}=i\frac{P_0}{2\pi}\int_{-\infty}^{\infty}\Theta^{(1)}(k)\,dk \tag{3.133}$$

$$ \overline{u}_2^{(1)}=\frac{P_0}{2\pi}\int_{-\infty}^{\infty}\Omega^{(1)}(k)\,dk \tag{3.134}$$

$$ \overline{\sigma}_{11}^{(1)}=\frac{P_0}{2\pi}\int_{-\infty}^{\infty}A^{(1)}(k)\,dk \tag{3.135}$$

$$ \overline{\sigma}_{22}^{(1)}=\frac{P_0}{2\pi}\int_{-\infty}^{\infty}B^{(1)}(k)\,dk \tag{3.136}$$

$$ \overline{\sigma}_{12}^{(1)}=i\frac{P_0}{2\pi}\int_{-\infty}^{\infty}C^{(1)}(k)\,dk \tag{3.137}$$

$$ \overline{u}_1^{(2)}=i\frac{P_0}{2\pi}\int_{-\infty}^{\infty}\Theta^{(2)}(k)\,dk \tag{3.138}$$

$$ \overline{u}_2^{(2)}=\frac{P_0}{2\pi}\int_{-\infty}^{\infty}\Omega^{(2)}(k)\,dk \tag{3.139}$$

$$ \overline{\sigma}_{11}^{(2)}=\frac{P_0}{2\pi}\int_{-\infty}^{\infty}A^{(2)}(k)\,dk \tag{3.140}$$

$$ \overline{\sigma}_{22}^{(2)}=\frac{P_0}{2\pi}\int_{-\infty}^{\infty}B^{(2)}(k)\,dk \tag{3.141}$$

$$ \overline{\sigma}_{12}^{(2)}=i\frac{P_0}{2\pi}\int_{-\infty}^{\infty}C^{(2)}(k)\,dk \tag{3.142}$$

elde edilir. (a) durumu için,

$0<k<k_L^{(1)}/\sqrt{1+\eta_1^{(1)}}$, $k_L^{(1)}/\sqrt{1+\eta_1^{(1)}}<k<k_T^{(1)}/\sqrt{1+\eta_2^{(1)}}$, $k_T^{(1)}/\sqrt{1+\eta_2^{(1)}}<k<k_L^{(2)}/\sqrt{1+\eta_1^{(2)}}$,

$k_L^{(2)}/\sqrt{1+\eta_1^{(2)}}<k<k_T^{(2)}/\sqrt{1+\eta_2^{(2)}}$, ve $k_T^{(2)}/\sqrt{1+\eta_2^{(2)}}<k$ aralıklarının (c) durumu için de,

$0<k<k_L^{(1)}/\sqrt{1+\eta_1^{(1)}}$, $k_L^{(1)}/\sqrt{1+\eta_1^{(1)}}<k<k_L^{(2)}/\sqrt{1+\eta_1^{(2)}}$, $k_L^{(2)}/\sqrt{1+\eta_1^{(2)}}<k<k_T^{(1)}/\sqrt{1+\eta_2^{(1)}}$,

$k_T^{(1)}/\sqrt{1+\eta_2^{(1)}}<k<k_T^{(2)}/\sqrt{1+\eta_2^{(2)}}$, ve $k_T^{(2)}/\sqrt{1+\eta_2^{(2)}}<k$ aralıklarının her birisinde bu ${}^i\Theta^{(m)}(k)$, ${}^i\Omega^{(m)}(k)$, ${}^iA^{(m)}(k)$, ${}^iB^{(m)}(k)$ ve ${}^iC^{(m)}(k)$,i=1,5, m=1,2 fonksiyonlarının yapısı belirlenmelidir. Bu işlemler Ek2-Ek.7 'de yapılmış ve sonuçlar verilmiştir. Buna göre (a) durumu için,

$$\bar{u}_i^{(m)}={}^1\bar{u}_i^{(m)}+{}^2\bar{u}_i^{(m)}+{}^6\bar{u}_i^{(m)}+{}^4\bar{u}_i^{(m)}+{}^5\bar{u}_i^{(m)} \tag{3.143}$$

$$\bar{\sigma}_{ij}^{(m)}={}^1\sigma_{ij}^{(m)}+{}^2\sigma_{ij}^{(m)}+{}^6\sigma_{ij}^{(m)}+{}^4\sigma_{ij}^{(m)}+{}^5\sigma_{ij}^{(m)} \tag{3.144}$$

ve (c) durumu için de,

$$\bar{u}_i^{(m)}={}^1\bar{u}_i^{(m)}+{}^2\bar{u}_i^{(m)}+{}^3\bar{u}_i^{(m)}+{}^4\bar{u}_i^{(m)}+{}^5\bar{u}_i^{(m)} \tag{3.145}$$

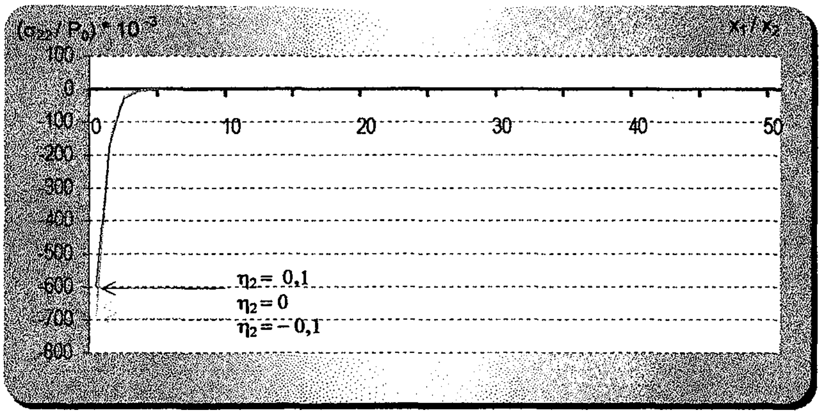
$$\bar{\sigma}_{ij}^{(m)}={}^1\sigma_{ij}^{(m)}+{}^2\sigma_{ij}^{(m)}+{}^3\sigma_{ij}^{(m)}+{}^4\sigma_{ij}^{(m)}+{}^5\sigma_{ij}^{(m)} \tag{3.146}$$

eşitlikleri yazılır. $\{{}^1\bar{u}_i^{(m)},\ {}^1\bar{\sigma}_{ij}^{(m)}\}\ m=1,2$ ifadeleri Ek.2 'de, $\{{}^2\bar{u}_i^{(m)},\ {}^2\bar{\sigma}_{ij}^{(m)}\}\ m=1,2$ ifadeleri Ek.3 'de, $\{{}^3\bar{u}_i^{(m)},\ {}^3\bar{\sigma}_{ij}^{(m)}\}\ m=1,2$ ifadeleri Ek.4 'de, $\{{}^4\bar{u}_i^{(m)},\ {}^4\bar{\sigma}_{ij}^{(m)}\}\ m=1,2$ ifadeleri Ek.5 'de, $\{{}^5\bar{u}_i^{(m)},\ {}^5\bar{\sigma}_{ij}^{(m)}\}\ m=1,2$ ifadeleri Ek.6'da ve $\{{}^6\bar{u}_i^{(m)},\ {}^6\bar{\sigma}_{ij}^{(m)}\}\ m=1,2$ ifadeleri Ek.7'de verilmiştir

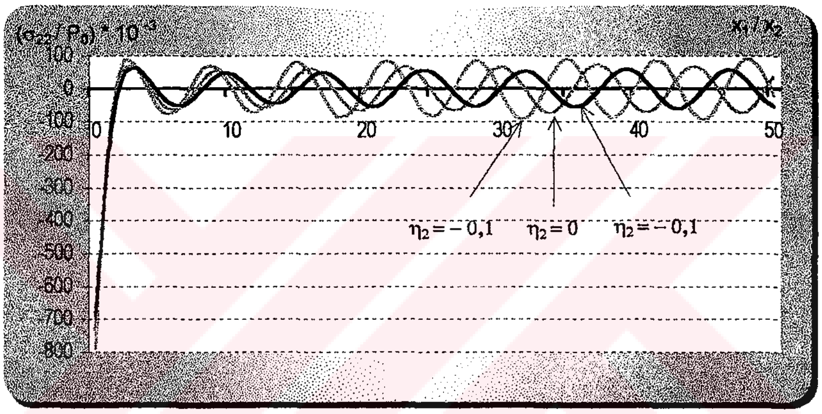
3.3. Sayısal İncelemeler.

3.3.1. Öngerilmeli homogen yarım düzlem özel durumuna ait sayısal incelemeler.

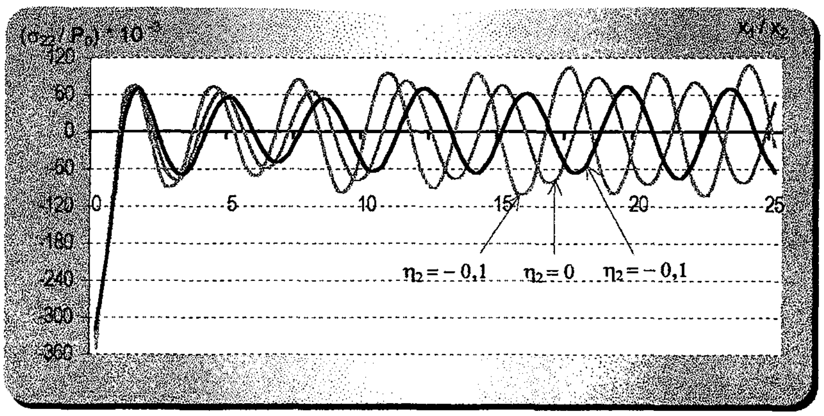
Bu kısımda homogen yarım düzlem durumunda oluşan gerilme dağılımlarına basınç ve çekme öngerilmelerinin etkileri incelenmiştir. Malzeme bir önceki bölümdeki sayısal incelemelerdeki gibi alınmıştır. Ω nın, statik duruma yaklaşık olarak eşdeğer olan 10 ve 250-25.000 arasında olmak üzere toplam 15 ayrı değeri için çizilen boyutsuz diagramlar Şekil 3.2-Şekil 3.16 da verilmiştir. Bir önceki bölümde anlatıldığı gibi bu boyutsuz diagramlar kullanılarak Ω nın diğer değerleri için de gerilmelerin bulunması mümkündür. Öngerilmeli yarım düzlem için stasyoner olmayan Lamb problemi Koshman (1981b)'de incelenmiştir. Ancak bu çalışmada bazı basit durumlara ait analitik ifadeler dışında önemli sayısal sonuçlar bulunmamaktadır. Diğer taraftan bu makalede verilen sonuçlar tezde ele alınan dış etkinin tipine uymamaktadır. Bu nedenle bu alt bölümde ayrıntılı bir inceleme yapılmıştır. Buradan elde edilecek sonuçlar üzerinde öngerilmeli bir tabakası bulunan yarım düzlemin sayısal sonuçlarının yorumlanmasında da yol gösterici olacaktır. Bu alt bölümde verilen diagramların yorumlanması sonuçlar bölümünde yapılmıştır. Hesaplarda öngerilmenin şiddeti η=0,10 olarak verilmiştir. η katsayısı ise s_0/n olarak tanımlanmaktadır. Poisson oranı 0,25 olarak alınmış olduğuna göre, öngerilmenin şiddetinin kayma modülünün %10 u kadar alınması, elastisite modülünün %4 ü kadar alınmasına karşı gelmektedir.



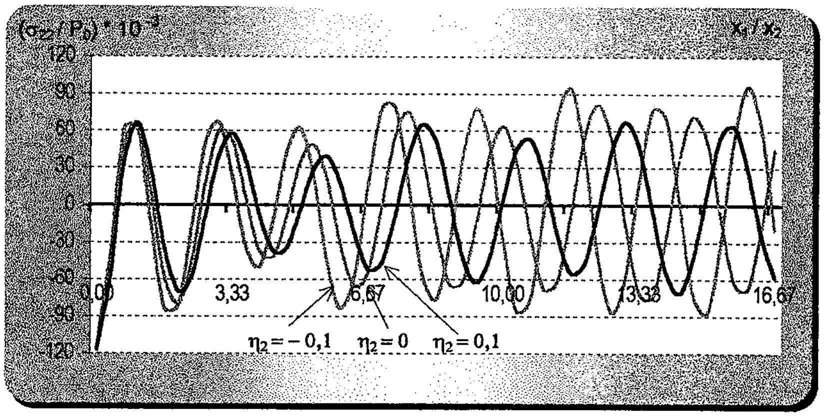
Şekil 3.2 Homogen yarım düzlemde $\Omega=10^{m/sn}$ için σ_{22} dağılımına farklı işaretteki öngerilme durumlarının etkisi.



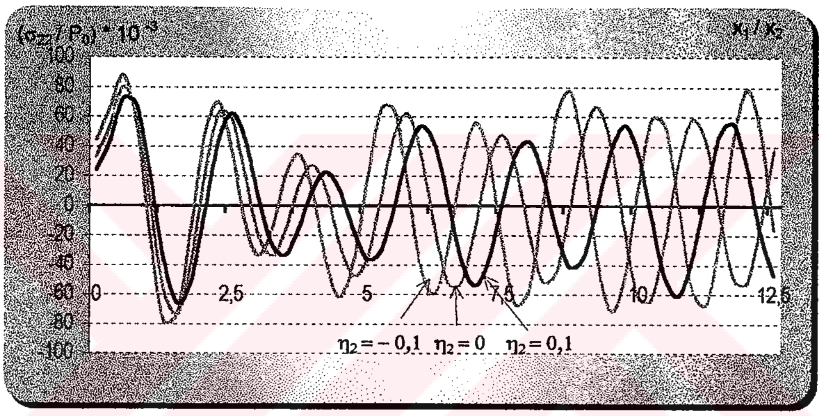
Şekil 3.3 Homogen yarım düzlemde $\Omega=250^{m/sn}$ için σ_{22} dağılımına farklı işaretteki öngerilme durumlarının etkisi



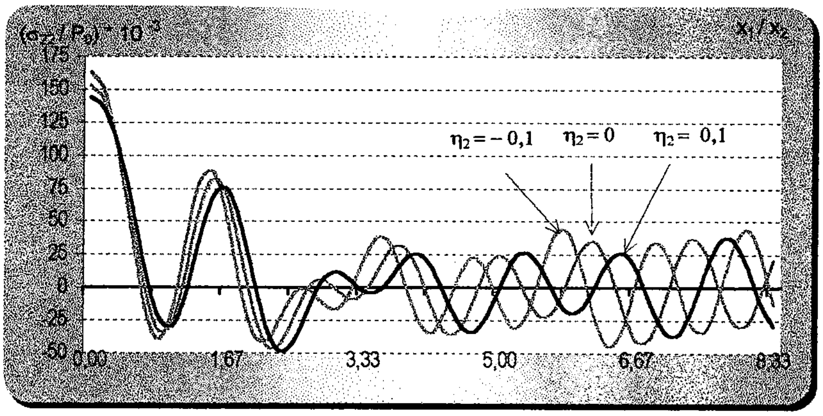
Şekil 3.4 Homogen yarım düzlemde $\Omega=500^{m/sn}$ için σ_{22} dağılımına farklı işaretteki öngerilme durumlarının etkisi



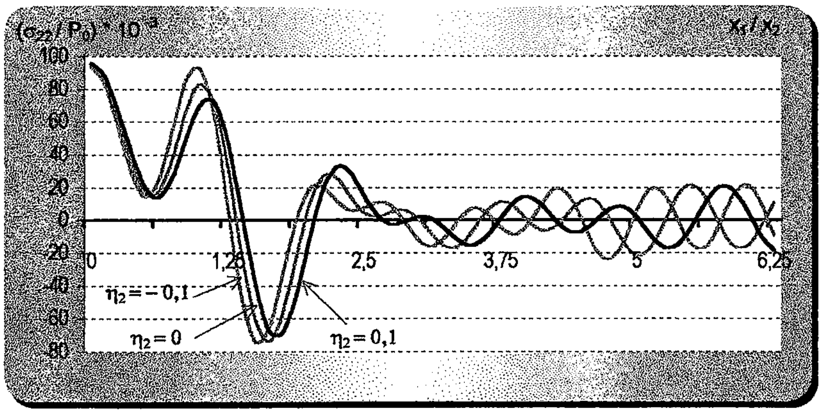
Şekil 3.5 Homogen yarım düzlemde Ω=750 $^{m/sn}$ için σ_{22} dağılımına farklı işaretteki öngerilme durumlarının etkisi



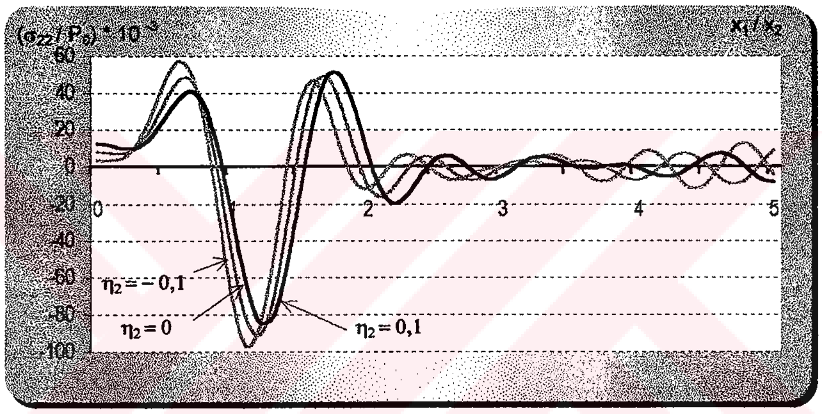
Şekil 3.6 Homogen yarım düzlemde Ω=1000 $^{m/sn}$ için σ_{22} dağılımına farklı işaretteki öngerilme durumlarının etkisi



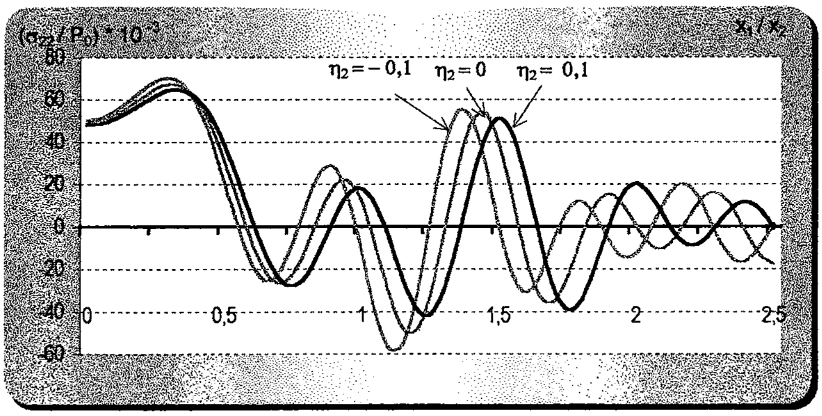
Şekil 3.7 Homogen yarım düzlemde Ω=1500 $^{m/sn}$ için σ_{22} dağılımına farklı işaretteki öngerilme durumlarının etkisi



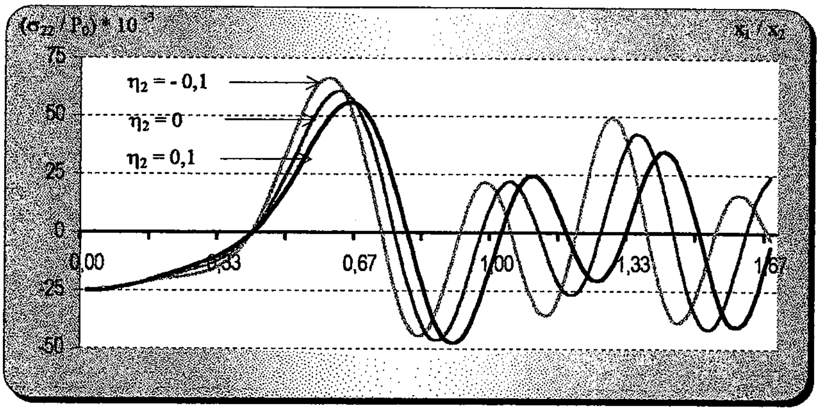
Şekil 3.8 Homogen yarım düzlemde Ω=2000 $^{m/sn}$ için σ_{22} dağılımına farklı işaretteki öngerilme durumlarının etkisi



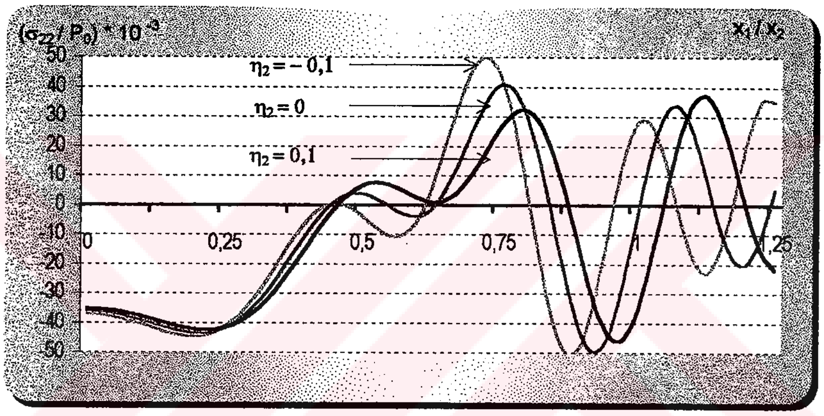
Şekil 3.9 Homogen yarım düzlemde Ω=2500 $^{m/sn}$ için σ_{22} dağılımına farklı işaretteki öngerilme durumlarının etkisi



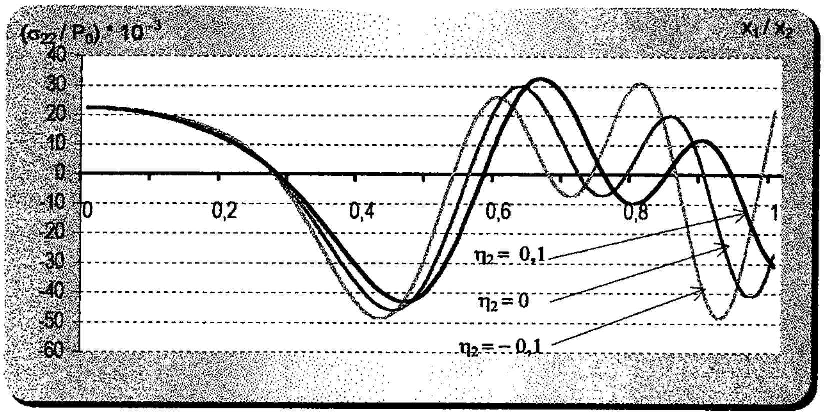
Şekil 3.10 Homogen yarım düzlemde Ω=5000 $^{m/sn}$ için σ_{22} dağılımına farklı işaretteki öngerilme durumlarının etkisi



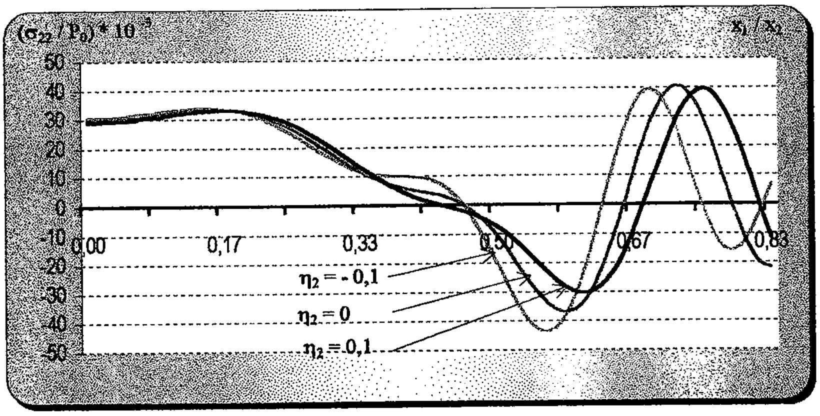
Şekil 3.11 Homogen yarım düzlemde Ω=7500 $^{m/sn}$ için σ_{22} dağılımına farklı işaretteki öngerilme durumlarının etkisi



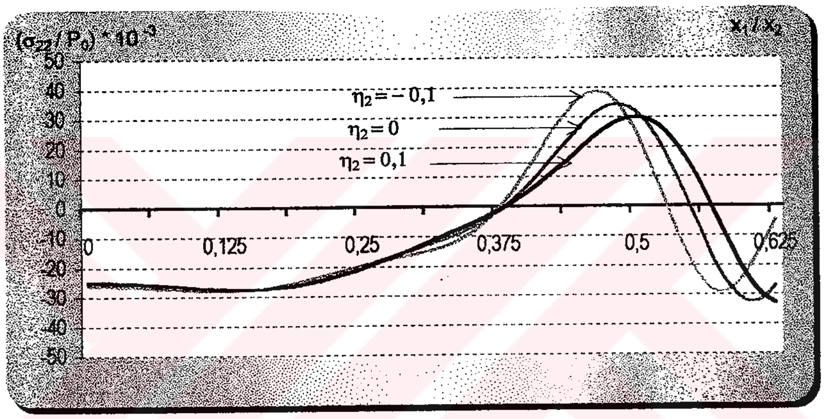
Şekil 3.12 Homogen yarım düzlemde Ω=10.000 $^{m/sn}$ için σ_{22} dağılımına farklı işaretteki öngerilme durumlarının etkisi



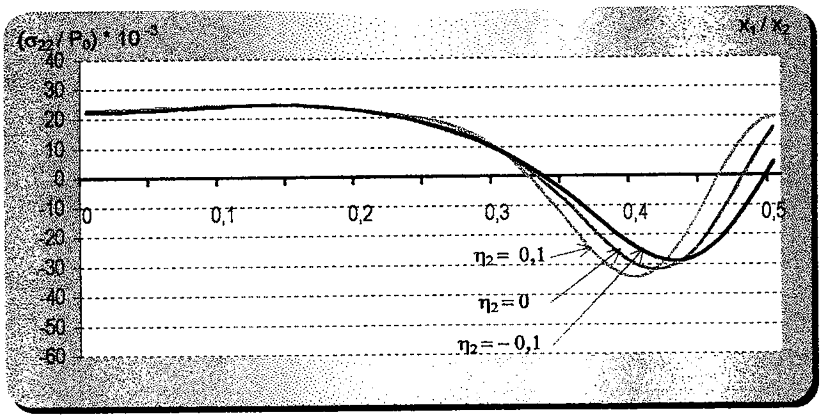
Şekil 3.13 Homogen yarım düzlemde Ω=12.500 $^{m/sn}$ için σ_{22} dağılımına farklı işaretteki öngerilme durumlarının etkisi



Şekil 3.14 Homogen yarım düzlemde Ω=15.000 $^{m/sn}$ için σ_{22} dağılımına farklı işaretteki öngerilme durumlarının etkisi



Şekil 3.15 Homogen yarım düzlemde Ω=20.000 $^{m/sn}$ için σ_{22} dağılımına farklı işaretteki öngerilme durumlarının etkisi



Şekil 3.16 Homogen yarım düzlemde Ω=25.000 $^{m/sn}$ için σ_{22} dağılımına farklı işaretteki öngerilme durumlarının etkisi

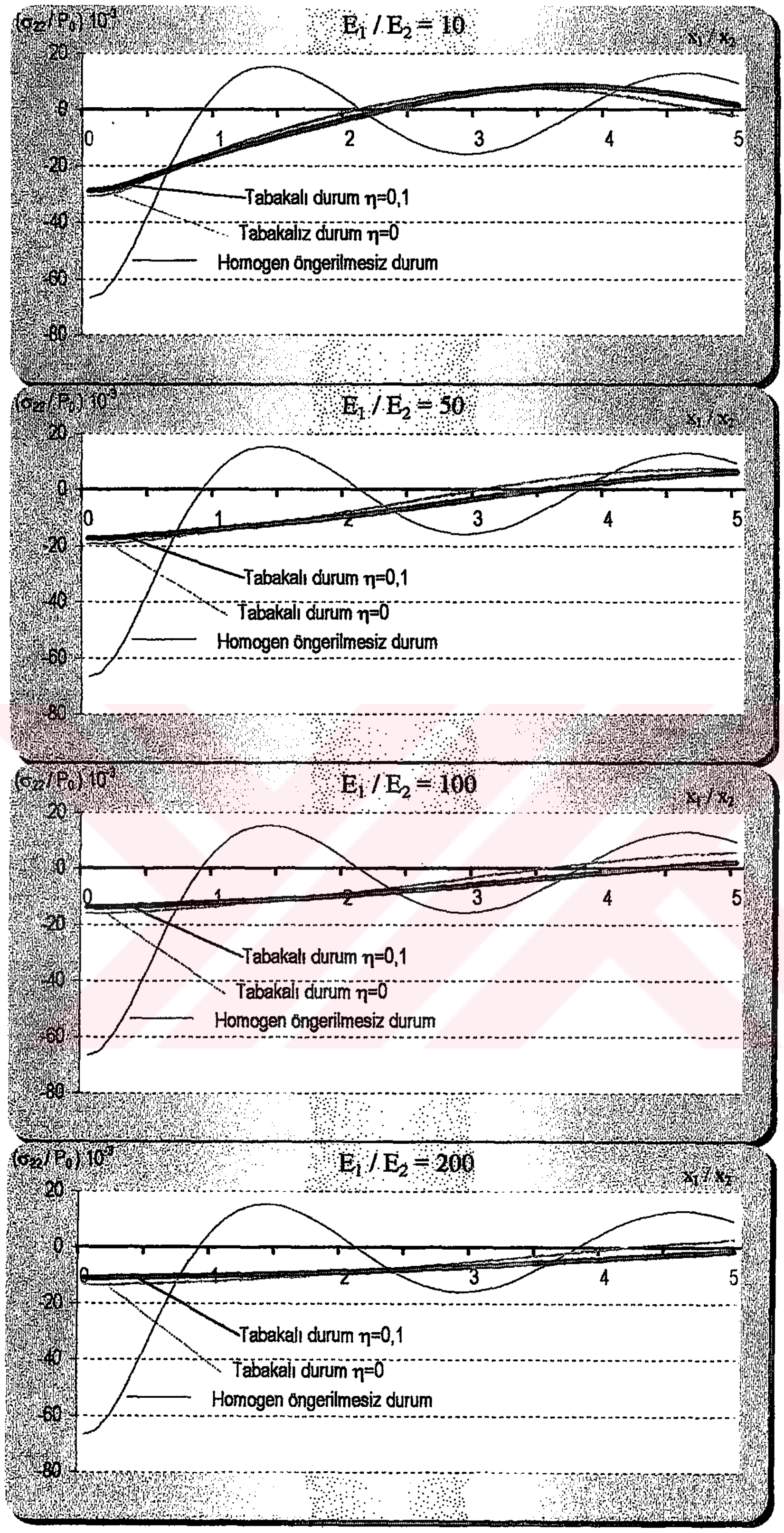
3.3.2. Tabaka ve yarım düzlem sınırında oluşan $\overline{\sigma}_{22}$ dağılımlarına ait sayısal incelemeler.

Bu bölümde üzerinde kendisinden daha sert bir tabakası bulunan öngerilme etkisindeki bir yarım düzlemde, yarım düzlem ve tabaka sınırında oluşan $\overline{\sigma}_{22}$ dağılımları sayısal olarak elde edilmiştir. Öncelikle tabakanın varlığının yarım düzlemdeki gerilmelere ne ölçüde etkidiğinin araştırılması daha sonra da öngerilmelerin bu dağılıma nasıl etki gösterdiklerinin araştırılması amaçlanmıştır. Ayrıca tabaka kalınlığının olaya etki eden önemli bir parametre olacağı açık olduğu için tabaka kalınlığındaki değişimin etkileri de araştırılmıştır. Diğer taraftan yoğunluklar ve Poisson oranları eşit kalmak üzere tabaka ve yarım düzlemin elastisite modüllerinin oranı da olayı belirleyen diğer bir parametre olduğu için her adımda farklı elastisite modülleri oranları için hesap yapılmıştır. Şekil 3.17-Şekil 3.21 deki grafiklerde bütün bu incelemelerin sonuçları görülmektedir. Bu incelemede frekansın ayrıca değişken olarak alınmamasının nedeni bir önceki bölümde açıklandığı gibi uygun Ω değerleri kullanılarak başka frekanslardaki gerilme yayılımlarının da elde edilebilmesidir.

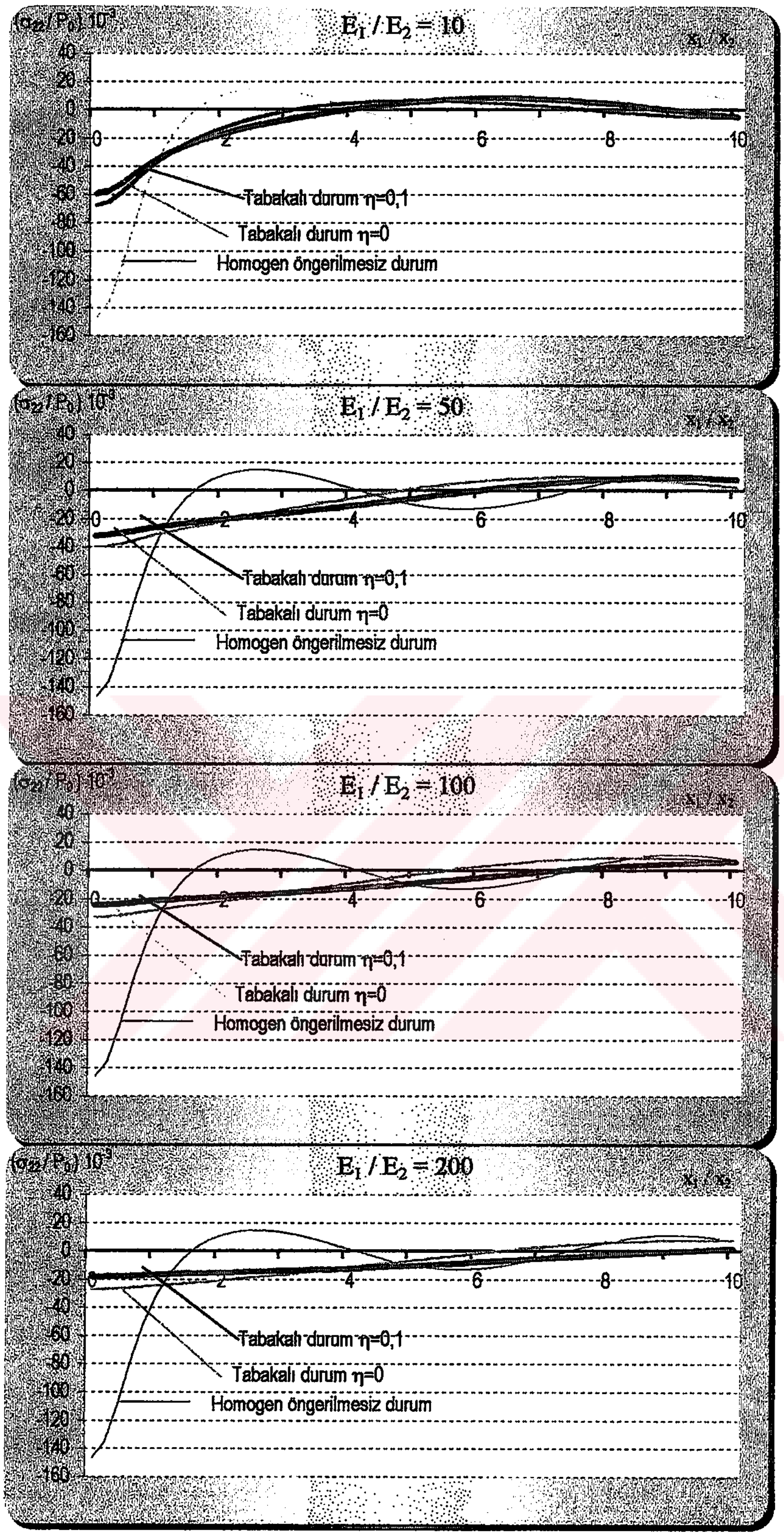
Daha sonra kendisinden daha sert ama nispeten ince bir tabakası bulunan yarım düzlemin Lamb problemi daha ayrıntılı olarak ele alınmıştır. Bu incelemenin uygulamada ayrı bir yeri olduğu düşünülmektedir. Şekil 3.22-Şekil 3.29 da verilen diagramlarda iki farklı tabaka kalınlığı ve dört farklı elastisite modülü oranı için tabakaya uygulanan çeşitli şiddetteki öngerilme durumları altında gerilme yayılımları ve Şekil 3.30-Şekil 3.41 de ise frekanstaki değişimin öngerilmelerin etkisine olan etkisi incelenmiştir.

Bölüm 3.3.1 de yapılan sayısal uygulamalardan öngerilmelerin etkisinin sadece gerilme yayılımının genliğin değil x_1 ekseni boyunca aldığı şekli de etkilediği görülmektedir. Dolayısıyla tabaka içindeki gerilmelerin x_2 ekseni boyunca yayılımının da öngerilmelerden nasıl etkilendiğinin araştırılmasının, öngerilmelerin etkisinin fiziksel yorumunun daha iyi yapılmasına yardımcı olacağı düşüncesiyle Şekil 3.42-Şekil 3.45 de sonuçları gösterilen sayısal uygulamalar yapılmıştır.

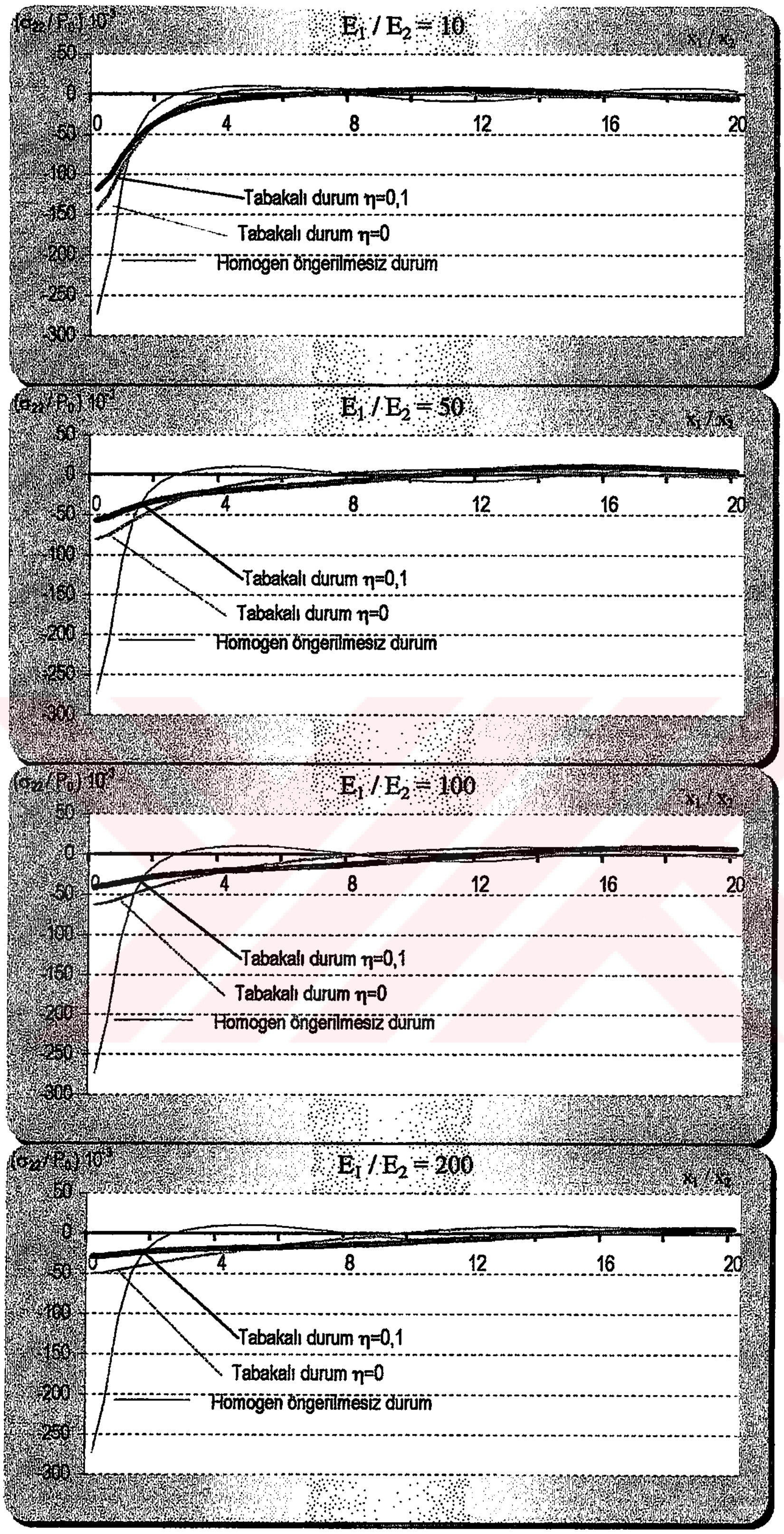
Bu alt bölümde elde edilen ve benzerleri literatürde bulunmayan sonuçların yorumlanması sonuçlar bölümünde yapılmıştır.



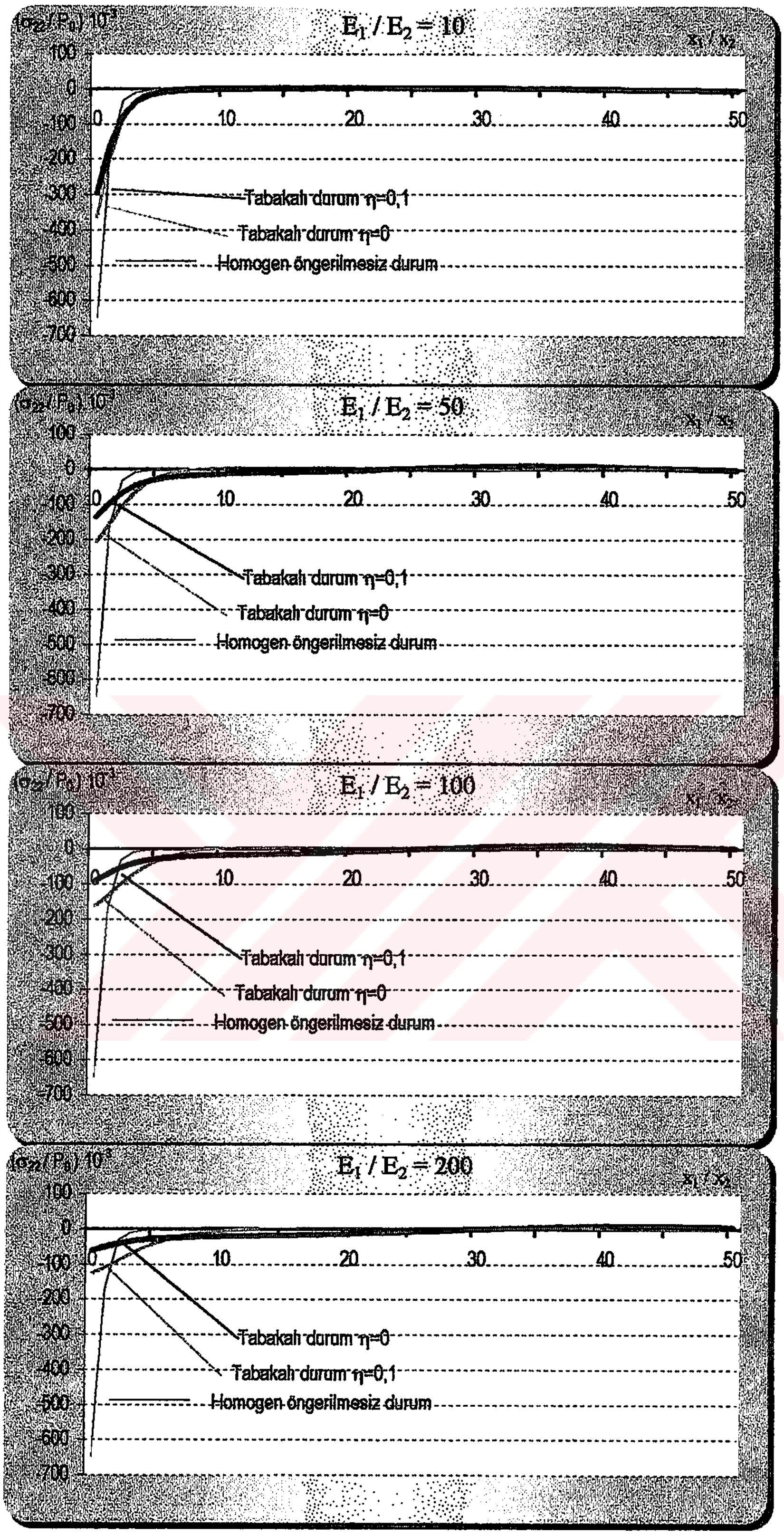
Şekil 3.17 $\Omega = 5.000\ ^{1/sn}\,.\ 0{,}1^{m} = 500\ ^{m/sn}$, $h = 0{,}1\ ^{m}$ ve farklı E_1/E_2 oranları için σ_{22} yayılımına öngerilmelerin etkisi.



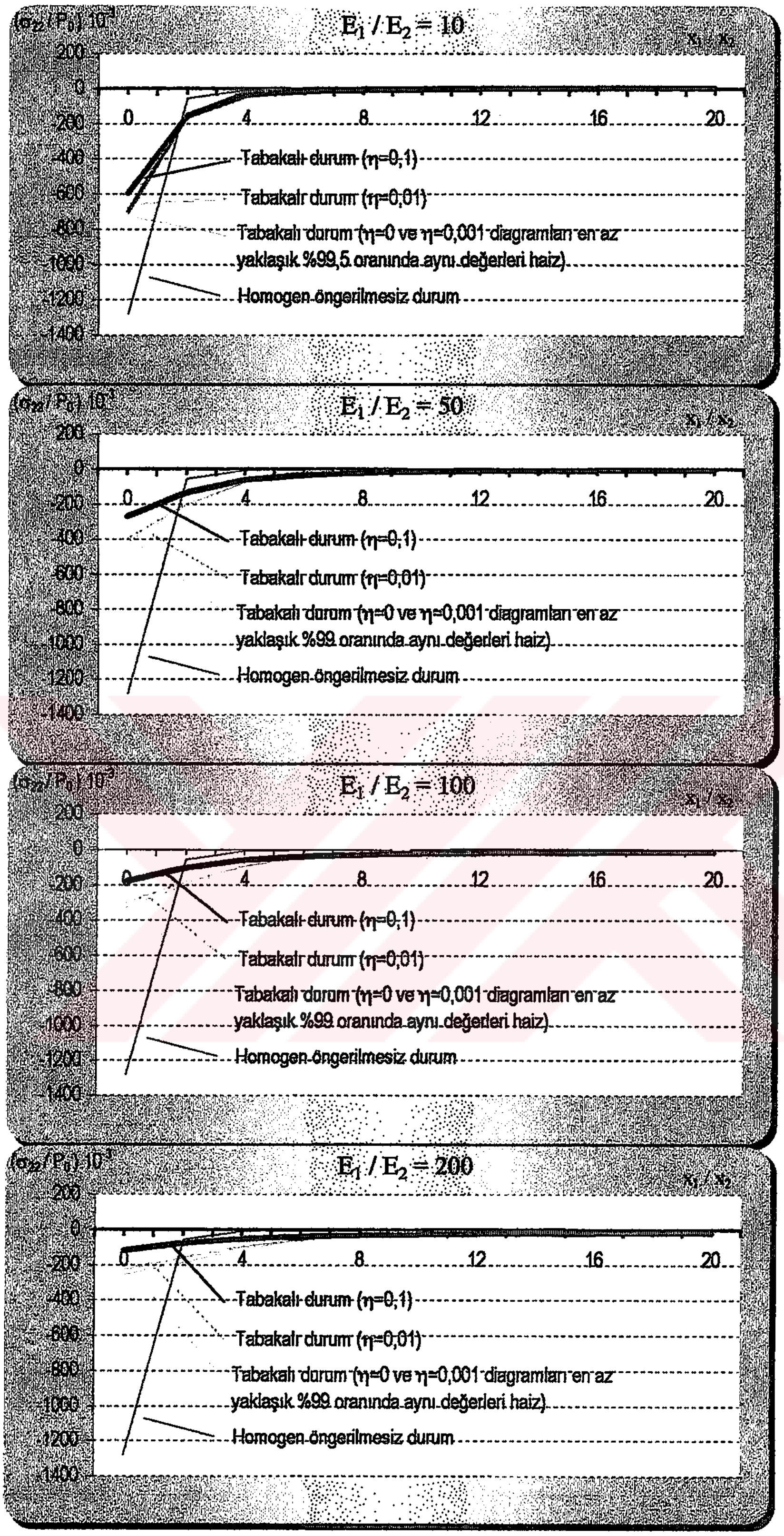
Şekil 3.18 $\Omega = 5.000\ ^{1/sn} .\ 0,05^{m} = 250\ ^{m/sn}$, $h = 0,05\ ^{m}$ ve farklı E_1/E_2 oranları için σ_{22} yayılımına öngerilmelerin etkisi.



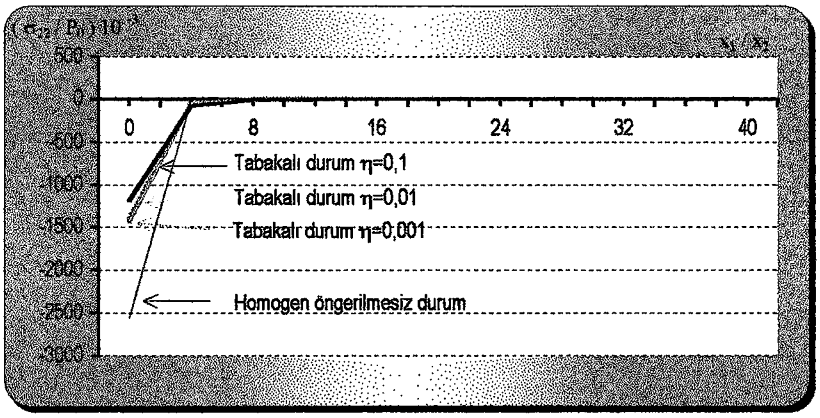
Şekil 3.19 $\Omega = 5.000\ ^{1/sn}.\ 0{,}025^{m} = 125\ ^{m/sn}$, $h = 0{,}025\ ^{m}$ ve farklı E_1/E_2 oranları için σ_{22} yayılımına öngerilmelerin etkisi.



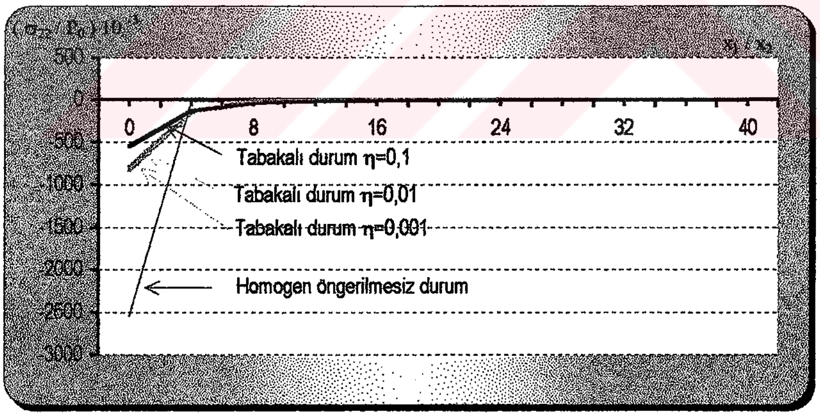
Şekil 3.20 $\Omega = 5.000^{\ 1/sn} .\ 0{,}01^{m} = 50^{m/sn}$, $h=0{,}01^{m}$ ve farklı E_1/E_2 oranları için σ_{22} yayılımına öngerilmelerin etkisi.



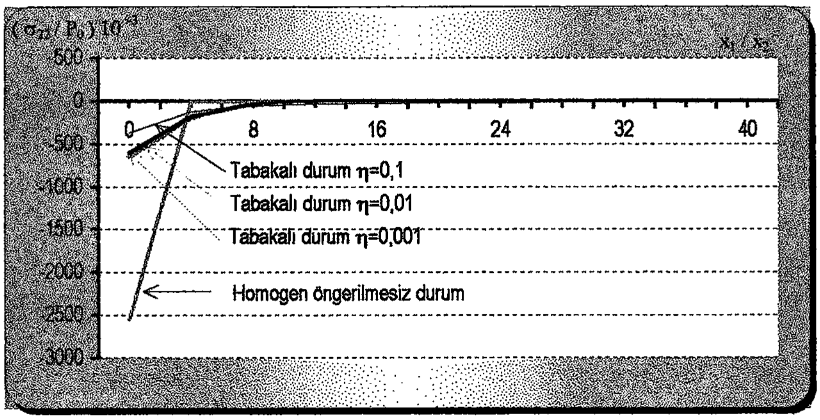
Şekil 3.21 $\Omega = 5.000\ ^{1/sn} .\ 0{,}005^{m} = 25\ ^{m/sn}$, h=0,005 ^{m} ve farklı E_1/E_2 oranları için σ_{22} yayılımına farklı şiddetteki öngerilmelerin etkisi.



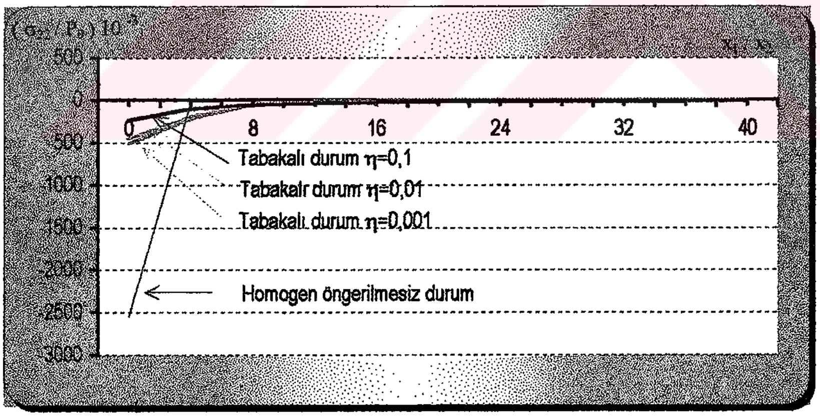
Şekil 3.22 $\Omega = 5.000\ ^{1/sn} \cdot 0{,}0025\ ^{m} = 12{,}5\ ^{m/sn}$, $h = 0.0025^{m}$ ve $E_1/E_2=10$ oranı için σ_{22} yayılımına, farklı şiddetteki öngerilmelerin etkisi.



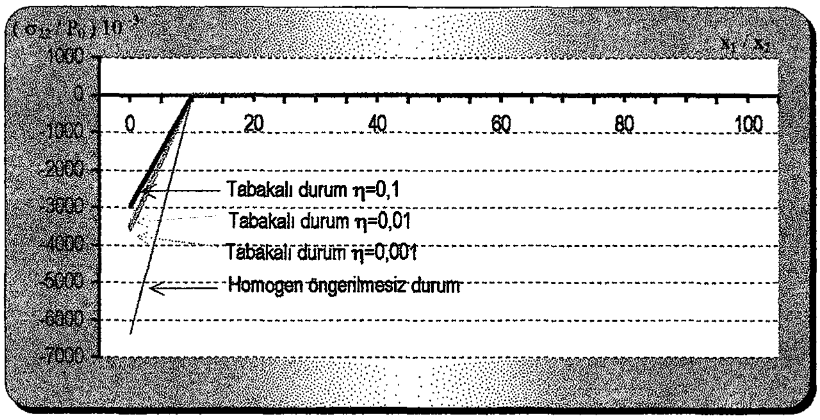
Şekil 3.23 $\Omega = 5.000\ ^{1/sn} \cdot 0{,}0025\ ^{m} = 12{,}5\ ^{m/sn}$, $h = 0.0025^{m}$ ve $E_1/E_2=50$ oranı için σ_{22} yayılımına, farklı şiddetteki öngerilmelerin etkisi.



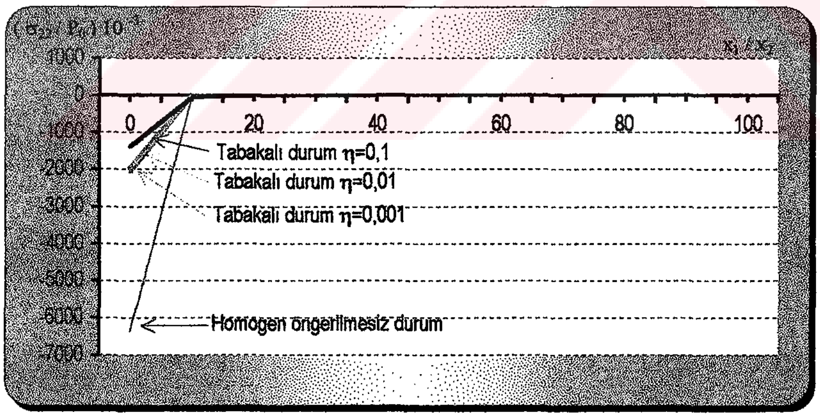
Şekil 3.24 $\Omega = 5.000\ ^{1/sn} \cdot 0{,}0025\ ^{m} = 12{,}5\ ^{m/sn}$, $h = 0.0025^{m}$ ve E_1/E_2=100 oranı için σ_{22} yayılımına, farklı şiddetteki öngerilmelerin etkisi.



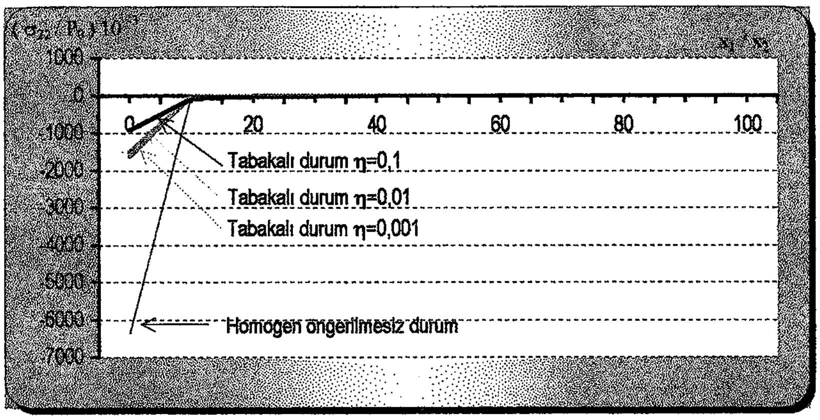
Şekil 3.25 $\Omega = 5.000\ ^{1/sn} \cdot 0{,}0025\ ^{m} = 12{,}5\ ^{m/sn}$, $h = 0.0025^{m}$ ve E_1/E_2=200 oranı için σ_{22} yayılımına, farklı şiddetteki öngerilmelerin etkisi.



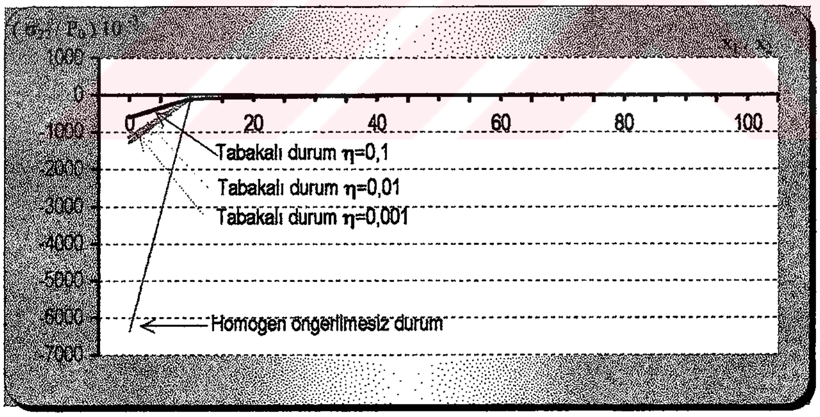
Şekil 3.26 $\Omega = 5.000\ ^{1/sn}\cdot 0{,}001\ ^{m} = 5\ ^{m/sn}$, $h = 0.001\ ^{m}$ ve E_1/E_2=10 oranı için σ_{22} yayılımına, farklı şiddetteki öngerilmelerin etkisi.



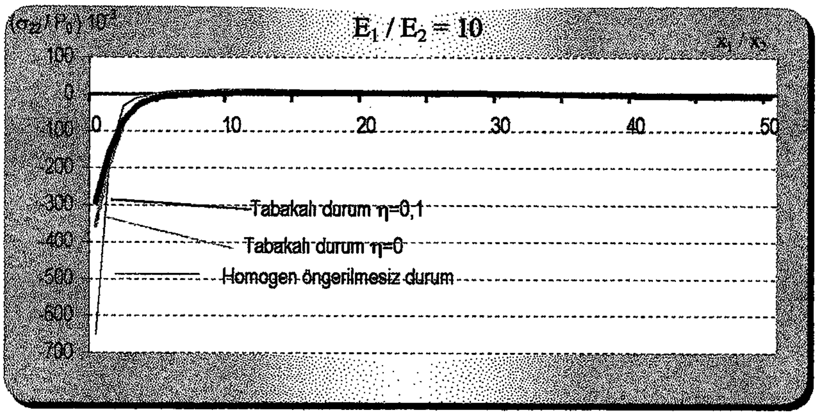
Şekil 3.27 $\Omega = 5.000\ ^{1/sn}\cdot 0{,}001\ ^{m} = 5\ ^{m/sn}$, $h = 0.001\ ^{m}$ ve E_1/E_2=50 oranı için σ_{22} yayılımına, farklı şiddetteki öngerilmelerin etkisi.



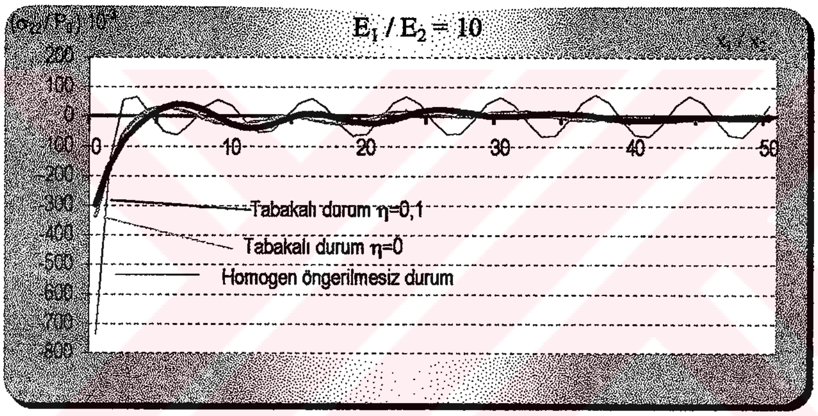
Şekil 3.28 $\Omega = 5.000\ ^{1/sn.}\ 0{,}001\ ^{m} = 5\ ^{m/sn}$, $h = 0.001\ ^{m}$ ve $E_1/E_2=100$ oranı için σ_{22} yayılımına, farklı şiddetteki öngerilmelerin etkisi.



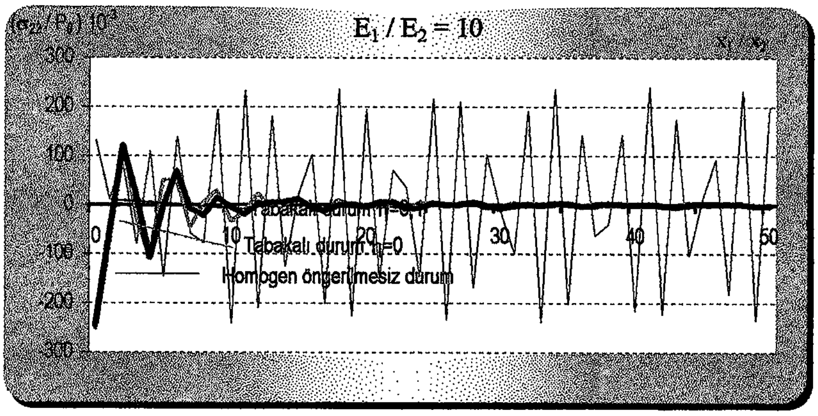
Şekil 3.29 $\Omega = 5.000\ ^{1/sn.}\ 0{,}001\ ^{m} = 5\ ^{m/sn}$, $h = 0.001\ ^{m}$ ve $E_1/E_2=200$ oranı için σ_{22} yayılımına, farklı şiddetteki öngerilmelerin etkisi.



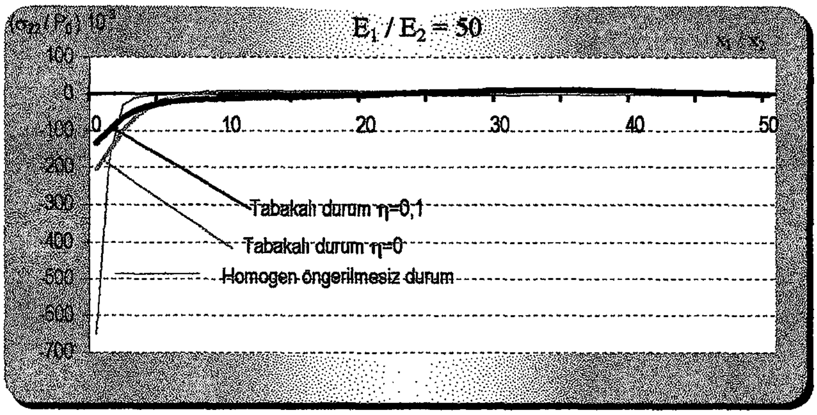
Şekil 3.30 Ω=5.000$^{1/sn}$.0,01^{m}=50$^{m/sn}$, h=0,01^{m} ve E_1/E_2=10 durumu için farklı öngerilmelerin σ_{22} yayılımına etkisi.



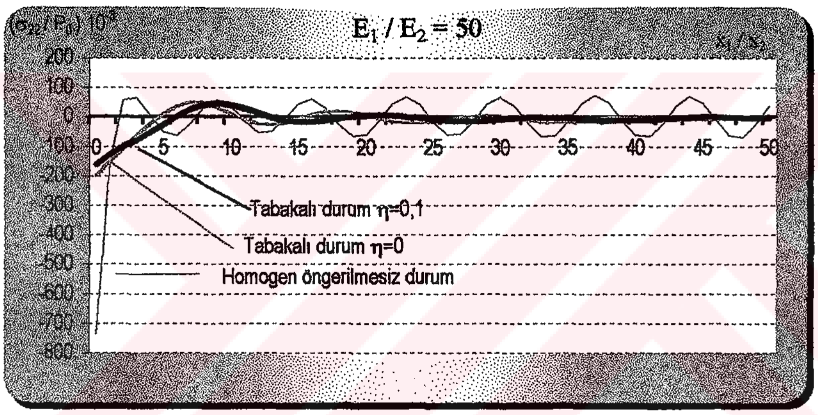
Şekil 3.31 Ω=25.000$^{1/sn}$.0,01^{m}=250$^{m/sn}$, h=0,01^{m} ve E_1/E_2=10 durumu için farklı öngerilmelerin σ_{22} yayılımına etkisi.



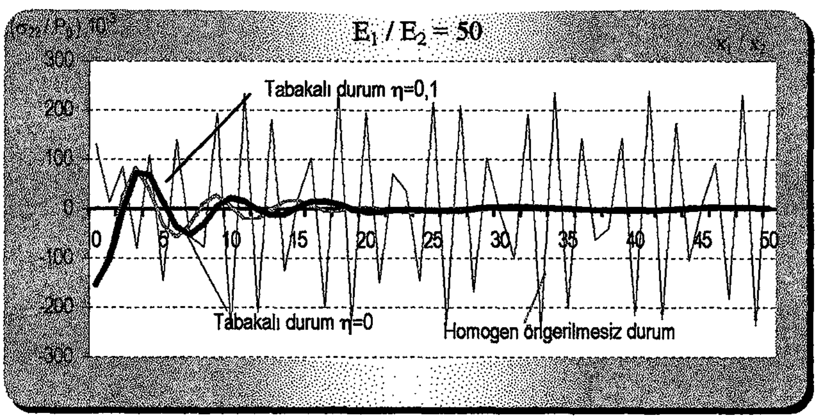
Şekil 3.32 Ω=100.000$^{1/sn}$.0,01^{m}=1000$^{m/sn}$, h=0,01^{m} ve E_1/E_2=10 durumu için farklı öngerilmelerin σ_{22} yayılımına etkisi.



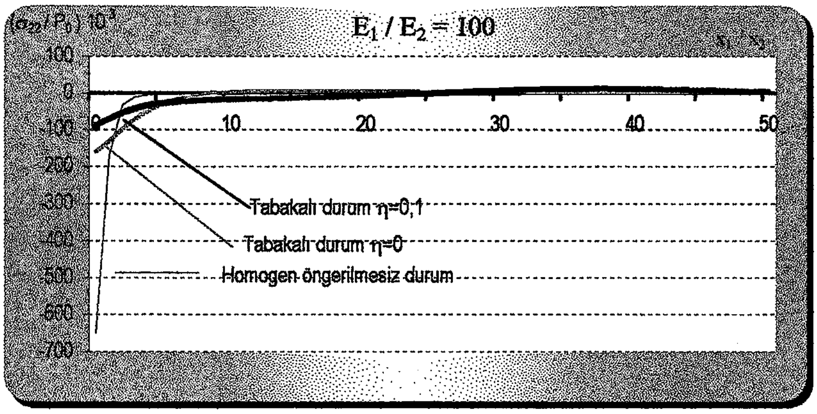
Şekil 3.33 $\Omega=5.000^{1/sn}.0,01^{m}=50^{m/sn}$, $h=0,01^{m}$ ve $E_1/E_2=50$ durumu için farklı öngerilmelerin σ_{22} yayılımına etkisi.



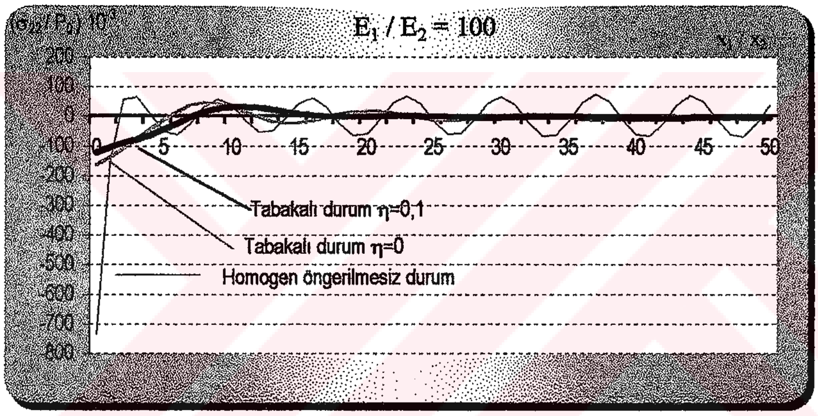
Şekil 3.34 $\Omega=25.000^{1/sn}.0,01^{m}=250^{m/sn}$, $h=0,01^{m}$ ve $E_1/E_2=50$ durumu için farklı öngerilmelerin σ_{22} yayılımına etkisi.



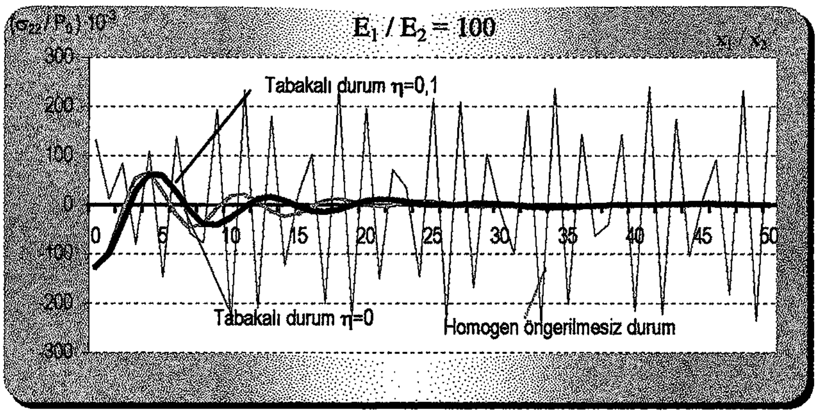
Şekil 3.35 $\Omega=100.000^{1/sn}.0,01^{m}=1000^{m/sn}$, $h=0,01^{m}$ ve $E_1/E_2=50$ durumu için farklı öngerilmelerin σ_{22} yayılımına etkisi.



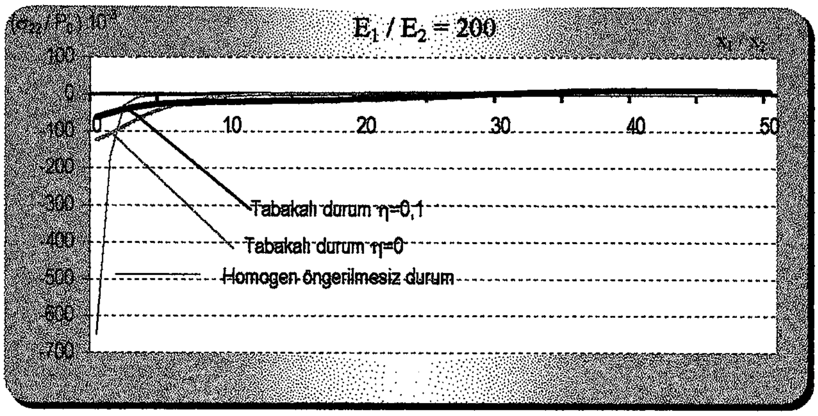
Şekil 3.36 $\Omega=5.000^{1/sn}.0{,}01^{m}=50^{m/sn}$, $h=0{,}01^{m}$ ve $E_1/E_2=100$ durumu için farklı öngerilmelerin σ_{22} yayılımına etkisi.



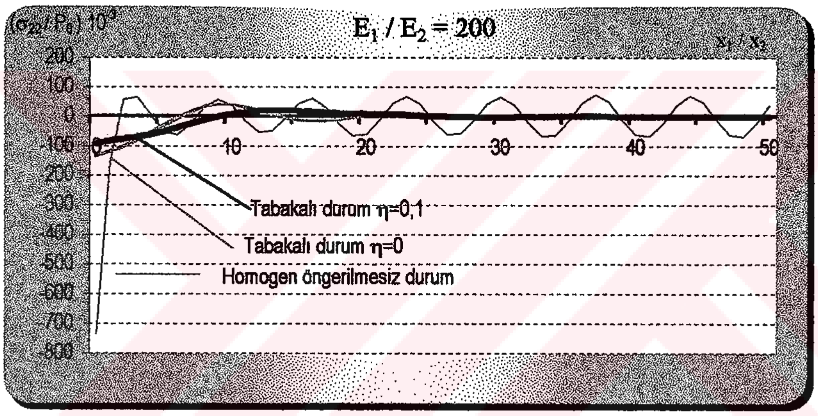
Şekil 3.37 $\Omega=25.000^{1/sn}.0{,}01^{m}=250^{m/sn}$, $h=0{,}01^{m}$ ve $E_1/E_2=100$ durumu için farklı öngerilmelerin σ_{22} yayılımına etkisi.



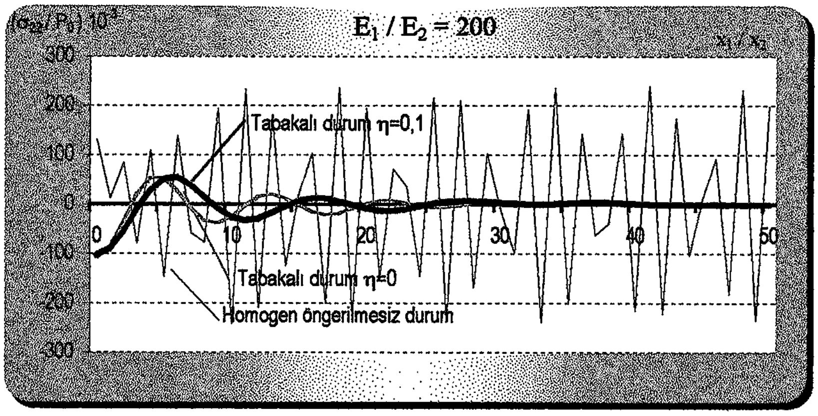
Şekil 3.38 $\Omega=100.000^{1/sn}.0{,}01^{m}=1000^{m/sn}$, $h=0{,}01^{m}$ ve $E_1/E_2=100$ durumu için farklı öngerilmelerin σ_{22} yayılımına etkisi.



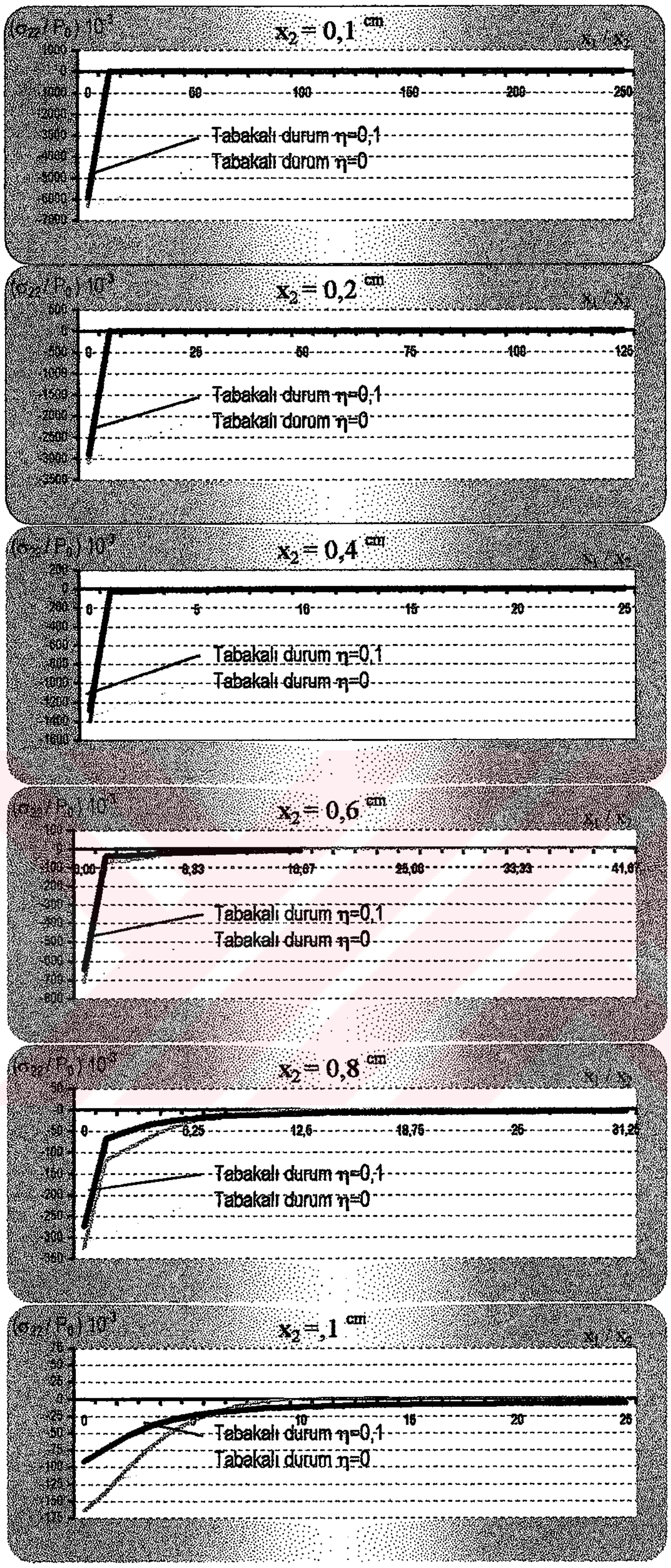
Şekil 3.39 Ω=5.000$^{1/sn}$.0,01^m=50$^{m/sn}$, h=0,01^m ve E_1/E_2=200 durumu için farklı öngerilmelerin σ_{22} yayılımına etkisi.



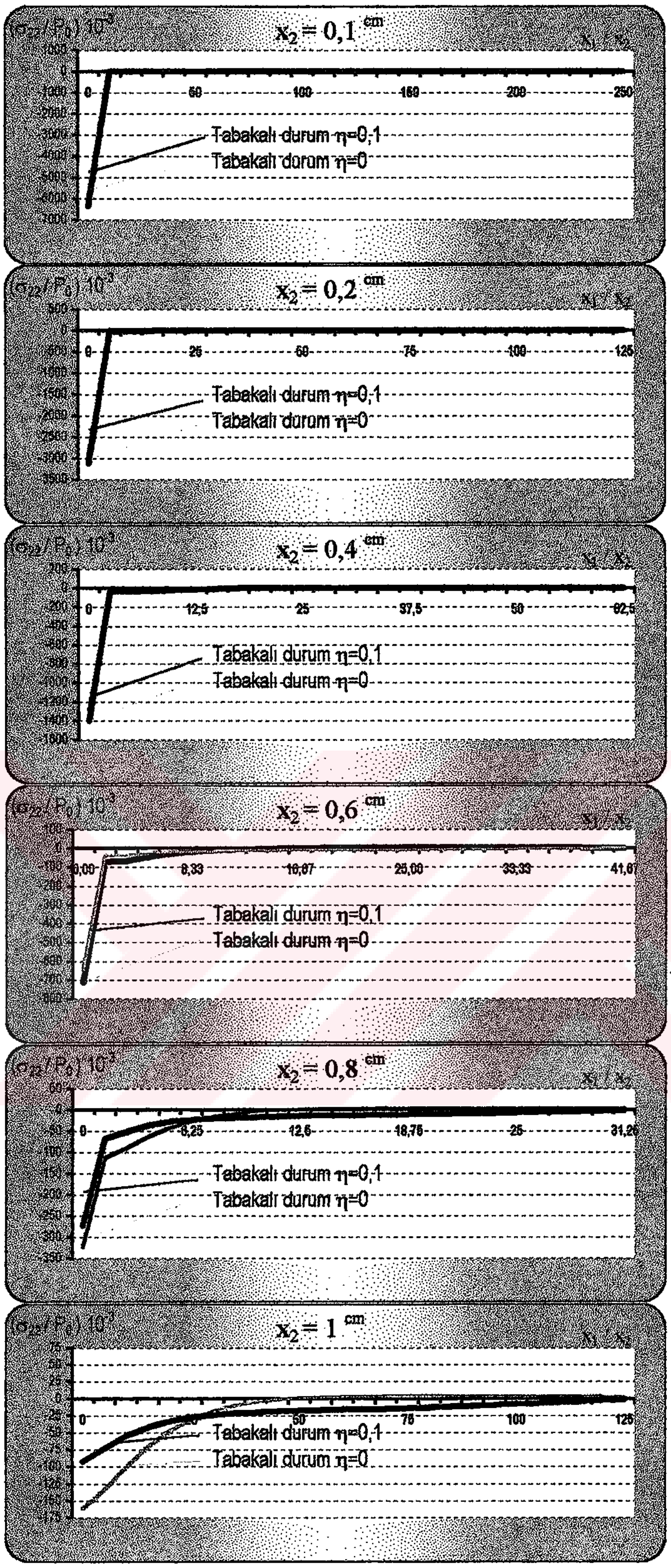
Şekil 3.40 Ω=25.000$^{1/sn}$.0,01^m=250$^{m/sn}$, h=0,01^m ve E_1/E_2=200 durumu için farklı öngerilmelerin σ_{22} yayılımına etkisi.



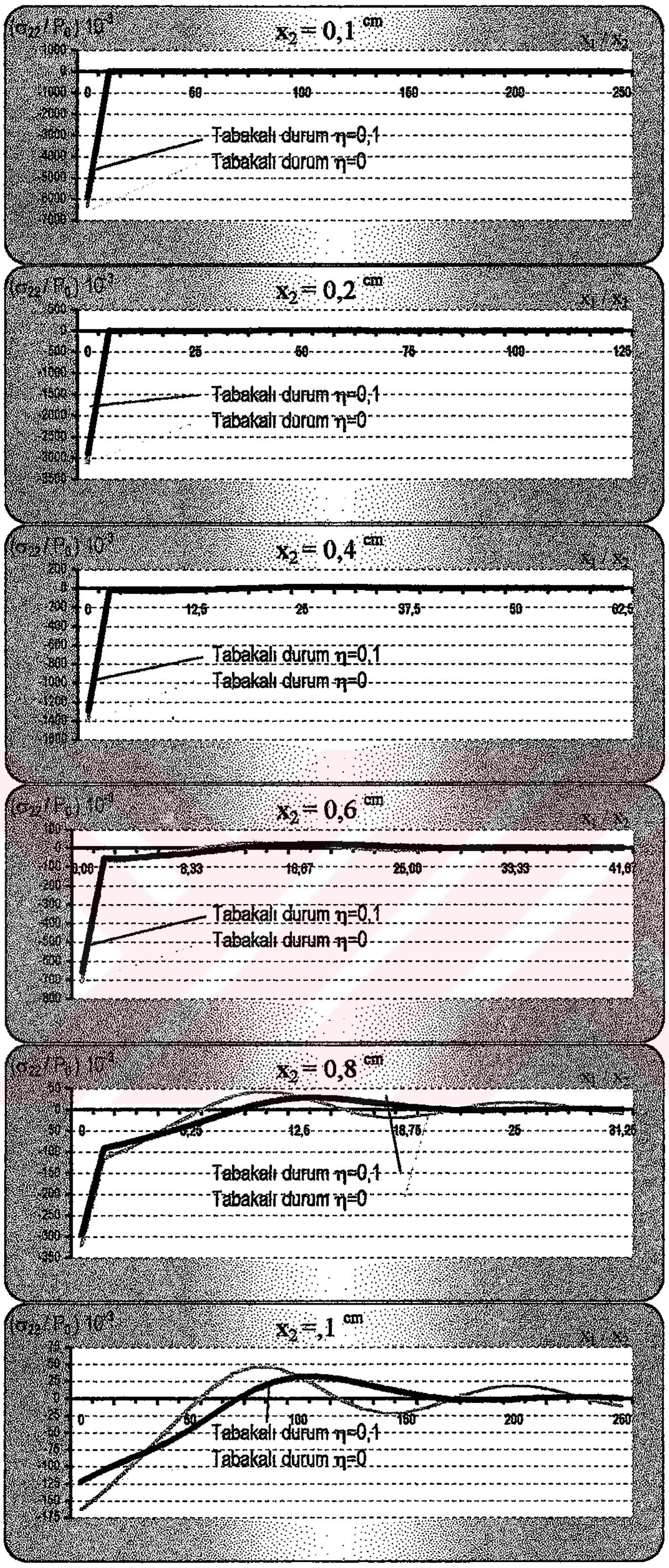
Şekil 3.41 Ω=100.000$^{1/sn}$.0,01^m=1000$^{m/sn}$, h=0,01^m ve E_1/E_2=200 durumu için farklı öngerilmelerin σ_{22} yayılımına etkisi.



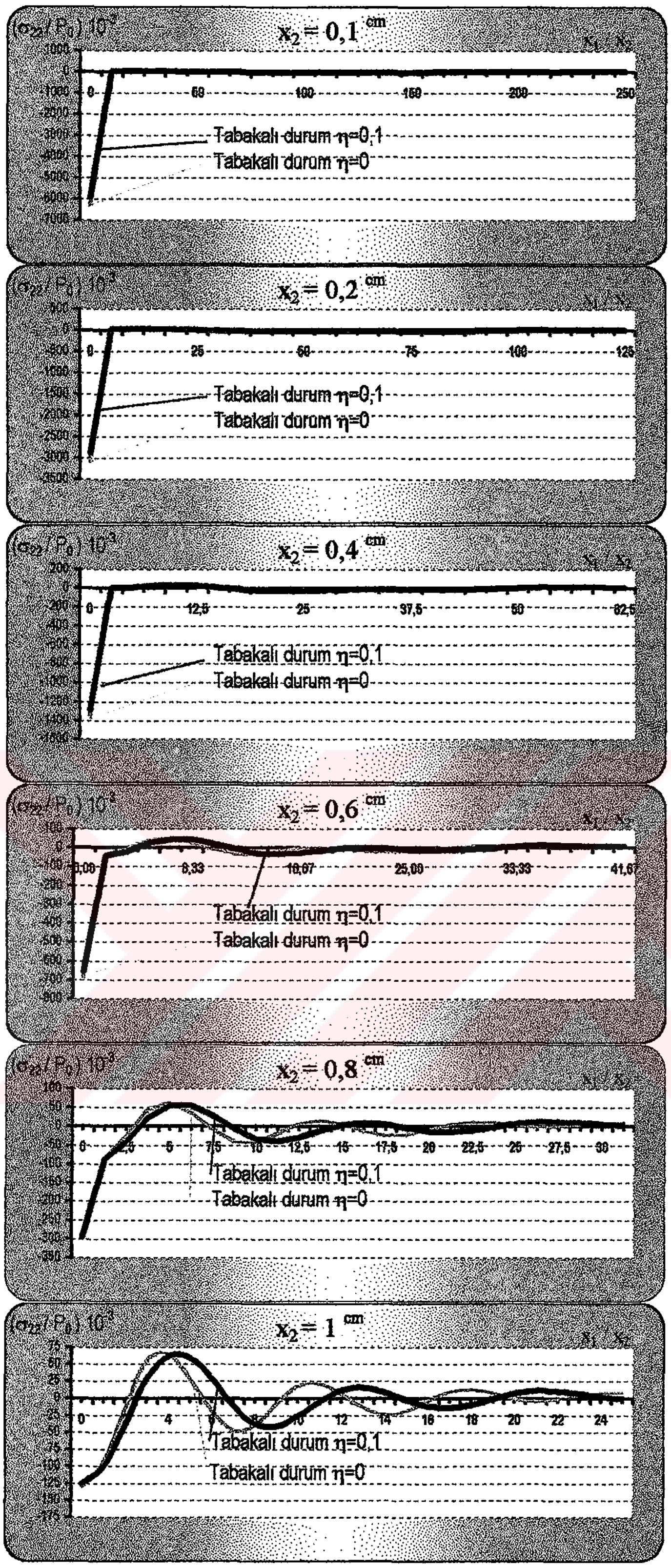
Şekil 3.42 h=1^{cm} kalınlığında tabaka kaplı yarım düzlemde ω=2500$^{1/sn}$ durumunda, tabaka içindeki çeşitli derinliklerdeki σ_{22} yayılımına tabakaya uygulanan öngerilmenin etkisinin E_1/E_2=10 için incelenmesi.



Şekil 3.43 $h=1^{cm}$ kalınlığında tabaka kaplı yarım düzlemde $\omega=5000^{1/sn}$ durumunda, tabaka içindeki çeşitli derinliklerdeki σ_{22} yayılımına tabakaya uygulanan öngerilmenin etkisinin $E_1/E_2=50$ için incelenmesi.



Şekil 3.44 h=1^{cm} kalınlığında tabaka kaplı yarım düzlemde ω=$25.000^{1/sn}$ durumunda, tabaka içindeki çeşitli derinliklerdeki σ_{22} yayılımına tabakaya uygulanan öngerilmenin etkisinin E_1/E_2=100 için incelenmesi.



Şekil 3.45 h=1^{cm} kalınlığında tabaka kaplı yarım düzlemde ω=$100.000^{1/sn}$ durumunda, tabaka içindeki çeşitli derinliklerdeki σ_{22} yayılımına tabakaya uygulanan öngerilmenin etkisinin E_1/E_2=200 için incelenmesi.

4. SONUÇLAR.

Bu çalışmada öngerilme etksindeki tabakalı yarım düzlemde, serbest yüzeye normal doğrultuda etkiyen harmonik bir çizgisel yükten dolayı oluşan gerilme ve yerdeğiştirme dağılımlarının incelenmesi için, Bölüm 3.1 'de açıklanan matematik model kullanılarak yapılan formülasyon ve bu formülasyona dayalı olarak geliştirilen bilgisayar programı yardımı ile elde edilen sayısal sonuçlara yer verilmiştir. Homogen bir yarım düzlemde, yüzeyde normal doğrultuda etkiyen harmonik bir çizgisel yükten dolayı oluşan gerilme ve yerdeğiştirme yayılımlarının özelliklerinin, yarım düzleme etkiyen öngerilmelere bağımlı olacağı çok açıktır. Gerek tabii halde öngerilme etkisinde bulunan gerekse üzerine yapay olarak öngerilme etkitilen ve yarım düzlem olarak kabul edilebilen bir sürekli ortamda bu öngerilmelerin etkisinin bilinmesi mühendislik sonuçları bakımından mutlaka incelenmesi gerekli olan bir konu olmasına rağmen literatürde büyük bir boşluk göze çarpmaktadır. Diğer taraftan mühendislik amacı, verilmiş bir harmonik yükten dolayı oluşan gerilmelerin şiddetini azaltmak ise, bunu başarmanın yolunun yarım düzleme öngerilme etkitmek olabileceği gibi üzerine daha sert bir tabaka –zırh- koymak olacağı da açıktır. Eğer bu iki yöntem birarada kullanılırsa öngerilme etkisindeki tabakalı yarım düzlem adı verilebilen sürekli ortama ulaşılır. Literatürde homogen ve öngerilmesiz yarım düzlemin Lamb problemine ait yeterince sayısal sonuç yokken, öngerilme etkisinde ve üzerinde kendisinden daha sert bir tabaka bulunan yarım düzleme ait Lamb probleminin incelenmesinin ise, mühendislik amaçlarından da önce literatürdeki boşluğu tamamlamak açısından öneminin olacağı görülmektedir. Kaldı ki homogen-öngerilmesiz, homogen-öngerilmeli ve tabakalı-öngerilmesiz yarım düzlem durumlarının hepsi bu durumdan birer özel hal olarak elde edilebilmektedir. Ayrıca yine mühendislik sonuçlarından önce öngerilmelerin, üzerinde tabakası bulunan bir yarım düzlemin Lamb probleminin sonuçlarını ne yönde etkilediğinin bilinmesi ve bunun fiziksel açıklamasının yapılması gereklidir. Zaten bütün bu işlemler yapıldıktan sonra öngerilmelerin, bir tabakanın varlığı ile beraber mühendislik amaçları yönünden nasıl kullanılması gerektiği hakkında sağlam bir fikre ulaşmak mümkün olur.

Bütün bu anlatılanların ışığında, bu çalışmanın ilk bölümünde elastodinamiğin klasik Lamb problemi σ_{22} gerilmelerinin yayılımı açısından dikkatle incelenmiştir. Yüzeyde etkiyen çizgisel yükün şiddetinin zamanla değişmediği –statik- durumda, gerilme dağılımının izotrop malzemenin fiziksel özelliklerine bağımlı olmadığı anlaşılmaktadır. Ancak malzeme, dinamik durumda gerilme yayılımına etki eden önemli bir parametre olmaktadır. Statik ve dinamik durumlar arasındaki diğer bir önemli fark ise statik durumda yarım düzlemin keyfi bir noktasında σ_{22} gerilmesi basınç olarak oluşurken dinamik durumda şiddeti zamana göre harmonik olmakta ve dolayısıyla hem basınç hem de çekme gerilmeleri oluşabilmektedir. Dinamik durumda σ_{22} gerilmelerin genliklerini veren Şekil 2.6-Şekil 2.9 'dan açık olarak görüldüğü gibi ayrıca dinamik durumda aynı anda yarım düzlemin bir noktasında basınç varken diğer bir noktasında çekme oluşabilmektedir. Yapılan sürekli ortam kabulü uyarınca çizgisel yükün altına yaklaştıkça σ_{22} gerilmesinin genliği sınırsız olarak artmaktadır. Bu nedenle fiziksel anlam taşıması bakımından çizgisel yükten yeterli bir uzaklıktaki noktalarda hesap yapılmasını gerekli olduğu görülmektedir. Diğer taraftan yarım düzleme ait Lamb probleminin sayısal sonuçları, σ_{22} gerilmelerinin yüzeye yakın bölgelerde serbest yüzeye paralel doğrultuda sönmediği görülmektedir. Ancak yüzeyden x_2 yönünde uzaklaşılması durumunda bu periyodik kısmın genliğinin giderek küçüldüğü ve sadece x_1=0 doğrusunun dar bir komşuluğunda gerilmelerin oluştuğu görülmektedir. Bu durum Rayleigh dalgalarının varlığı ile açıklanır

Öngerilmeli homogen ve öngerilmeli tabakalı durumlara ait yapılan sayısal incelemelerden elde edilen mekanik önemi olan sonuçlar aşağıda sıralanmıştır.

1. Öncelikle tabakasız öngerilmeli duruma ait Şekil 3.2-Şekil 3.16 da, sonuçları diyagramlarla gösterilen uygulamalar yapılmıştır. Bu diagramlarda çekme öngerilmelerinin gerilme yayılımlarının genliklerini azalttığı, basınç öngerilmelerinin ise artırdığı genel olarak göze çarpmaktadır. Öngerilmelerin bu etkisi ortamın sertliğinin öngerilmelerden dolayı değiştiği düşünülürse açıklık kazanmaktadır. Ayrıca çekme öngerilmelerinin gerilme yayılımı eğrilerinin sıfırdan geçtiği noktalar arasındaki uzaklığı artırırken, basınç öngerilmelerinin azalttığı izlenmektedir. Diğer taraftan öngerilmelerin etkisinin x_1 ekseni boyunca yayılımının her frekans için öyle bir yapı kazandığı görülmektedir ki, uygun yerlerde maximum gerilmeler öngerilmelerin işaretine bağlı olarak azaltılmakta veya artırılmaktadır.

2. Öngerilmeli tabakalı duruma ait öncelikle Şekil 3.17-Şekil 3.21 de sonuçları verilen uygulamalar yapılmış ve tabaka kalınlığının azalması ve/veya tabakanın elastisite modülünün yarım düzlemin elastisite modülüne olan oranının artması ile öngerilmelerin etkisinin arttığı anlaşılmıştır. Bu diagramlardan ayrıca tabakada öngerilme olmasa da tabakanın varlığının gerilme durumunu son derece önemli ölçüde etkilediği görülmektedir. Öyle ki yükün tam altındaki ve yüke yakın bir bölgedeki tabaka sınırındaki gerilmeler, tabakanın varlığı ile elastisite modüllerinin oranına bağlı olarak 1/6 oranına varan azalmalar gösterebilmektedir. Ayrıca bu şekillerden E_1/E_2 oranının yükselmesi ile gerilme dağılımının sanki rijit bir

tabakanın varlığı durumunda oluşan sıfira eşit gerilme dağılımına yaklaştığı diğer bir deyişle $E_1/E_2 \to \infty$ limit durumuna ulaşacak bir yapı kazandığı görülmektedir. Şekil 3.21 den ise öngerilmenin etkisinin öngerilmenin şiddeti ile monoton olarak arttığı izlenmektedir. Tabakaya uygulanan öngerilmeler ise öngerilmesiz duruma göre η=0,1 için ($\eta = \sigma_0^{(1)} / \mu^{(1)}$ veya ν=0,25 olduğuna göre $\eta' = 0{,}04 = \sigma_0^{(1)} / E^{(1)}$) %100 ü aşan bir azaltma yapabildikleri de yine Şekil 3.21 den görülmektedir. Bu sonuçlardan öngerilmelerin tabaka ve yarım düzlem sınırındaki gerilmeyi 100 ü aşan miktarda azalatabildiği görülmektedir. Bunun ise tabaka ve yarım düzlem arasındaki adhezyon mukavemetinin korunmasında önemli bir rol oynadığı açıktır.

3. Şekil 3.21-Şekil 3.29 da sonuçları verilen uygulamalarda öngerilmenin şiddetindeki değişimin etkisi daha detaylı bir biçimde incelenmiştir. Şekil 3.21 den tabakadaki η=0.001 değerini haiz öngerilmenin öngerilmesiz tabakalı duruma ait gerilme durumunu yaklaşık %2 den daha fazla değiştirmediği görülmektedir. Şekil 3.17-Şekil 3.29 daki sonuçlar toplu olarak değerlendirildiğinde öngerilmelerin etkisinin x_1=0 doğrusunun küçük bir komşuluğunda daha büyük boyutlarda olduğu görülmektedir.

4. Şekil 3.30-Şekil 3.41 de yapılan uygulamalarda ise frekansın değişiminin öngerilmelerin etkisine olan etkisi incelenmiş ve frekans arttıkça bu etkinin azaldığı görülmüştür. Ayrıca her bir frekans için rijitlik oranının artmasının bu etkiyi arttırdığı da genel olarak izlenmektedir. Bu etkiler daha önce söylenildiği gibi hem en büyük gerilmelerin değerini azaltıcı yönde hem de gerilme eğrisinin sıfirdan geçtiği noktalar arasındaki uzaklığı artırıcı yöndedir.

5. Ve son olarak Şekil 3.42-Şekil 3.45 de sonuçları verilen sayısal uygulamalarda öngerilmenin etkisinin bu sefer tabaka yüksekliği diğer bir deyişle x_2 ekseni boyunca değişimi incelenmiş ve tabaka içinde altı noktada dört frekans değeri için ayrı ayrı incelemeler yapılmıştır. Bunlardan, öngerilmenin etkisinin tekillikten uzaklaşıp tabaka ve yarım düzlemin sınırına doğru yaklaştıkça arttığı izlenmektedir. Diğer taraftan bu diagramların incelenmesi ile gerilme yayılımlarının genliklerinin x_2 nin büyümesi ile söndüğü izlenmektedir. Bunun sebebi arayüzey boyunca Stoneley dalgalarının oluşmamasındandır. Stoneley dalgaları bilindiği gibi tabakalı ortamlarda ortamların kayma modüllerinin birbirine çok yakın olması durumunda oluşmakta olup, burada elastisite modüllerinin oranının en küçük 10 olarak alınması nedeniyle bu tür dalgalar oluşmamaktadır. Bu oranın 10'dan küçük alınmamasının nedeni ise, daha küçük oplması durumunda öngerilmelerin etkisinin oldukça azalmasıdır.

KAYNAKLAR

Achenbach, J.D., (1973), Wave Propogation in Elastic Solids, North-Holland, Amsterdam.

Bagno, A.M., ve Guz, A.N., (1997), "Elastic Waves in Prestresses Bodies Interacting with Fluid" (Survey), Int. Appl. Mech., 435-465.

Belward, I.A., (1972), "Elastic Waves in a Prestressed Money Material", Bull. Austral. Math. Soc., 7 (1): 135-160.

Belward, I.A., (1976), "The Propogation of Small Amplitude Waves in Prestressed Incomprressible Elastic Cylinders", Int. J. Eng. Sci., 14 (8): 647-659.

Bergman, R.N., Shanbender, R.A., (1958), "Effects of Statically-Applied Stresses on the Velocities of Propogation of Ultrasonic Waves", J. Appl. Phys., 29 (12): 1736-1739.

Bhattacharya, R.C., (1977), "A Theory for Longitudinal Elastic Wave Propogation in a Solid Cylinder Under an Initial Stress", Int. Adv. Nondestruc. Test., 5: 321-325.

Bhattacharya, S. Ve Sengupta, P.R., (1978), "Effects of Initial Stresses on Reflection and Refraction of Plane Waves at a Plane Interface of Two Elastic Solid Media", Gerlands Beitrt. Geophys., 87 (5): 395-402.

Biot, M.A., (1965), Mechanics of Incremental Deformations, Willey, New York.

Bradford, L.J. ve Dong, S., (1975), "Elastodynamic Behaviour of Laminted Orthotrophic Plates Under Initial Stress", Int. J. Solids and Struc., 11 (2): 213-230.

Brunelle, E.J., (1973), "Surface Wave Propogation under Initial Tension or Compression", Bull. Seismol. Soc. Amer., 63 (6) Pt.1, 1895-1899.

Chadwick, P. ve Jarvis, D.A., (1979a), "Surface Waves in a Prestressed Elastic Body", Proc. Roy. Soc. London, A, 366 (1727): 517-536.

Chadwick, P. ve Jarvis, D.A., (1979b), "Interfacial Waves in a Prestrained non-Hookean Body I-Biaxial States of Strain", Quart. J. Mech. And Appl. Math., 32 (4): 387-399.

Chadwick, P. ve Jarvis, D.A., (1979c), "Interfacial Waves in a Prestrained non-Hookean Body II-Triaxial States of Strain", Quart. J. Mech. And Appl. Math., 32 (4): 401-418.

Chakraborty, S.K. ve De R.K., (1982), "The Propogation of Disturbance Due to a Normal Pressure over the Boundary of an Initially Stressed Anisotropic Elastic Solid", Rev. Roum. Sci. Tech. Ser. Mech. Appl. 27 (1): 129-136.

Chakraborty, S.K. ve Dey. S., (1980), "Reflection and Refraction of Plane Shear Waves in Initially Stressed Orthotropic Elastic Media", Gerlands Beitrt. Geophys., 89 (5): 425-436.

Chakraborty, S.K. ve Dey. S., (1982), "The Disturbance due to Plane and Line Sources in Prestressed Semi-Infinite Elastic Solid", Int. J. Solids and Struc., 18 (12), 1153-1164.

Chattopadhyay, A. ve Kar, B.K., (1981), "Love Waves due to a Point Source in an Isotropic Elastic Medium under Initial Stress", Int. J. Non-Linear Mech., 16 (3/4): 247-258.

Chattopadhyay, A., Pal, A.K. ve Kushawha, V., (1982), "Generation of Love Waves under Initial Stress due to a Momentary Point Source", Indian J. Pure and Appl. Math., 13 (7): 817-828.

Chen, W.T. ve Wright, T.W., (1966), Frequency Equations for Wave Propogation in an Initially Stressed Circular Cylinder", J. Accoust. Soc. Amer., 39 (5), Pt.1: 847-848.

Crecraft, D.I., (1962), "Ultrasonic Wave Velocities in Stressed Nickel Steel", Nature, 195 (4847): 1193-1194.

Demiray, H. ve Şuhubi, E.S., (1970), Small Torsional Oscilations in Initially Twisted Circular Rubber Cylinder", Int. J. Eng. Sci.8 (1).

Dey, S., (1971), "Wave Propogation in Two Layered Medium under Initial Stresses", Pure and Appl. Geophys., 90 (7): 38-52.

Dey. S. ve Addy, S.K., (1977), "Reflection of Plane Waves under Initial Stresses at a Free Surface", Int. J. Non-Linear Mech., 12 (6): 371-381.

Eringen, A.C., (1964), Non-linear Theory of Continuous Media, Mc Graw Hill, New York.

Eringen, A.C. ve Şuhubi, E.S. (1974a), Elastodynamics I-Finite Motions, Academic Press, New-York.

Eringen, A.C. ve Şuhubi, E.S. (1974b), Elastodynamics II, Academic Press, New-York.

Ewing, M. E., Jardetzky, W.S. ve Press, E., (1957), Elastic Waves in Layered Media, McGraw-Hill, New York.

Fine, A.D., ve Shield R.T., (1966), "Second-Order Effects in The Propogation of Elastic Waves", Int. J. Solids and Struct., 2 (4): 605-620.

Green, A.E., (1963), "A Note on Wave Propogation in Initially Deformed Bodies", J. Mech. And Phys. Solids, 11 (2): 119-126.

Green, A.E., ve Atkins, J.E., (1960), Large Elastic Deformations and Non-Linear Continuum Mechanics, Oxford Press, ?.

Green, A.E., Rivlin, R.S. ve Shield, R.T., (1952), "General Theory of Small Elastic Deformations Superposed on Finite Elastic Deformations", Proc. Roy. Soc. Ser. A., 211 (1104): 128-154.

Guz, A.N., (1986a), Elastic Waves in Solids with Initial Stresses, Vol.1-General Theory, Naukowa-Dumka, Kiev.

Guz, A.N., (1986b), Elastic Waves in Solids with Initial Stresses, Vol.2-Propogation Regularities, Naukowa-Dumka, Kiev.

Guz, A.N., (1998), "Surface Waves in Bodies with Initial Stresses and an Ultrasonic Nondestructive Method for Determining Stresses in Near-Surface Layers of Solids", Int. Appl. Mech., 315-326.

Guz, A.N., (1999), Fundamentals of the Three-Dimensional Theory of Stability of Deformable Bodies, Springer-Verlag, New-York.

Hayes, M. ve Rivlin, R.S., (1961a), "Propogation of a Plane Wave in an Isotropic Elastic Material subjected to Pure Homogenous Deformation" Arch. Rat. Mech. And Anal., 8 (1): 15-22.

Hayes, M. ve Rivlin, R.S., (1961b), "Surface Waves in Deformed Materials" Arch. Rat. Mech. And Anal., 5: 358-380.

Kladinoga, V.S., ve Bogdanov, V.L., (2000), "Vibrations of Shallow Non-circular Membranes with Initial Stresses", Int. Appl. Mech., 829-837.

Koshman, V.P., (1981a), "Dynamics of an Incompressible Half-Plane With Initial Strains", Soviet Appl. Mech., 817-821.

Koshman, V.P., (1981b), "Lamb's Plane Problem for a Compressible Half-Space With Initial Stresses", Soviet Appl. Mech., 912-917.

Kostrov, B.V. ve Nikitin, L.V., (1969), "The Propogation of Seismic Waves in Prestressed Elastic Medium", C.R. Union Geodes. et Geophys. Internat., (15), Pt.1: 98.

Kurashige, M., (1972), "Shear Waves Guided by a Cylindrical Hole in a Finitely Deformed Elastic Solid, Trans. ASME. E., 39 (3): 703-708.

Kurashige, M., (1974), Radial propogation of Axial Shear Waves in a Finitely Deformed Elastic Solid, Trans. ASME. E., 41 (1): 83-88.

Mott. G., (1971), "Equations of Elastic Motion of an Isotropic Medium in the Presence of Body Forces and Static Stresses", J. Acoust. Soc. Amer., 50 (3) Pt.2: 859-868.

Pan, U.C., (1979), "SH-Waves in two layered inhomogenous Medium under Initial Stresses", Bull. Calcutta. Math. Soc., 71 (2): 86-93.

Payton, R.G., (1969), "Two-dimensional Wave-front Shaped Induced in a Homogenously-Strained Elastic Body by a Point Perturbing Body Force", Arch. Rat. Mech. And Anal., 32 (4): 311-330.

Payton, R.G., (1971), "Two dimensional Anisotropic Elastic Waves Emanating from a Point Source", Proc. Cambr. Phil. Soc. 70 (1): 191-210.

Payton, R.G., (1972), "Wave-front Singularities for two-dimensional Anisotropic Elastic Waves", Proc. Camb. Phil. Soc. 72 (2): 105-116.

Ramakanth, J., (1964), "Longitudinal Vibrations of Prestressed Circular Cylinder", Bull. Acad. Pol. Sci. Ser. Tech., 12 (11): 495-503.

Reismann, H. ve Pawlik, P.S., (1979), "Dynamics of Initially-Stressed Hyperelactic Solids", ZAMM, 59 (4): 145-155.

Rogers, J., Moodie, T.B., ve Clements, D.L., (1976), "Radial Propogation of Rotationary Shear Waves in an Initially-Stressed non-Hookean Material", J. Mech., 15 (4): 595-614.

Sawyers, K.N. ve Rivlin, R.S., (1978), "On the Speed of Propogation of Waves in a Deformed Compressible Elastic Material, ZAMP. 29 (2): 245-251.

Smith R.T., (1963), "Stress-Induced Anisotropy in Solids, -The Accustoelastic Effect-", Ultrasonics, 1 (3): 135-147.

Surhendu, D., (1972), "Torsional Waves under Initial Stress", Pure and Appl. Geophys., 94 (2).

Şuhubi, E.S., (1965), "Small Longitudinal Vibrations of an Initially-Stressed Circular Cylinder", Int. J. Eng. Sci., 2 (5): 509-515.

Tang, S., (1967), "Wave Propogation in Initially-Stressed Elastic Solids", Acta Mech., 4 (1): 92-106.

Toupin, R.A.ve Bernstein, B., (1961). "Sound Waves in Deformed Perfectly Elastic Materials: Acoustoelastic Effect", J. Acoust. Soc.Amer., 33 (2): 216-225..

Trusdell, C., (1961), "General and Exact Theory of Waves in Finite Elastic Strain", Arch. Rat. Mech. And Anal., 8 (4): 263-296.

Wagh, D.K., (1974), "Effect on Constant Initial Stress on the Love Wave Propogation, Acta Geophys., 22 (1): 3-9.

Walton, K. (1973), "Seismic Waves in Prestrained Media", Geophys. Astron. Roy. Soc., 31 (4): 373-394.

Weitsman, Y., (1973), "On the Reflection of Harmonic Waves in Fiberreinforced Materials". J. Sound and Vibr., 26 (1): 73-89.

Wilson, A.J., (1972), "Wave Propogation in Uniaxially-Stressed Elastic Media", Pure and Appl. Geophys., 93 (1): 5-18.

Wilson, A.J., (1973a), "Surface and Plate Waves in Biaxially-Stressed Elastic Media", Pure and Appl. Geophys., 102 (1): 182-192.

Wilson, A.J., (1973b), "SH Waves in Stressed Elastic Plates", Pure and Appl. Geophys., 110 (9): 1977-1981.

Wilson, A.J., (1977), "Wave Propogation in Prestressed Thin Elastic Plates", Int. J. Eng. Sci., 15 (4): 245-251.

EKLER

EK 1. Lineer Denklem Sisteminin Çözümünde Kullanılan Kısaltmalar.

$$\chi_1 = (\lambda^{(1)} + 2\mu^{(1)})\upsilon^{(1)2} - \lambda^{(1)}k^2 \tag{E1.1}$$
$$\chi_2 = 2\mu^{(1)}k\upsilon'^{(1)} \tag{E1.2}$$
$$\chi_3 = 2\mu^{(1)}k\upsilon^{(1)} \tag{E1.3}$$
$$\chi_4 = \mu^{(1)}\left[\left(2+\eta_2^{(1)}\right)k^2 - k_T^{(1)2}\right] \tag{E1.4}$$
$$\chi_5 = (\lambda^{(2)} + 2\mu^{(2)})\upsilon^{(2)2} - \lambda^{(2)}k^2 \tag{E1.5}$$
$$\chi_6 = 2\mu^{(2)}k\upsilon'^{(2)} \tag{E1.6}$$
$$\chi_7 = 2\mu^{(2)}k\upsilon^{(2)} \tag{E1.7}$$
$$\chi_8 = \mu^{(2)}\left[\left(2+\eta_2^{(2)}\right)k^2 - k_T^{(2)2}\right] \tag{E1.8}$$
$$\chi_9 = -(\lambda^{(1)} + 2\mu^{(1)})k^2 + \lambda^{(1)}\upsilon^{(1)2} \tag{E1.9}$$
$$\chi_{10} = -(\lambda^{(2)} + 2\mu^{(2)})k^2 + \lambda^{(2)}\upsilon^{(2)2} \tag{E1.10}$$

$$H_1 = e^{-\upsilon^{(1)}x_2} \tag{E1.11}$$
$$H_2 = e^{\upsilon^{(1)}x_2} \tag{E1.12}$$
$$H_3 = e^{-\upsilon'^{(1)}x_2} \tag{E1.13}$$
$$H_4 = e^{\upsilon'^{(1)}x_2} \tag{E1.14}$$
$$H_5 = e^{-\upsilon^{(2)}x_2} \tag{E1.15}$$
$$H_6 = e^{-\upsilon'^{(2)}x_2} \tag{E1.16}$$

$$\overline{H}_1 = e^{-\upsilon^{(1)}h} \tag{E1.17}$$
$$\overline{H}_2 = e^{\upsilon^{(1)}h} \tag{E1.18}$$
$$\overline{H}_3 = e^{-\upsilon'^{(1)}h} \tag{E1.19}$$
$$\overline{H}_4 = e^{\upsilon'^{(1)}h} \tag{E1.20}$$
$$\overline{H}_5 = e^{-\upsilon^{(2)}h} \tag{E1.21}$$
$$\overline{H}_6 = e^{-\upsilon'^{(2)}h} \tag{E1.22}$$

$$\begin{aligned}\Gamma_1 = {} & k^2(\chi_5\chi_8 + \chi_2\chi_7 + \chi_1\chi_4 + \chi_2\chi_3 - \chi_6\chi_7 - \chi_4\chi_5 - \chi_1\chi_8 - \chi_3\chi_6) \\ & + k(\chi_4\chi_6 - \chi_2\chi_8)(\upsilon^{(1)} + \upsilon^{(2)}) \\ & + k(\chi_1\chi_7 + \chi_3\chi_5)(\upsilon'^{(2)} - \upsilon'^{(1)}) \\ & + \upsilon'^{(1)}\upsilon^{(1)}(\chi_5\chi_8 - \chi_6\chi_7) + \upsilon'^{(1)}\upsilon^{(2)}(\chi_1\chi_8 + \chi_3\chi_6) \\ & + \upsilon'^{(2)}\upsilon^{(1)}(\chi_2\chi_7 - \chi_4\chi_5) - \upsilon'^{(2)}\upsilon^{(2)}(\chi_1\chi_4 + \chi_2\chi_3)\end{aligned} \tag{E1.23}$$

$$\begin{aligned}\Gamma_2 = {} & 2\upsilon'^{(1)}k(\chi_6\chi_7 - \chi_5\chi_8) + 2\upsilon'^{(1)}\chi_4(k\chi_5 - \upsilon^{(2)}\chi_6) \\ & + 2k^2(\chi_2\chi_8 - \chi_2\chi_4) + 2\upsilon'^{(2)}\chi_2(\upsilon^{(2)}\chi_4 - k\chi_7)\end{aligned} \tag{E1.24}$$

$$\begin{aligned}\Gamma_3 = {} & k^2(\chi_5\chi_8 + \chi_1\chi_4 - \chi_6\chi_7 - \chi_2\chi_7 - \chi_4\chi_5 - \chi_1\chi_8 - \chi_2\chi_3 - \chi_3\chi_6) \\ & + k(\chi_2\chi_8 + \chi_4\chi_6)(\upsilon^{(1)} + \upsilon^{(2)}) \\ & + k(\chi_1\chi_7 + \chi_3\chi_5)(\upsilon'^{(1)} + \upsilon'^{(2)}) \\ & + \upsilon'^{(1)}\upsilon^{(1)}(\chi_6\chi_7 - \chi_5\chi_8) - \upsilon'^{(1)}\upsilon^{(2)}(\chi_1\chi_8 + \chi_3\chi_6) \\ & - \upsilon'^{(2)}\upsilon^{(1)}(\chi_2\chi_7 + \chi_4\chi_5) + \upsilon'^{(2)}\upsilon^{(2)}(\chi_2\chi_3 - \chi_1\chi_4)\end{aligned} \tag{E1.25}$$

$$\begin{aligned}\Gamma_4 = {} & k^2(\chi_5\chi_8 + \chi_1\chi_4 + \chi_3\chi_6 + \chi_2\chi_3 - \chi_4\chi_5 - \chi_1\chi_8 - \chi_2\chi_7 - \chi_6\chi_7) \\ & + k(\chi_2\chi_8 + \chi_4\chi_6)(\upsilon^{(2)} - \upsilon^{(1)}) \\ & + k(\chi_1\chi_7 - \chi_3\chi_5)(\upsilon'^{(1)} + \upsilon'^{(2)}) \\ & + \upsilon'^{(1)}\upsilon^{(1)}(\chi_5\chi_8 - \chi_6\chi_7) + \upsilon'^{(1)}\upsilon^{(2)}(\chi_3\chi_6 - \chi_1\chi_8) \\ & + \upsilon'^{(2)}\upsilon^{(1)}(\chi_2\chi_7 + \chi_4\chi_5) - \upsilon'^{(2)}\upsilon^{(2)}(\chi_2\chi_3 + \chi_1\chi_4)\end{aligned} \tag{E1.26}$$

$$\begin{aligned}\Gamma_5 = {} & k^2(\chi_5\chi_8 + \chi_1\chi_4 + \chi_3\chi_6 - \chi_2\chi_3 - \chi_4\chi_5 - \chi_1\chi_8 + \chi_2\chi_7 - \chi_6\chi_7) \\ & + k(\chi_2\chi_8 - \chi_4\chi_6)(\upsilon^{(1)} - \upsilon^{(2)}) \\ & + k(\chi_3\chi_5 - \chi_1\chi_7)(\upsilon'^{(1)} - \upsilon'^{(2)}) \\ & + \upsilon'^{(1)}\upsilon^{(1)}(\chi_6\chi_7 - \chi_8\chi_5) + \upsilon'^{(1)}\upsilon^{(2)}(\chi_1\chi_8 - \chi_3\chi_6) \\ & + \upsilon'^{(2)}\upsilon^{(1)}(\chi_4\chi_5 - \chi_2\chi_7) + \upsilon'^{(2)}\upsilon^{(2)}(\chi_2\chi_3 - \chi_1\chi_4)\end{aligned} \tag{E1.27}$$

$$\begin{aligned}\Gamma_6 = {} & 2k\upsilon^{(1)}(\chi_6\chi_7 - \chi_5\chi_8) + 2k\chi_3(k\chi_5 - \upsilon^{(2)}\chi_6) \\ & + 2k\chi_1(\upsilon^{(1)}\chi_8 - k\chi_3) + 2\upsilon'^{(2)}\chi_1(\upsilon^{(2)}\chi_3 - \upsilon^{(1)}\chi_7)\end{aligned} \tag{E1.28}$$

$$\begin{aligned}\Gamma_7 = {} & 2\upsilon'^{(1)}k(\chi_1\chi_8 - \chi_1\chi_4) + 2\upsilon'^{(1)}\chi_6(k\chi_3 - \upsilon^{(1)}\chi_4) \\ & + 2k^2(\chi_2\chi_4 - \chi_2\chi_8) + 2\upsilon'^{(2)}\chi_2(\upsilon^{(1)}\chi_4 - k\chi_3)\end{aligned} \tag{E1.29}$$

$$\begin{aligned}\Gamma_8 = {} & 2\upsilon'^{(1)}k(\chi_1\chi_8 - \chi_3\chi_6) + 2\upsilon'^{(1)}\chi_4(\upsilon^{(1)}\chi_6 - k\chi_1) \\ & + 2k^2(\chi_2\chi_4 - \chi_2\chi_8) + 2\upsilon'^{(2)}\chi_2(k\chi_3 - \upsilon^{(1)}\chi_4)\end{aligned} \tag{E1.30}$$

$$\begin{aligned}\Gamma_9 = {} & -2\upsilon^{(1)}k(\chi_2\chi_8 + \chi_4\chi_6) + 2\upsilon'^{(2)}\chi_1(\upsilon^{(1)}\chi_4 - k\chi_3) \\ & + 2k^2(\chi_3\chi_2 + \chi_3\chi_6) + 2\upsilon'^{(1)}\chi_1(\upsilon^{(1)}\chi_8 - k\chi_3)\end{aligned} \tag{E1.31}$$

$$\begin{aligned}\Gamma_{10} = {} & 2\upsilon^{(1)}k(\chi_2\chi_8 - \chi_4\chi_6) + 2\upsilon'^{(2)}\chi_1(\upsilon^{(1)}\chi_4 - k\chi_3) \\ & + 2k^2(\chi_3\chi_6 - \chi_3\chi_2) + 2\upsilon'^{(1)}\chi_1(k\chi_3 - \upsilon^{(1)}\chi_8)\end{aligned} \tag{E1.32}$$

$$\begin{aligned}\Gamma_{11} = {} & -2k\upsilon'^{(1)}(\chi_3\chi_5 + \chi_1\chi_7) + 2k^2(\chi_2\chi_7 + \chi_2\chi_3) \\ & - 2k\chi_2(\upsilon^{(1)}\chi_4 + \upsilon^{(2)}\chi_4) + 2\upsilon'^{(1)}\chi_4(\upsilon^{(1)}\chi_5 + \upsilon^{(2)}\chi_1)\end{aligned} \tag{E1.33}$$

$$\begin{aligned}\Gamma_{12} = {} & 2k\upsilon'^{(1)}(\chi_3\chi_5 - \chi_1\chi_7) + 2k^2(\chi_2\chi_7 - \chi_2\chi_3) \\ & + 2\upsilon'^{(1)}\chi_4(\upsilon^{(2)}\chi_1 - \upsilon^{(1)}\chi_5) + 2k\chi_2(\upsilon^{(1)}\chi_4 - \upsilon^{(2)}\chi_4)\end{aligned} \tag{E1.34}$$

$$\begin{aligned}\Gamma_{13} = {} & -2k\upsilon^{(1)}(\chi_4\chi_5 + \chi_2\chi_7) + 2k\chi_3(k\chi_5 + \upsilon^{(2)}\chi_2) \\ & + 2\upsilon'^{(1)}\chi_1(\upsilon^{(1)}\chi_7 - \upsilon^{(2)}\chi_3) + 2k\chi_1(\upsilon^{(1)}\chi_4 - k\chi_3)\end{aligned} \tag{E1.35}$$

$$\begin{aligned}\Gamma_{14} = {} & 2k\upsilon^{(1)}(\chi_2\chi_7 - \chi_4\chi_5) + 2k\chi_3(k\chi_5 - \upsilon^{(2)}\chi_2) \\ & + 2\upsilon'^{(1)}\chi_1(\upsilon^{(2)}\chi_3 - \upsilon^{(1)}\chi_7) + 2k\chi_1(\upsilon^{(1)}\chi_4 - k\chi_3)\end{aligned} \tag{E1.36}$$

$$\Delta_1 = \begin{vmatrix} -P_0 & \chi_1 & -i\chi_2 & i\chi_2 & 0 & 0 \\ 0 & -i\chi_3 & \chi_4 & \chi_4 & 0 & 0 \\ 0 & -ik\overline{H}_2 & -\upsilon'^{(1)}\overline{H}_3 & \upsilon'^{(1)}\overline{H}_4 & ik\overline{H}_5 & \upsilon'^{(2)}\overline{H}_6 \\ 0 & \upsilon^{(1)}\overline{H}_2 & ik\overline{H}_3 & ik\overline{H}_4 & \upsilon^{(2)}\overline{H}_5 & -ik\overline{H}_6 \\ 0 & \chi_1\overline{H}_2 & -i\chi_2\overline{H}_3 & i\chi_2\overline{H}_4 & -\chi_5\overline{H}_5 & i\chi_6\overline{H}_6 \\ 0 & -i\chi_3\overline{H}_2 & \chi_4\overline{H}_3 & \chi_4\overline{H}_4 & -i\chi_7\overline{H}_5 & -\chi_8\overline{H}_6 \end{vmatrix} \tag{E1.37}$$

$$= -P_0\overline{H}_5\overline{H}_6\left[\chi_4\overline{H}_2(\overline{H}_3\Gamma_1 - \overline{H}_4\Gamma_3) + \chi_3\overline{H}_3\overline{H}_4\Gamma_2\right]$$

$$\Delta_2 = \begin{vmatrix} \chi_1 & -P_0 & -i\chi_2 & i\chi_2 & 0 & 0 \\ i\chi_3 & 0 & \chi_4 & \chi_4 & 0 & 0 \\ -ik\overline{H}_1 & 0 & -\upsilon'^{(1)}\overline{H}_3 & \upsilon'^{(1)}\overline{H}_4 & ik\overline{H}_5 & \upsilon'^{(2)}\overline{H}_6 \\ -\upsilon^{(1)}\overline{H}_1 & 0 & ik\overline{H}_3 & ik\overline{H}_4 & \upsilon^{(2)}\overline{H}_5 & -ik\overline{H}_6 \\ \chi_1\overline{H}_1 & 0 & -i\chi_2\overline{H}_3 & i\chi_2\overline{H}_4 & -\chi_5\overline{H}_5 & i\chi_6\overline{H}_6 \\ i\chi_3\overline{H}_1 & 0 & \chi_4\overline{H}_3 & \chi_4\overline{H}_4 & -i\chi_7\overline{H}_5 & -\chi_8\overline{H}_6 \end{vmatrix} \tag{E1.38}$$

$$= P_0\overline{H}_5\overline{H}_6\left[\chi_4\overline{H}_1(\overline{H}_3\Gamma_5 - \overline{H}_4\Gamma_4) - \chi_3\overline{H}_3\overline{H}_4\Gamma_2\right]$$

$$\Delta_3 = \begin{vmatrix} \chi_1 & \chi_1 & -P_0 & i\chi_2 & 0 & 0 \\ i\chi_3 & -i\chi_3 & 0 & \chi_4 & 0 & 0 \\ -ik\overline{H}_1 & -ik\overline{H}_2 & 0 & \upsilon'^{(1)}\overline{H}_4 & ik\overline{H}_5 & \upsilon'^{(2)}\overline{H}_6 \\ -\upsilon^{(1)}\overline{H}_1 & \upsilon^{(1)}\overline{H}_2 & 0 & ik\overline{H}_4 & \upsilon^{(2)}\overline{H}_5 & -ik\overline{H}_6 \\ \chi_1\overline{H}_1 & \chi_1\overline{H}_2 & 0 & i\chi_2\overline{H}_4 & -\chi_5\overline{H}_5 & i\chi_6\overline{H}_6 \\ i\chi_3\overline{H}_1 & -i\chi_3\overline{H}_2 & 0 & \chi_4\overline{H}_4 & -i\chi_7\overline{H}_5 & -\chi_8\overline{H}_6 \end{vmatrix} \tag{E1.39}$$

$$= -iP_0\overline{H}_5\overline{H}_6\left[\chi_3\overline{H}_4(\overline{H}_2\Gamma_3 + \overline{H}_1\Gamma_4) + \chi_4\overline{H}_1\overline{H}_2\Gamma_6\right]$$

$$\Delta_4 = \begin{vmatrix} \chi_1 & \chi_1 & -i\chi_2 & -P_0 & 0 & 0 \\ i\chi_3 & -i\chi_3 & \chi_4 & 0 & 0 & 0 \\ -ik\overline{H}_1 & -ik\overline{H}_2 & -\upsilon'^{(1)}\overline{H}_3 & 0 & ik\overline{H}_5 & \upsilon'^{(2)}\overline{H}_6 \\ -\upsilon^{(1)}\overline{H}_1 & \upsilon^{(1)}\overline{H}_2 & ik\overline{H}_3 & 0 & \upsilon^{(2)}\overline{H}_5 & -ik\overline{H}_6 \\ \chi_1\overline{H}_1 & \chi_1\overline{H}_2 & -i\chi_2\overline{H}_3 & 0 & -\chi_5\overline{H}_5 & i\chi_6\overline{H}_6 \\ i\chi_3\overline{H}_1 & -i\chi_3\overline{H}_2 & \chi_4\overline{H}_3 & 0 & -i\chi_7\overline{H}_5 & -\chi_8\overline{H}_6 \end{vmatrix} \tag{E1.40}$$

$$= iP_0\overline{H}_5\overline{H}_6\left[\chi_3\overline{H}_3(\overline{H}_2\Gamma_1 + \overline{H}_1\Gamma_5) + \chi_4\overline{H}_1\overline{H}_2\Gamma_6\right]$$

$$\Delta_5=\begin{vmatrix}\chi_1 & \chi_1 & -i\chi_2 & i\chi_2 & -P_0 & 0\\ i\chi_3 & -i\chi_3 & \chi_4 & \chi_4 & 0 & 0\\ -ik\overline{H}_1 & -ik\overline{H}_2 & -\upsilon'^{(1)}\overline{H}_3 & \upsilon'^{(1)}\overline{H}_4 & 0 & \upsilon'^{(2)}\overline{H}_6\\ -\upsilon^{(1)}\overline{H}_1 & \upsilon^{(1)}\overline{H}_2 & ik\overline{H}_3 & ik\overline{H}_4 & 0 & -ik\overline{H}_6\\ \chi_1\overline{H}_1 & \chi_1\overline{H}_2 & -i\chi_2\overline{H}_3 & i\chi_2\overline{H}_4 & 0 & i\chi_6\overline{H}_6\\ i\chi_3\overline{H}_1 & -i\chi_3\overline{H}_2 & \chi_4\overline{H}_3 & \chi_4\overline{H}_4 & 0 & -\chi_8\overline{H}_6\end{vmatrix} \tag{E1.41}$$

$$=P_0\overline{H}_6\left[\chi_3\overline{H}_3\overline{H}_4(\overline{H}_2\Gamma_7+\overline{H}_1\Gamma_8)+\chi_4\overline{H}_1\overline{H}_2(\overline{H}_3\Gamma_{10}-\overline{H}_4\Gamma_9)\right]$$

$$\Delta_6=\begin{vmatrix}\chi_1 & \chi_1 & -i\chi_2 & i\chi_2 & 0 & -P_0\\ i\chi_3 & -i\chi_3 & \chi_4 & \chi_4 & 0 & 0\\ -ik\overline{H}_1 & -ik\overline{H}_2 & -\upsilon'^{(1)}\overline{H}_3 & \upsilon'^{(1)}\overline{H}_4 & ik\overline{H}_5 & 0\\ -\upsilon^{(1)}\overline{H}_1 & \upsilon^{(1)}\overline{H}_2 & ik\overline{H}_3 & ik\overline{H}_4 & \upsilon^{(2)}\overline{H}_5 & 0\\ \chi_1\overline{H}_1 & \chi_1\overline{H}_2 & -i\chi_2\overline{H}_3 & i\chi_2\overline{H}_4 & -\chi_5\overline{H}_5 & 0\\ i\chi_3\overline{H}_1 & -i\chi_3\overline{H}_2 & \chi_4\overline{H}_3 & \chi_4\overline{H}_4 & -i\chi_7\overline{H}_5 & 0\end{vmatrix} \tag{E1.42}$$

$$=iP_0\overline{H}_5\left[\chi_3\overline{H}_3\overline{H}_4(\overline{H}_2\Gamma_{11}+\overline{H}_1\Gamma_{12})+\chi_4\overline{H}_1\overline{H}_2(\overline{H}_4\Gamma_{13}-\overline{H}_3\Gamma_{14})\right]$$

$$\alpha_1=\frac{\Delta_1}{\Delta}=-P_0\frac{\chi_4\overline{H}_2(\overline{H}_3\Gamma_1-\overline{H}_4\Gamma_3)+\chi_3\overline{H}_3\overline{H}_4\Gamma_2}{\begin{array}{c}\left[(\chi_2\chi_3+\chi_1\chi_4)(\overline{H}_2\overline{H}_3\Gamma_1+\overline{H}_1\overline{H}_4\Gamma_4)+(\chi_2\chi_3-\chi_1\chi_4)(\overline{H}_1\overline{H}_3\Gamma_5+\overline{H}_2\overline{H}_4\Gamma_3)\right.\\ \left.+2(\chi_1\chi_3\overline{H}_3\overline{H}_4\Gamma_2+\chi_2\chi_4\overline{H}_1\overline{H}_2\Gamma_6)\right]\end{array}} \tag{E1.43}$$

$$\alpha_2=\frac{\Delta_2}{\Delta}=P_0\frac{\chi_4\overline{H}_1(\overline{H}_3\Gamma_5-\overline{H}_4\Gamma_4)-\chi_3\overline{H}_3\overline{H}_4\Gamma_2}{\begin{array}{c}\left[(\chi_2\chi_3+\chi_1\chi_4)(\overline{H}_2\overline{H}_3\Gamma_1+\overline{H}_1\overline{H}_4\Gamma_4)+(\chi_2\chi_3-\chi_1\chi_4)(\overline{H}_1\overline{H}_3\Gamma_5+\overline{H}_2\overline{H}_4\Gamma_3)\right.\\ \left.+2(\chi_1\chi_3\overline{H}_3\overline{H}_4\Gamma_2+\chi_2\chi_4\overline{H}_1\overline{H}_2\Gamma_6)\right]\end{array}} \tag{E1.44}$$

$$\beta_1=\frac{\Delta_3}{\Delta}=-iP_0\frac{\chi_3\overline{H}_4(\overline{H}_2\Gamma_3+\overline{H}_1\Gamma_4)+\chi_4\overline{H}_1\overline{H}_2\Gamma_6}{\begin{array}{c}\left[(\chi_2\chi_3+\chi_1\chi_4)(\overline{H}_2\overline{H}_3\Gamma_1+\overline{H}_1\overline{H}_4\Gamma_4)+(\chi_2\chi_3-\chi_1\chi_4)(\overline{H}_1\overline{H}_3\Gamma_5+\overline{H}_2\overline{H}_4\Gamma_3)\right.\\ \left.+2(\chi_1\chi_3\overline{H}_3\overline{H}_4\Gamma_2+\chi_2\chi_4\overline{H}_1\overline{H}_2\Gamma_6)\right]\end{array}} \tag{E1.45}$$

$$\beta_2=\frac{\Delta_4}{\Delta}=iP_0\frac{\chi_3\overline{H}_3(\overline{H}_2\Gamma_1+\overline{H}_1\Gamma_5)+\chi_4\overline{H}_1\overline{H}_2\Gamma_6}{\begin{array}{c}\left[(\chi_2\chi_3+\chi_1\chi_4)(\overline{H}_2\overline{H}_3\Gamma_1+\overline{H}_1\overline{H}_4\Gamma_4)+(\chi_2\chi_3-\chi_1\chi_4)(\overline{H}_1\overline{H}_3\Gamma_5+\overline{H}_2\overline{H}_4\Gamma_3)\right.\\ \left.+2(\chi_1\chi_3\overline{H}_3\overline{H}_4\Gamma_2+\chi_2\chi_4\overline{H}_1\overline{H}_2\Gamma_6)\right]\end{array}} \tag{E1.46}$$

$$\alpha_3=\frac{\Delta_5}{\Delta}=P_0\frac{\chi_3\overline{H}_3\overline{H}_4(\overline{H}_2\Gamma_7+\overline{H}_1\Gamma_8)+\chi_4\overline{H}_1\overline{H}_2(\overline{H}_3\Gamma_{10}-\overline{H}_4\Gamma_9)}{\begin{array}{c}\overline{H}_5\left[(\chi_2\chi_3+\chi_1\chi_4)(\overline{H}_2\overline{H}_3\Gamma_1+\overline{H}_1\overline{H}_4\Gamma_4)+(\chi_2\chi_3-\chi_1\chi_4)(\overline{H}_1\overline{H}_3\Gamma_5+\overline{H}_2\overline{H}_4\Gamma_3)\right.\\ \left.+2(\chi_1\chi_3\overline{H}_3\overline{H}_4\Gamma_2+\chi_2\chi_4\overline{H}_1\overline{H}_2\Gamma_6)\right]\end{array}} \tag{E1.47}$$

$$\beta_3=\frac{\Delta_6}{\Delta}=iP_0\frac{\chi_3\overline{H}_3\overline{H}_4(\overline{H}_2\Gamma_{11}+\overline{H}_1\Gamma_{12})+\chi_4\overline{H}_1\overline{H}_2(\overline{H}_4\Gamma_{13}-\overline{H}_3\Gamma_{14})}{\begin{array}{c}\overline{H}_6\left[(\chi_2\chi_3+\chi_1\chi_4)(\overline{H}_2\overline{H}_3\Gamma_1+\overline{H}_1\overline{H}_4\Gamma_4)+(\chi_2\chi_3-\chi_1\chi_4)(\overline{H}_1\overline{H}_3\Gamma_5+\overline{H}_2\overline{H}_4\Gamma_3)\right.\\ \left.+2(\chi_1\chi_3\overline{H}_3\overline{H}_4\Gamma_2+\chi_2\chi_4\overline{H}_1\overline{H}_2\Gamma_6)\right]\end{array}} \tag{E1.48}$$

$$\overline{\alpha}_1=\frac{\chi_4\overline{H}_2(\overline{H}_3\Gamma_1-\overline{H}_4\Gamma_3)+\chi_3\overline{H}_3\overline{H}_4\Gamma_2}{\begin{array}{c}\left[(\chi_2\chi_3+\chi_1\chi_4)(\overline{H}_2\overline{H}_3\Gamma_1+\overline{H}_1\overline{H}_4\Gamma_4)+(\chi_2\chi_3-\chi_1\chi_4)(\overline{H}_1\overline{H}_3\Gamma_5+\overline{H}_2\overline{H}_4\Gamma_3)\right.\\ \left.+2(\chi_1\chi_3\overline{H}_3\overline{H}_4\Gamma_2+\chi_2\chi_4\overline{H}_1\overline{H}_2\Gamma_6)\right]\end{array}} \tag{E1.49}$$

$$\overline{\alpha}_2=\frac{\chi_4\overline{\overline{H}}_1(\overline{\overline{H}}_3\Gamma_5-\overline{\overline{H}}_4\Gamma_4)-\chi_3\overline{\overline{H}}_3\overline{\overline{H}}_4\Gamma_2}{\begin{array}{c}\left[(\chi_2\chi_3+\chi_1\chi_4)(\overline{\overline{H}}_2\overline{\overline{H}}_3\Gamma_1+\overline{\overline{H}}_1\overline{\overline{H}}_4\Gamma_4)+(\chi_2\chi_3-\chi_1\chi_4)(\overline{\overline{H}}_1\overline{\overline{H}}_3\Gamma_5+\overline{\overline{H}}_2\overline{\overline{H}}_4\Gamma_3)\right.\\ \left.+2(\chi_1\chi_3\overline{\overline{H}}_3\overline{\overline{H}}_4\Gamma_2+\chi_2\chi_4\overline{\overline{H}}_1\overline{\overline{H}}_2\Gamma_6)\right]\end{array}} \tag{E1.50}$$

$$\overline{\beta}_1=\frac{\chi_3\overline{\overline{H}}_4(\overline{\overline{H}}_2\Gamma_3+\overline{\overline{H}}_1\Gamma_4)+\chi_4\overline{\overline{H}}_1\overline{\overline{H}}_2\Gamma_6}{\begin{array}{c}\left[(\chi_2\chi_3+\chi_1\chi_4)(\overline{\overline{H}}_2\overline{\overline{H}}_3\Gamma_1+\overline{\overline{H}}_1\overline{\overline{H}}_4\Gamma_4)+(\chi_2\chi_3-\chi_1\chi_4)(\overline{\overline{H}}_1\overline{\overline{H}}_3\Gamma_5+\overline{\overline{H}}_2\overline{\overline{H}}_4\Gamma_3)\right.\\ \left.+2(\chi_1\chi_3\overline{\overline{H}}_3\overline{\overline{H}}_4\Gamma_2+\chi_2\chi_4\overline{\overline{H}}_1\overline{\overline{H}}_2\Gamma_6)\right]\end{array}} \tag{E1.51}$$

$$\overline{\beta}_2=\frac{\chi_3\overline{\overline{H}}_3(\overline{\overline{H}}_2\Gamma_1+\overline{\overline{H}}_1\Gamma_5)+\chi_4\overline{\overline{H}}_1\overline{\overline{H}}_2\Gamma_6}{\begin{array}{c}\left[(\chi_2\chi_3+\chi_1\chi_4)(\overline{\overline{H}}_2\overline{\overline{H}}_3\Gamma_1+\overline{\overline{H}}_1\overline{\overline{H}}_4\Gamma_4)+(\chi_2\chi_3-\chi_1\chi_4)(\overline{\overline{H}}_1\overline{\overline{H}}_3\Gamma_5+\overline{\overline{H}}_2\overline{\overline{H}}_4\Gamma_3)\right.\\ \left.+2(\chi_1\chi_3\overline{\overline{H}}_3\overline{\overline{H}}_4\Gamma_2+\chi_2\chi_4\overline{\overline{H}}_1\overline{\overline{H}}_2\Gamma_6)\right]\end{array}} \tag{E1.52}$$

$$\overline{\alpha}_3=\frac{\chi_3\overline{\overline{H}}_3\overline{\overline{H}}_4(\overline{\overline{H}}_2\Gamma_7+\overline{\overline{H}}_1\Gamma_8)+\chi_4\overline{\overline{H}}_1\overline{\overline{H}}_2(\overline{\overline{H}}_3\Gamma_{10}-\overline{\overline{H}}_4\Gamma_9)}{\begin{array}{c}\overline{\overline{H}}_5\left[(\chi_2\chi_3+\chi_1\chi_4)(\overline{\overline{H}}_2\overline{\overline{H}}_3\Gamma_1+\overline{\overline{H}}_1\overline{\overline{H}}_4\Gamma_4)+(\chi_2\chi_3-\chi_1\chi_4)(\overline{\overline{H}}_1\overline{\overline{H}}_3\Gamma_5+\overline{\overline{H}}_2\overline{\overline{H}}_4\Gamma_3)\right.\\ \left.+2(\chi_1\chi_3\overline{\overline{H}}_3\overline{\overline{H}}_4\Gamma_2+\chi_2\chi_4\overline{\overline{H}}_1\overline{\overline{H}}_2\Gamma_6)\right]\end{array}} \tag{E1.53}$$

$$\overline{\beta}_3=\frac{\chi_3\overline{\overline{H}}_3\overline{\overline{H}}_4(\overline{\overline{H}}_2\Gamma_{11}+\overline{\overline{H}}_1\Gamma_{12})+\chi_4\overline{\overline{H}}_1\overline{\overline{H}}_2(\overline{\overline{H}}_4\Gamma_{13}-\overline{\overline{H}}_3\Gamma_{14})}{\begin{array}{c}\overline{\overline{H}}_6\left[(\chi_2\chi_3+\chi_1\chi_4)(\overline{\overline{H}}_2\overline{\overline{H}}_3\Gamma_1+\overline{\overline{H}}_1\overline{\overline{H}}_4\Gamma_4)+(\chi_2\chi_3-\chi_1\chi_4)(\overline{\overline{H}}_1\overline{\overline{H}}_3\Gamma_5+\overline{\overline{H}}_2\overline{\overline{H}}_4\Gamma_3)\right.\\ \left.+2(\chi_1\chi_3\overline{\overline{H}}_3\overline{\overline{H}}_4\Gamma_2+\chi_2\chi_4\overline{\overline{H}}_1\overline{\overline{H}}_2\Gamma_6)\right]\end{array}} \tag{E1.54}$$

EK 2. $0 < k < k_L^{(1)} / \sqrt{1+\eta_1^{(1)}}$ **Durumu.**

Bu durumda (3.52) ve (3.53) uyarınca,

$$\upsilon^{(1)} = \left[k^2(1+\eta_1^{(1)}) - k_L^{(1)2}\right]^{1/2} = i\left[k_L^{(1)2} - k^2(1+\eta_1^{(1)})\right]^{1/2} = i\,\hat{\upsilon}^{(1)} \tag{E2.1}$$
$$\upsilon'^{(1)} = \left[k^2(1+\eta_2^{(1)}) - k_T^{(1)2}\right]^{1/2} = i\left[k_T^{(1)2} - k^2(1+\eta_2^{(1)})\right]^{1/2} = i\,\hat{\upsilon}'^{(1)} \tag{E2.2}$$
$$\upsilon^{(2)} = \left[k^2(1+\eta_1^{(2)}) - k_L^{(2)2}\right]^{1/2} = i\left[k_L^{(2)2} - k^2(1+\eta_1^{(2)})\right]^{1/2} = i\,\hat{\upsilon}^{(2)} \tag{E2.3}$$
$$\upsilon'^{(2)} = \left[k^2(1+\eta_2^{(2)}) - k_T^{(2)2}\right]^{1/2} = i\left[k_T^{(2)2} - k^2(1+\eta_2^{(2)})\right]^{1/2} = i\,\hat{\upsilon}'^{(2)} \tag{E2.4}$$

eşitlikleri kullanılır. Burada,

$$\hat{\upsilon}^{(1)} = \left[k_L^{(1)2} - k^2(1+\eta_1^{(1)})\right]^{1/2} \tag{E2.5}$$
$$\hat{\upsilon}'^{(1)} = \left[k_T^{(1)2} - k^2(1+\eta_2^{(1)})\right]^{1/2} \tag{E2.6}$$
$$\hat{\upsilon}^{(2)} = \left[k_L^{(2)2} - k^2(1+\eta_1^{(2)})\right]^{1/2} \tag{E2.7}$$
$$\hat{\upsilon}'^{(2)} = \left[k_T^{(2)2} - k^2(1+\eta_2^{(2)})\right]^{1/2} \tag{E2.8}$$

kısaltmaları yapılmıştır. Buna göre,

$$\hat{\chi}_1 = -(\lambda^{(1)} + 2\mu^{(1)})\hat{\upsilon}^{(1)2} - \lambda^{(1)}k^2 \tag{E2.9}$$
$$\hat{\chi}_2 = 2i\mu^{(1)}k\hat{\upsilon}'^{(1)} = i\hat{\chi}'_2 \tag{E2.10}$$
$$\hat{\chi}_3 = 2i\mu^{(1)}k\hat{\upsilon}^{(1)} = i\hat{\chi}'_3 \tag{E2.11}$$
$$\hat{\chi}_5 = -(\lambda^{(2)} + 2\mu^{(2)})\hat{\upsilon}^{(2)2} - \lambda^{(2)}k^2 \tag{E2.12}$$
$$\hat{\chi}_6 = 2i\mu^{(2)}k\hat{\upsilon}'^{(2)} = i\hat{\chi}'_6 \tag{E2.13}$$
$$\hat{\chi}_7 = 2i\mu^{(2)}k\hat{\upsilon}^{(2)} = i\hat{\chi}'_7 \tag{E2.14}$$
$$\hat{\chi}_9 = -(\lambda^{(1)} + 2\mu^{(1)})k^2 - \lambda^{(1)}\hat{\upsilon}^{(1)2} \tag{E2.15}$$
$$\hat{\chi}_{10} = -(\lambda^{(2)} + 2\mu^{(2)})k^2 - \lambda^{(2)}\hat{\upsilon}^{(2)2} \tag{E2.16}$$

ve,

$$\hat{\chi}'_2 = 2\mu^{(1)}k\hat{\upsilon}'^{(1)} \tag{E2.17}$$
$$\hat{\chi}'_3 = 2\mu^{(1)}k\hat{\upsilon}^{(1)} \tag{E2.18}$$
$$\hat{\chi}'_6 = 2\mu^{(2)}k\hat{\upsilon}'^{(2)} \tag{E2.19}$$
$$\hat{\chi}'_7 = 2\mu^{(2)}k\hat{\upsilon}^{(2)} \tag{E2.20}$$

kısaltmaları yapılarak,

$$^1\chi_1 = \hat{\chi}_1 \tag{E2.21}$$
$$^1\chi_2 = i\hat{\chi}'_2 \tag{E2.22}$$
$$^1\chi_3 = i\hat{\chi}'_3 \tag{E2.23}$$
$$^1\chi_4 = \chi_4 \tag{E2.24}$$
$$^1\chi_5 = \hat{\chi}_5 \tag{E2.25}$$
$$^1\chi_6 = i\hat{\chi}'_6 \tag{E2.26}$$
$$^1\chi_7 = i\hat{\chi}'_7 \tag{E2.27}$$

$$^{1}\chi_8 = \chi_8 \tag{E2.28}$$
$$^{1}\chi_9 = \hat{\chi}_9 \tag{E2.29}$$
$$^{1}\chi_{10} = \hat{\chi}_{10} \tag{E2.30}$$

bulunur. Ayrıca,

$$^{1}H_1 = \hat{H}_1 = e^{-i\hat{\upsilon}^{(1)}x_2} \tag{E2.31}$$
$$^{1}H_2 = \hat{H}_2 = e^{i\hat{\upsilon}^{(1)}x_2} \tag{E2.32}$$
$$^{1}H_3 = \hat{H}_3 = e^{-i\hat{\upsilon}'^{(1)}x_2} \tag{E2.33}$$
$$^{1}H_4 = \hat{H}_4 = e^{i\hat{\upsilon}'^{(1)}x_2} \tag{E2.34}$$
$$^{1}H_5 = \hat{H}_5 = e^{-i\hat{\upsilon}^{(2)}x_2} \tag{E2.35}$$
$$^{1}H_6 = \hat{H}_6 = e^{-i\hat{\upsilon}'^{(2)}x_2} \tag{E2.36}$$

ve,

$$^{1}\overline{H}_1 = \hat{\overline{H}}_1 = e^{-i\hat{\upsilon}^{(1)}h} \tag{E2.37}$$
$$^{1}\overline{H}_2 = \hat{\overline{H}}_2 = e^{i\hat{\upsilon}^{(1)}h} \tag{E2.38}$$
$$^{1}\overline{H}_3 = \hat{\overline{H}}_3 = e^{-i\hat{\upsilon}'^{(1)}h} \tag{E2.39}$$
$$^{1}\overline{H}_4 = \hat{\overline{H}}_4 = e^{i\hat{\upsilon}'^{(1)}h} \tag{E2.40}$$
$$^{1}\overline{H}_5 = \hat{\overline{H}}_5 = e^{-i\hat{\upsilon}^{(2)}h} \tag{E2.41}$$
$$^{1}\overline{H}_6 = \hat{\overline{H}}_6 = e^{-i\hat{\upsilon}'^{(2)}h} \tag{E2.42}$$

olup bütün bu eşitlikler (E1.23)-(E1.36) eşitliklerinde kullanılırsa,

$$\begin{aligned}
^{1}\Gamma_1 = {} & k^2(\hat{\chi}_5\chi_8 - \hat{\chi}_2'\hat{\chi}_7' + \hat{\chi}_1\chi_4 - \hat{\chi}_2'\hat{\chi}_3' + \hat{\chi}_6'\hat{\chi}_7' - \chi_4\hat{\chi}_5 - \hat{\chi}_1\chi_8 + \hat{\chi}_3'\hat{\chi}_6') \\
& - k(\chi_4\hat{\chi}_6' - \hat{\chi}_2'\chi_8)(\hat{\upsilon}^{(1)} + \hat{\upsilon}^{(2)}) \\
& + k(\hat{\chi}_1\hat{\chi}_7' + \hat{\chi}_3'\hat{\chi}_5)(\hat{\upsilon}'^{(1)} - \hat{\upsilon}'^{(2)}) \\
& - \hat{\upsilon}'^{(1)}\hat{\upsilon}^{(1)}(\hat{\chi}_5\chi_8 + \hat{\chi}_6'\hat{\chi}_7') - \hat{\upsilon}'^{(1)}\hat{\upsilon}^{(2)}(\hat{\chi}_1\chi_8 - \hat{\chi}_3'\hat{\chi}_6') \\
& + \hat{\upsilon}'^{(2)}\hat{\upsilon}^{(1)}(\hat{\chi}_2'\hat{\chi}_7' + \chi_4\hat{\chi}_5) + \hat{\upsilon}'^{(2)}\hat{\upsilon}^{(2)}(\hat{\chi}_1\chi_4 - \hat{\chi}_2'\hat{\chi}_3')
\end{aligned} \tag{E2.43}$$

$$\begin{aligned}
^{1}\Gamma_2 = {} & i\left[-2\hat{\upsilon}'^{(1)}k(\hat{\chi}_6'\hat{\chi}_7' + \hat{\chi}_5\chi_8) + 2\hat{\upsilon}'^{(1)}\chi_4(k\hat{\chi}_5 + \hat{\upsilon}^{(2)}\hat{\chi}_6') \right. \\
& \left. + 2k^2(\hat{\chi}_2'\chi_8 - \hat{\chi}_2'\chi_4) - 2\hat{\upsilon}'^{(2)}\hat{\chi}_2'(\hat{\upsilon}^{(2)}\chi_4 - k\hat{\chi}_7')\right] \\
= {} & i\ {}^{1}\Gamma_2'
\end{aligned} \tag{E2.44}$$

$$\begin{aligned}
^{1}\Gamma_3 = {} & k^2(\hat{\chi}_5\chi_8 + \hat{\chi}_1\chi_4 + \hat{\chi}_6'\hat{\chi}_7' + \hat{\chi}_2'\hat{\chi}_7' - \chi_4\hat{\chi}_5 - \hat{\chi}_1\chi_8 + \hat{\chi}_2'\hat{\chi}_3' + \hat{\chi}_3'\hat{\chi}_6') \\
& - k(\hat{\chi}_2'\chi_8 + \chi_4\hat{\chi}_6')(\hat{\upsilon}^{(1)} + \hat{\upsilon}^{(2)}) \\
& - k(\hat{\chi}_1\hat{\chi}_7' + \hat{\chi}_3'\hat{\chi}_5)(\hat{\upsilon}'^{(1)} + \hat{\upsilon}'^{(2)}) \\
& + \hat{\upsilon}'^{(1)}\hat{\upsilon}^{(1)}(\hat{\chi}_6'\hat{\chi}_7' + \hat{\chi}_5\chi_8) + \hat{\upsilon}'^{(1)}\hat{\upsilon}^{(2)}(\hat{\chi}_1\chi_8 - \hat{\chi}_3'\hat{\chi}_6') \\
& + \hat{\upsilon}'^{(2)}\hat{\upsilon}^{(1)}(-\hat{\chi}_2'\hat{\chi}_7' + \chi_4\hat{\chi}_5) + \hat{\upsilon}'^{(2)}\hat{\upsilon}^{(2)}(\hat{\chi}_2'\hat{\chi}_3' + \hat{\chi}_1\chi_4)
\end{aligned} \tag{E2.45}$$

$$\begin{aligned}
{}^{1}\Gamma_4 = {} & k^2(\hat{\chi}_5\chi_8+\hat{\chi}_1\chi_4-\hat{\chi}'_3\hat{\chi}'_6-\hat{\chi}'_2\hat{\chi}'_3-\chi_4\hat{\chi}_5-\hat{\chi}_1\chi_8+\hat{\chi}'_2\hat{\chi}'_7+\hat{\chi}'_6\hat{\chi}'_7) \\
& +k(\hat{\chi}'_2\chi_8+\chi_4\hat{\chi}'_6)(\hat{\upsilon}^{(1)}-\hat{\upsilon}^{(2)}) \\
& -k(\hat{\chi}_1\hat{\chi}'_7-\hat{\chi}'_3\hat{\chi}_5)(\hat{\upsilon}'^{(1)}+\hat{\upsilon}'^{(2)}) \\
& -\hat{\upsilon}'^{(1)}\hat{\upsilon}^{(1)}(\hat{\chi}_5\chi_8+\hat{\chi}'_6\hat{\chi}'_7)+\hat{\upsilon}'^{(1)}\hat{\upsilon}^{(2)}(\hat{\chi}'_3\hat{\chi}'_6+\hat{\chi}_1\chi_8) \\
& +\hat{\upsilon}'^{(2)}\hat{\upsilon}^{(1)}(\hat{\chi}'_2\hat{\chi}'_7-\chi_4\hat{\chi}_5)+\hat{\upsilon}'^{(2)}\hat{\upsilon}^{(2)}(-\hat{\chi}'_2\hat{\chi}'_3+\hat{\chi}_1\chi_4)
\end{aligned} \tag{E2.46}$$

$$\begin{aligned}
{}^{1}\Gamma_5 = {} & k^2(\hat{\chi}_5\chi_8+\hat{\chi}_1\chi_4-\hat{\chi}'_3\hat{\chi}'_6+\hat{\chi}'_2\hat{\chi}'_3-\chi_4\hat{\chi}_5-\hat{\chi}_1\chi_8-\hat{\chi}'_2\hat{\chi}'_7+\hat{\chi}'_6\hat{\chi}'_7) \\
& +k(\hat{\chi}'_2\chi_8-\chi_4\hat{\chi}'_6)(\hat{\upsilon}^{(2)}-\hat{\upsilon}^{(1)}) \\
& +k(\hat{\chi}'_3\hat{\chi}_5-\hat{\chi}_1\hat{\chi}'_7)(\hat{\upsilon}'^{(2)}-\hat{\upsilon}'^{(1)}) \\
& +\hat{\upsilon}'^{(1)}\hat{\upsilon}^{(1)}(\hat{\chi}'_6\hat{\chi}'_7+\chi_8\hat{\chi}_5)-\hat{\upsilon}'^{(1)}\hat{\upsilon}^{(2)}(\hat{\chi}_1\chi_8+\hat{\chi}'_3\hat{\chi}'_6) \\
& -\hat{\upsilon}'^{(2)}\hat{\upsilon}^{(1)}(\chi_4\hat{\chi}_5+\hat{\chi}'_2\hat{\chi}'_7)+\hat{\upsilon}'^{(2)}\hat{\upsilon}^{(2)}(\hat{\chi}'_2\hat{\chi}'_3+\hat{\chi}_1\chi_4)
\end{aligned} \tag{E2.47}$$

$$\begin{aligned}
{}^{1}\Gamma_6 = {} & i\big[-2k\hat{\upsilon}^{(1)}(\hat{\chi}'_6\hat{\chi}'_7+\hat{\chi}_5\chi_8)+2k\hat{\chi}'_3(k\hat{\chi}_5+\hat{\upsilon}^{(2)}\hat{\chi}'_6) \\
& +2k\hat{\chi}_1(\hat{\upsilon}^{(1)}\chi_8-k\hat{\chi}'_3)+2\hat{\upsilon}'^{(2)}\hat{\chi}_1(-\hat{\upsilon}^{(2)}\hat{\chi}'_3+\hat{\upsilon}^{(1)}\hat{\chi}'_7)\big] \\
= {} & i\ {}^{1}\Gamma'_6
\end{aligned} \tag{E2.48}$$

$$\begin{aligned}
{}^{1}\Gamma_7 = {} & i\big[2\hat{\upsilon}'^{(1)}k(\hat{\chi}_1\chi_8-\hat{\chi}_1\chi_4)-2\hat{\upsilon}'^{(1)}\hat{\chi}'_6(k\hat{\chi}'_3-\hat{\upsilon}^{(1)}\chi_4) \\
& +2k^2(\hat{\chi}'_2\chi_4-\hat{\chi}'_2\chi_8)-2\hat{\upsilon}'^{(2)}\hat{\chi}'_2(\hat{\upsilon}^{(1)}\chi_4-k\hat{\chi}'_3)\big] \\
= {} & i\ {}^{1}\Gamma'_7
\end{aligned} \tag{E2.49}$$

$$\begin{aligned}
{}^{1}\Gamma_8 = {} & i\big[2\hat{\upsilon}'^{(1)}k(\hat{\chi}_1\chi_8+\hat{\chi}'_3\hat{\chi}'_6)-2\hat{\upsilon}'^{(1)}\chi_4(\hat{\upsilon}^{(1)}\hat{\chi}'_6+k\hat{\chi}_1) \\
& +2k^2(\hat{\chi}'_2\chi_4-\hat{\chi}'_2\chi_8)-2\hat{\upsilon}'^{(2)}\hat{\chi}'_2(k\hat{\chi}'_3-\hat{\upsilon}^{(1)}\chi_4)\big] \\
= {} & i\ {}^{1}\Gamma'_8
\end{aligned} \tag{E2.50}$$

$$\begin{aligned}
{}^{1}\Gamma_9 = {} & 2\hat{\upsilon}^{(1)}k(\hat{\chi}'_2\chi_8+\chi_4\hat{\chi}'_6)-2\hat{\upsilon}'^{(2)}\hat{\chi}_1(\hat{\upsilon}^{(1)}\chi_4-k\hat{\chi}'_3) \\
& -2k^2(\hat{\chi}'_3\hat{\chi}'_2+\hat{\chi}'_3\hat{\chi}'_6)-2\hat{\upsilon}'^{(1)}\hat{\chi}_1(\hat{\upsilon}^{(1)}\chi_8-k\hat{\chi}'_3)
\end{aligned} \tag{E2.51}$$

$$\begin{aligned}
{}^{1}\Gamma_{10} = {} & -2\hat{\upsilon}^{(1)}k(\hat{\chi}'_2\chi_8-\chi_4\hat{\chi}'_6)-2\hat{\upsilon}'^{(2)}\hat{\chi}_1(\hat{\upsilon}^{(1)}\chi_4-k\hat{\chi}'_3) \\
& -2k^2(\hat{\chi}'_3\hat{\chi}'_6-\hat{\chi}'_3\hat{\chi}'_2)-2\hat{\upsilon}'^{(1)}\hat{\chi}_1(k\hat{\chi}'_3-\hat{\upsilon}^{(1)}\chi_8)
\end{aligned} \tag{E2.52}$$

$$\begin{aligned}
{}^{1}\Gamma_{11} = {} & 2k\hat{\upsilon}'^{(1)}(\hat{\chi}'_3\hat{\chi}_5+\hat{\chi}_1\hat{\chi}'_7)-2k^2(\hat{\chi}'_2\hat{\chi}'_7+\hat{\chi}'_2\hat{\chi}'_3) \\
& +2k\hat{\chi}'_2(\hat{\upsilon}^{(1)}\chi_4+\hat{\upsilon}^{(2)}\chi_4)-2\hat{\upsilon}'^{(1)}\chi_4(\hat{\upsilon}^{(1)}\hat{\chi}_5+\hat{\upsilon}^{(2)}\hat{\chi}_1)
\end{aligned} \tag{E2.53}$$

$$\begin{aligned}
{}^{1}\Gamma_{12} = {} & -2k\hat{\upsilon}'^{(1)}(\hat{\chi}'_3\hat{\chi}_5-\hat{\chi}_1\hat{\chi}'_7)-2k^2(\hat{\chi}'_2\hat{\chi}'_7-\hat{\chi}'_2\hat{\chi}'_3) \\
& -2\hat{\upsilon}'^{(1)}\chi_4(\hat{\upsilon}^{(2)}\hat{\chi}_1-\hat{\upsilon}^{(1)}\hat{\chi}_5)-2k\hat{\chi}'_2(\hat{\upsilon}^{(1)}\chi_4-\hat{\upsilon}^{(2)}\chi_4)
\end{aligned} \tag{E2.54}$$

$$\begin{aligned}
{}^{1}\Gamma_{13} = {} & i\big[-2k\hat{\upsilon}^{(1)}(\chi_4\hat{\chi}_5-\hat{\chi}'_2\hat{\chi}'_7)+2k\hat{\chi}'_3(k\hat{\chi}_5-\hat{\upsilon}^{(2)}\hat{\chi}'_2) \\
& -2\hat{\upsilon}'^{(1)}\hat{\chi}_1(\hat{\upsilon}^{(1)}\hat{\chi}'_7-\hat{\upsilon}^{(2)}\hat{\chi}'_3)+2k\hat{\chi}_1(\hat{\upsilon}^{(1)}\chi_4-k\hat{\chi}'_3)\big] \\
= {} & i\ {}^{1}\Gamma'_{13}
\end{aligned} \tag{E2.55}$$

$$\begin{aligned}
{}^{1}\Gamma_{14} = {} & i\big[-2k\hat{\upsilon}^{(1)}(\hat{\chi}'_2\hat{\chi}'_7+\chi_4\hat{\chi}_5)+2k\hat{\chi}'_3(k\hat{\chi}_5+\hat{\upsilon}^{(2)}\hat{\chi}'_2) \\
& -2\hat{\upsilon}'^{(1)}\hat{\chi}_1(\hat{\upsilon}^{(2)}\hat{\chi}'_3-\hat{\upsilon}^{(1)}\hat{\chi}'_7)+2k\hat{\chi}_1(\hat{\upsilon}^{(1)}\chi_4-k\hat{\chi}'_3)\big] \\
= {} & i\ {}^{1}\Gamma'_{14}
\end{aligned} \tag{E2.56}$$

bulunur. Bu durumda,

$$
\begin{aligned}
{}^1[\Delta] = & (-\hat{\chi}_2'\hat{\chi}_3' + \hat{\chi}_1\chi_4)(\hat{\bar{H}}_2\hat{\bar{H}}_3\,{}^1\Gamma_1 + \hat{\bar{H}}_1\hat{\bar{H}}_4\,{}^1\Gamma_4) \\
& - (\hat{\chi}_2'\hat{\chi}_3' + \hat{\chi}_1\chi_4)(\hat{\bar{H}}_1\hat{\bar{H}}_3\,{}^1\Gamma_5 + \hat{\bar{H}}_2\hat{\bar{H}}_4\,{}^1\Gamma_3) \\
& + 2i\,(\hat{\chi}_1\hat{\chi}_3'\hat{\bar{H}}_3\hat{\bar{H}}_4\,{}^1\Gamma_2 + \hat{\chi}_2'\chi_4\hat{\bar{H}}_1\hat{\bar{H}}_2\,{}^1\Gamma_6)
\end{aligned} \tag{E2.57}
$$

ve (E2.44), (E2.48) ve (E2.37)-(E2.42) eşitlikleri yardımı ile,

$$
\begin{aligned}
{}^1[\Delta] = & (-\hat{\chi}_2'\hat{\chi}_3' + \hat{\chi}_1\chi_4)(\hat{\bar{H}}_2\hat{\bar{H}}_3\,{}^1\Gamma_1 + \hat{\bar{H}}_1\hat{\bar{H}}_4\,{}^1\Gamma_4) \\
& - (\hat{\chi}_2'\hat{\chi}_3' + \hat{\chi}_1\chi_4)(\hat{\bar{H}}_1\hat{\bar{H}}_3\,{}^1\Gamma_5 + \hat{\bar{H}}_2\hat{\bar{H}}_4\,{}^1\Gamma_3) \\
& - 2(\hat{\chi}_1\hat{\chi}_3'\,{}^1\Gamma_2' + \hat{\chi}_2'\chi_4\,{}^1\Gamma_6')
\end{aligned} \tag{E2.58}
$$

olarak elde edilir. Görüldüğü gibi ${}^1[\Delta]$ in bu ifadesinin sanal parçası yoktur. Ancak $\hat{\bar{H}}_i$ ifadelerinin yapısı dikkate alındığında durumun aslında böyle olmadığı görülür ve yapılan ara işlemler sonucunda,

$$
\begin{aligned}
{}^1[\Delta] = & (-\hat{\chi}_2'\hat{\chi}_3' + \hat{\chi}_1\chi_4)\cos\left[(\hat{\upsilon}^{(1)} - \hat{\upsilon}'^{(1)})h\right]({}^1\Gamma_1 + {}^1\Gamma_4) \\
& - (\hat{\chi}_2'\hat{\chi}_3' + \hat{\chi}_1\chi_4)\cos\left[(\hat{\upsilon}^{(1)} + \hat{\upsilon}'^{(1)})h\right]({}^1\Gamma_3 + {}^1\Gamma_5) \\
& - 2(\hat{\chi}_1\hat{\chi}_3'\,{}^1\Gamma_2' + \hat{\chi}_2'\chi_4\,{}^1\Gamma_6') \\
& + i\left\{(-\hat{\chi}_2'\hat{\chi}_3' + \hat{\chi}_1\chi_4)\sin\left[(\hat{\upsilon}^{(1)} - \hat{\upsilon}'^{(1)})h\right]({}^1\Gamma_1 - {}^1\Gamma_4)\right. \\
& \left. - (\hat{\chi}_2'\hat{\chi}_3' + \hat{\chi}_1\chi_4)\sin\left[(\hat{\upsilon}^{(1)} + \hat{\upsilon}'^{(1)})h\right]({}^1\Gamma_3 - {}^1\Gamma_5)\right\}
\end{aligned} \tag{E2.59}
$$

bulunur. Bu kısımda yapılması gereken son işlem (3.122)-(3.131) eşitliklerinde kullanılmak üzere ${}^1\bar{\alpha}_i$ ve ${}^1\bar{\beta}_i$ ifadelerini elde etmektir. Bunun için (E1.49)-(E1.54) eşitlikleri yardımı ile,

$$
{}^1\bar{\alpha}_1 = \frac{\chi_4 e^{i\hat{\upsilon}^{(1)}h}(e^{-i\hat{\upsilon}'^{(1)}h}\,{}^1\Gamma_1 - e^{i\hat{\upsilon}'^{(1)}h}\,{}^1\Gamma_3) - \hat{\chi}_3'\,{}^1\Gamma_2'}{{}^1[\Delta]} \tag{E2.60}
$$

$$
{}^1\bar{\alpha}_2 = \frac{\chi_4 e^{-i\hat{\upsilon}^{(1)}h}(e^{-i\hat{\upsilon}'^{(1)}h}\,{}^1\Gamma_5 - e^{i\hat{\upsilon}'^{(1)}h}\,{}^1\Gamma_4) + \hat{\chi}_3'\,{}^1\Gamma_2'}{{}^1[\Delta]} \tag{E2.61}
$$

$$
{}^1\bar{\beta}_1 = i\frac{\hat{\chi}_3' e^{i\hat{\upsilon}'^{(1)}h}(e^{i\hat{\upsilon}^{(1)}h}\,{}^1\Gamma_3 + e^{-i\hat{\upsilon}^{(1)}h}\,{}^1\Gamma_4) + \chi_4\,{}^1\Gamma_6'}{{}^1[\Delta]} \tag{E2.62}
$$

$$
{}^1\bar{\beta}_2 = i\frac{\hat{\chi}_3' e^{-i\hat{\upsilon}'^{(1)}h}(e^{i\hat{\upsilon}^{(1)}h}\,{}^1\Gamma_1 + e^{-i\hat{\upsilon}^{(1)}h}\,{}^1\Gamma_5) + \chi_4\,{}^1\Gamma_6'}{{}^1[\Delta]} \tag{E2.63}
$$

$$
{}^1\bar{\alpha}_3 = \frac{-\hat{\chi}_3'(e^{i\hat{\upsilon}^{(1)}h}\,{}^1\Gamma_7' + e^{-i\hat{\upsilon}^{(1)}h}\,{}^1\Gamma_8') + \chi_4(e^{-i\hat{\upsilon}'^{(1)}h}\,{}^1\Gamma_{10} - e^{i\hat{\upsilon}'^{(1)}h}\,{}^1\Gamma_9)}{e^{-i\hat{\upsilon}^{(2)}h}\,{}^1[\Delta]} \tag{E2.64}
$$

$$
{}^1\bar{\beta}_3 = i\frac{\hat{\chi}_3'(e^{i\hat{\upsilon}^{(1)}h}\,{}^1\Gamma_{11} + e^{-i\hat{\upsilon}^{(1)}h}\,{}^1\Gamma_{12}) + \chi_4(e^{i\hat{\upsilon}'^{(1)}h}\,{}^1\Gamma_{13}' - e^{-i\hat{\upsilon}'^{(1)}h}\,{}^1\Gamma_{14}')}{e^{-i\hat{\upsilon}'^{(2)}h}\,{}^1[\Delta]} \tag{E2.65}
$$

bulunur. k dönüşüm parametresine göre,

$\hat{\chi}_1, \chi_4, \hat{\chi}_5, \chi_8, \hat{\chi}_9, \hat{\chi}_{10}, {}^1\Gamma_1, {}^1\Gamma_3, {}^1\Gamma_4, {}^1\Gamma_5, {}^1\Gamma_9, {}^1\Gamma_{10}, {}^1\Gamma_{11}, {}^1\Gamma_{12}, \hat{\upsilon}^{(1)}, \hat{\upsilon}'^{(1)}, \hat{\upsilon}^{(2)}, \hat{\upsilon}'^{(2)}$;çift,

$\hat{\chi}_2', \hat{\chi}_3', \hat{\chi}_6', \chi_7', {}^1\Gamma_2', {}^1\Gamma_6', {}^1\Gamma_7', {}^1\Gamma_8', {}^1\Gamma_{13}', {}^1\Gamma_{14}'$; tek fonksiyonlar olmaktadır. Buna göre,

$$
\begin{aligned}
{}^{1}[\Delta]_{G} = {} & (-\hat{\chi}_2'\hat{\chi}_3' + \hat{\chi}_1\chi_4)\cos\left[(\hat{\upsilon}^{(1)} - \hat{\upsilon}'^{(1)})h\right]({}^{1}\Gamma_1 + {}^{1}\Gamma_4) \\
& -(\hat{\chi}_2'\hat{\chi}_3' + \hat{\chi}_1\chi_4)\cos\left[(\hat{\upsilon}^{(1)} + \hat{\upsilon}'^{(1)})h\right]({}^{1}\Gamma_3 + {}^{1}\Gamma_5) \\
& -2(\hat{\chi}_1\hat{\chi}_3'\ {}^{1}\Gamma_2' + \hat{\chi}_2'\chi_4\ {}^{1}\Gamma_6')
\end{aligned}
\tag{E2.66}
$$

ve,

$$
\begin{aligned}
{}^{1}[\Delta]_{S} = {} & (-\hat{\chi}_2'\hat{\chi}_3' + \hat{\chi}_1\chi_4)\sin\left[(\hat{\upsilon}^{(1)} - \hat{\upsilon}'^{(1)})h\right]({}^{1}\Gamma_1 - {}^{1}\Gamma_4) \\
& -(\hat{\chi}_2'\hat{\chi}_3' + \hat{\chi}_1\chi_4)\sin\left[(\hat{\upsilon}^{(1)} + \hat{\upsilon}'^{(1)})h\right]({}^{1}\Gamma_3 - {}^{1}\Gamma_5)
\end{aligned}
\tag{E2.67}
$$

fonksiyonlarının çift oldukları görülür. Burada,

$$
{}^{1}[\Delta] = {}^{1}[\Delta]_{G} + i\ {}^{1}[\Delta]_{S}
\tag{E2.68}
$$

olmaktadır. (E2.60)-(E2.65) eşitliklerinin gerçel ve sanal kısımlarının ayrılması için bunların düzenlenerak pay ve paydalarının ${}^{1}[\Delta]$ ifadesinin karmaşık eşleniği ile çarpılması gerekmektedir. Bütün bu işlemler yapıldığında,

$$
{}^{1}\overline{\alpha}_{1PG} = \chi_4\left\{\cos\left[\left(\hat{\upsilon}^{(1)} - \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_1 - \cos\left[\left(\hat{\upsilon}^{(1)} + \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_3\right\} - \hat{\chi}_3'\ {}^{1}\Gamma_2'
\tag{E2.69}
$$

$$
{}^{1}\overline{\alpha}_{1PS} = \chi_4\left\{\sin\left[\left(\hat{\upsilon}^{(1)} - \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_1 - \sin\left[\left(\hat{\upsilon}^{(1)} + \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_3\right\}
\tag{E2.70}
$$

$$
{}^{1}\overline{\alpha}_{2PG} = \chi_4\left\{\cos\left[\left(\hat{\upsilon}^{(1)} + \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_5 - \cos\left[\left(\hat{\upsilon}^{(1)} - \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_4\right\} + \hat{\chi}_3'\ {}^{1}\Gamma_2'
\tag{E2.71}
$$

$$
{}^{1}\overline{\alpha}_{2PS} = \chi_4\left\{-\sin\left[\left(\hat{\upsilon}^{(1)} + \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_5 + \sin\left[\left(\hat{\upsilon}^{(1)} - \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_4\right\}
\tag{E2.72}
$$

$$
{}^{1}\overline{\beta}_{1PG} = \hat{\chi}_3'\left\{-\sin\left[\left(\hat{\upsilon}^{(1)} + \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_3 + \sin\left[\left(\hat{\upsilon}^{(1)} - \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_4\right\}
\tag{E2.73}
$$

$$
{}^{1}\overline{\beta}_{1PS} = \hat{\chi}_3'\left\{\cos\left[\left(\hat{\upsilon}^{(1)} + \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_3 + \cos\left[\left(\hat{\upsilon}^{(1)} - \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_4\right\} + \chi_4\ {}^{1}\Gamma_6'
\tag{E2.74}
$$

$$
{}^{1}\overline{\beta}_{2PG} = \hat{\chi}_3'\left\{-\sin\left[\left(\hat{\upsilon}^{(1)} - \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_1 + \sin\left[\left(\hat{\upsilon}^{(1)} + \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_5\right\}
\tag{E2.75}
$$

$$
{}^{1}\overline{\beta}_{2PS} = \hat{\chi}_3'\left\{\cos\left[\left(\hat{\upsilon}^{(1)} - \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_1 + \cos\left[\left(\hat{\upsilon}^{(1)} + \hat{\upsilon}'^{(1)}\right)h\right]{}^{1}\Gamma_5\right\} + \chi_4\ {}^{1}\Gamma_6'
\tag{E2.76}
$$

$$
{}^{1}\overline{\alpha}_{3PG} = -\hat{\chi}_3'\cos\left(\hat{\upsilon}^{(1)}h\right)({}^{1}\Gamma_7' + {}^{1}\Gamma_8') + \chi_4\cos\left(\hat{\upsilon}'^{(1)}h\right)({}^{1}\Gamma_{10} - {}^{1}\Gamma_9)
\tag{E2.77}
$$

$$
{}^{1}\overline{\alpha}_{3PS} = -\hat{\chi}_3'\sin\left(\hat{\upsilon}^{(1)}h\right)({}^{1}\Gamma_7' - {}^{1}\Gamma_8') - \chi_4\sin\left(\hat{\upsilon}'^{(1)}h\right)({}^{1}\Gamma_{10} + {}^{1}\Gamma_9)
\tag{E2.78}
$$

$$
{}^{1}\overline{\beta}_{3PG} = -\hat{\chi}_3'\sin\left(\hat{\upsilon}^{(1)}h\right)({}^{1}\Gamma_{11} - {}^{1}\Gamma_{12}) - \chi_4\sin\left(\hat{\upsilon}'^{(1)}h\right)({}^{1}\Gamma_{13}' + {}^{1}\Gamma_{14}')
\tag{E2.79}
$$

$$
{}^{1}\overline{\beta}_{3PS} = \hat{\chi}_3'\cos\left(\hat{\upsilon}^{(1)}h\right)({}^{1}\Gamma_{11} + {}^{1}\Gamma_{12}) + \chi_4\cos\left(\hat{\upsilon}'^{(1)}h\right)({}^{1}\Gamma_{13}' - {}^{1}\Gamma_{14}')
\tag{E2.80}
$$

ve,

$$
{}^{1}\overline{\alpha}_{1G} = \frac{{}^{1}[\Delta]_{G}\ {}^{1}\overline{\alpha}_{1PG} + {}^{1}[\Delta]_{S}\ {}^{1}\overline{\alpha}_{1PS}}{{}^{1}[\Delta]_{G}^{2} + {}^{1}[\Delta]_{S}^{2}}
\tag{E2.81}
$$

$$
{}^{1}\overline{\alpha}_{1S} = \frac{{}^{1}[\Delta]_{G}\ {}^{1}\overline{\alpha}_{1PS} - {}^{1}[\Delta]_{S}\ {}^{1}\overline{\alpha}_{1PG}}{{}^{1}[\Delta]_{G}^{2} + {}^{1}[\Delta]_{S}^{2}}
\tag{E2.82}
$$

$$
{}^{1}\overline{\alpha}_{2G} = \frac{{}^{1}[\Delta]_{G}\ {}^{1}\overline{\alpha}_{2PG} + {}^{1}[\Delta]_{S}\ {}^{1}\overline{\alpha}_{2PS}}{{}^{1}[\Delta]_{G}^{2} + {}^{1}[\Delta]_{S}^{2}}
\tag{E2.83}
$$

$$
{}^{1}\overline{\alpha}_{2S} = \frac{{}^{1}[\Delta]_{G}\ {}^{1}\overline{\alpha}_{2PS} - {}^{1}[\Delta]_{S}\ {}^{1}\overline{\alpha}_{2PG}}{{}^{1}[\Delta]_{G}^{2} + {}^{1}[\Delta]_{S}^{2}}
\tag{E2.84}
$$

$$ {}^1\overline{\beta}_{1G} = \frac{{}^1[\Delta]_G\,{}^1\overline{\beta}_{1PG} + {}^1[\Delta]_S\,{}^1\overline{\beta}_{1PS}}{{}^1[\Delta]_G^{\,2} + {}^1[\Delta]_S^{\,2}} \tag{E2.85} $$

$$ {}^1\overline{\beta}_{1S} = \frac{{}^1[\Delta]_G\,{}^1\overline{\beta}_{1PS} - {}^1[\Delta]_S\,{}^1\overline{\beta}_{1PG}}{{}^1[\Delta]_G^{\,2} + {}^1[\Delta]_S^{\,2}} \tag{E2.86} $$

$$ {}^1\overline{\beta}_{2G} = \frac{{}^1[\Delta]_G\,{}^1\overline{\beta}_{2PG} + {}^1[\Delta]_S\,{}^1\overline{\beta}_{2PS}}{{}^1[\Delta]_G^{\,2} + {}^1[\Delta]_S^{\,2}} \tag{E2.87} $$

$$ {}^1\overline{\beta}_{2S} = \frac{{}^1[\Delta]_G\,{}^1\overline{\beta}_{2PS} - {}^1[\Delta]_S\,{}^1\overline{\beta}_{2PG}}{{}^1[\Delta]_G^{\,2} + {}^1[\Delta]_S^{\,2}} \tag{E2.88} $$

$$ {}^1\tilde{\overline{\alpha}}_{3G} = \frac{{}^1[\Delta]_G\,{}^1\overline{\alpha}_{3PG} + {}^1[\Delta]_S\,{}^1\overline{\alpha}_{3PS}}{{}^1[\Delta]_G^{\,2} + {}^1[\Delta]_S^{\,2}} \tag{E2.89} $$

$$ {}^1\tilde{\overline{\alpha}}_{3S} = \frac{{}^1[\Delta]_G\,{}^1\overline{\alpha}_{3PS} - {}^1[\Delta]_S\,{}^1\overline{\alpha}_{3PG}}{{}^1[\Delta]_G^{\,2} + {}^1[\Delta]_S^{\,2}} \tag{E2.90} $$

$$ {}^1\tilde{\overline{\beta}}_{3G} = \frac{{}^1[\Delta]_G\,{}^1\overline{\beta}_{3PG} + {}^1[\Delta]_S\,{}^1\overline{\beta}_{3PS}}{{}^1[\Delta]_G^{\,2} + {}^1[\Delta]_S^{\,2}} \tag{E2.91} $$

$$ {}^1\tilde{\overline{\beta}}_{3S} = \frac{{}^1[\Delta]_G\,{}^1\overline{\beta}_{3PS} - {}^1[\Delta]_S\,{}^1\overline{\beta}_{3PG}}{{}^1[\Delta]_G^{\,2} + {}^1[\Delta]_S^{\,2}} \tag{E2.92} $$

olmak üzere,

$$ {}^1\overline{\alpha}_1 = {}^1\overline{\alpha}_{1G} + i\ {}^1\overline{\alpha}_{1S} \tag{E2.93} $$

$$ {}^1\overline{\alpha}_2 = {}^1\overline{\alpha}_{2G} + i\ {}^1\overline{\alpha}_{2S} \tag{E2.94} $$

$$ {}^1\overline{\beta}_1 = {}^1\overline{\beta}_{1G} + i\ {}^1\overline{\beta}_{1S} \tag{E2.95} $$

$$ {}^1\overline{\beta}_2 = {}^1\overline{\beta}_{2G} + i\ {}^1\overline{\beta}_{2S} \tag{E2.96} $$

$$ {}^1\overline{\alpha}_3 = \cos(\hat{\upsilon}^{(2)}h)\ {}^1\tilde{\overline{\alpha}}_{3G} - \sin(\hat{\upsilon}^{(2)}h)\ {}^1\tilde{\overline{\alpha}}_{3S} + i\left[\cos(\hat{\upsilon}^{(2)}h)\ {}^1\tilde{\overline{\alpha}}_{3S} + \sin(\hat{\upsilon}^{(2)}h)\ {}^1\tilde{\overline{\alpha}}_{3G}\right] \tag{E2.97} $$

$$ {}^1\overline{\beta}_3 = \cos(\hat{\upsilon}'^{(2)}h)\ {}^1\tilde{\overline{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^1\tilde{\overline{\beta}}_{3S} + i\left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^1\tilde{\overline{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^1\tilde{\overline{\beta}}_{3G}\right] \tag{E2.98} $$

eşitlikleri elde edilir. ${}^1\overline{\alpha}_{iPG}, {}^1\overline{\alpha}_{iPS}$ (i=1,3) fonksiyonları k'ya göre çift, ${}^1\overline{\beta}_{iPG}, {}^1\overline{\beta}_{iPS}$ (i=1,3) ise tek olup, ${}^1[\Delta]_G$ ve ${}^1[\Delta]_S$ fonksiyonları da çift olduklarına göre sonuç olarak ${}^1\overline{\alpha}_{iG}, {}^1\overline{\alpha}_{iS}$ (i=1,3) fonksiyonlarının çift, ${}^1\overline{\beta}_{iG}, {}^1\overline{\beta}_{iS}$ (i=1,3) fonksiyonlarının da tek oldukları görülür. (E3.93)-(E3.98) eşitlikleri (3.123)-(3.132) de kullanılırsa, (E2.1)-(E2.4) ve (E2.21)-(E2.36) eşitlikleri yardımı ile,

$$ \begin{aligned} {}^1\Theta^{(1)}(k) = {} & k\left[\left({}^1\overline{\alpha}_{1G} + i\ {}^1\overline{\alpha}_{1S}\right)e^{-i\left(\hat{\upsilon}^{(1)}x_2 + kx_1\right)} - \left({}^1\overline{\alpha}_{2G} + i\ {}^1\overline{\alpha}_{2S}\right)e^{i\left(\hat{\upsilon}^{(1)}x_2 - kx_1\right)}\right] \\ & + i\hat{\upsilon}'^{(1)}\left[\left({}^1\overline{\beta}_{1G} + i\ {}^1\overline{\beta}_{1S}\right)e^{-i\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right)} + \left({}^1\overline{\beta}_{2G} + i\ {}^1\overline{\beta}_{2S}\right)e^{i\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right)}\right] \end{aligned} \tag{E2.99} $$

$$ \begin{aligned} {}^1\Omega^{(1)}(k) = {} & i\hat{\upsilon}^{(1)}\left[\left({}^1\overline{\alpha}_{1G} + i\ {}^1\overline{\alpha}_{1S}\right)e^{-i\left(\hat{\upsilon}^{(1)}x_2 + kx_1\right)} + \left({}^1\overline{\alpha}_{2G} + i\ {}^1\overline{\alpha}_{2S}\right)e^{i\left(\hat{\upsilon}^{(1)}x_2 - kx_1\right)}\right] \\ & + k\left[\left({}^1\overline{\beta}_{1G} + i\ {}^1\overline{\beta}_{1S}\right)e^{-i\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right)} - \left({}^1\overline{\beta}_{2G} + i\ {}^1\overline{\beta}_{2S}\right)e^{i\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right)}\right] \end{aligned} \tag{E2.100} $$

$$ \begin{aligned} {}^1A^{(1)}(k) = {} & \hat{\chi}_9\left[-\left({}^1\overline{\alpha}_{1G} + i\ {}^1\overline{\alpha}_{1S}\right)e^{-i\left(\hat{\upsilon}^{(1)}x_2 + kx_1\right)} + \left({}^1\overline{\alpha}_{2G} + i\ {}^1\overline{\alpha}_{2S}\right)e^{i\left(\hat{\upsilon}^{(1)}x_2 - kx_1\right)}\right] \\ & + i\hat{\chi}'_2\left[\left({}^1\overline{\beta}_{1G} + i\ {}^1\overline{\beta}_{1S}\right)e^{-i\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right)} + \left({}^1\overline{\beta}_{2G} + i\ {}^1\overline{\beta}_{2S}\right)e^{i\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right)}\right] \end{aligned} \tag{E2.101} $$

$$
\begin{aligned}
{}^1B^{(1)}(k) = \hat{\chi}_1\Big[&-\left({}^1\overline{\alpha}_{1G} + i\ {}^1\overline{\alpha}_{1S}\right)e^{-i\left(\hat{\upsilon}^{(1)}x_2+kx_1\right)} + \left({}^1\overline{\alpha}_{2G} + i\ {}^1\overline{\alpha}_{2S}\right)e^{i\left(\hat{\upsilon}^{(1)}x_2-kx_1\right)}\Big] \\
&-i\hat{\chi}'_2\Big[\left({}^1\overline{\beta}_{1G} + i\ {}^1\overline{\beta}_{1S}\right)e^{-i\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right)} + \left({}^1\overline{\beta}_{2G} + i\ {}^1\overline{\beta}_{2S}\right)e^{i\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)}\Big]
\end{aligned} \tag{E2.102}
$$

$$
\begin{aligned}
{}^1C^{(1)}(k) = -i\hat{\chi}'_3\Big[&\left({}^1\overline{\alpha}_{1G} + i\ {}^1\overline{\alpha}_{1S}\right)e^{-i\left(\hat{\upsilon}^{(1)}x_2+kx_1\right)} + \left({}^1\overline{\alpha}_{2G} + i\ {}^1\overline{\alpha}_{2S}\right)e^{i\left(\hat{\upsilon}^{(1)}x_2-kx_1\right)}\Big] \\
&+\chi_4\Big[-\left({}^1\overline{\beta}_{1G} + i\ {}^1\overline{\beta}_{1S}\right)e^{-i\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right)} + \left({}^1\overline{\beta}_{2G} + i\ {}^1\overline{\beta}_{2S}\right)e^{i\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)}\Big]
\end{aligned} \tag{E2.103}
$$

$$
{}^1\Theta^{(2)}(k) = -k\left({}^1\overline{\alpha}_{3G} + i\ {}^1\overline{\alpha}_{3S}\right)e^{-i\left(\hat{\upsilon}^{(2)}x_2+kx_1\right)} - i\hat{\upsilon}'^{(2)}\left({}^1\overline{\beta}_{3G} + i\ {}^1\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)} \tag{E2.104}
$$

$$
{}^1\Omega^{(2)}(k) = -i\hat{\upsilon}^{(2)}\left({}^1\overline{\alpha}_{3G} + i\ {}^1\overline{\alpha}_{3S}\right)e^{-i\left(\hat{\upsilon}^{(2)}x_2+kx_1\right)} - k\left({}^1\overline{\beta}_{3G} + i\ {}^1\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)} \tag{E2.105}
$$

$$
{}^1A^{(2)}(k) = \hat{\chi}_{10}\left({}^1\overline{\alpha}_{3G} + i\ {}^1\overline{\alpha}_{3S}\right)e^{-i\left(\hat{\upsilon}^{(2)}x_2+kx_1\right)} - i\hat{\chi}'_6\left({}^1\overline{\beta}_{3G} + i\ {}^1\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)} \tag{E2.106}
$$

$$
{}^1B^{(2)}(k) = \hat{\chi}_5\left({}^1\overline{\alpha}_{3G} + i\ {}^1\overline{\alpha}_{3S}\right)e^{-i\left(\hat{\upsilon}^{(2)}x_2+kx_1\right)} + i\hat{\chi}'_6\left({}^1\overline{\beta}_{3G} + i\ {}^1\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)} \tag{E2.107}
$$

$$
{}^1C^{(2)}(k) = i\hat{\chi}'_7\left({}^1\overline{\alpha}_{3G} + i\ {}^1\overline{\alpha}_{3S}\right)e^{-i\left(\hat{\upsilon}^{(2)}x_2+kx_1\right)} + \chi_8\left({}^1\overline{\beta}_{3G} + i\ {}^1\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)} \tag{E2.108}
$$

bulunur. (3.133)-(3.142) uyarınca yukarıdaki eşitliklerden (E2.99), (E2.103), (E2.104) ve (E2.108) in sanal, (E2.100), (E2.101), (E2.102), (E2.105), (E2.106) ve (E2.107) 'nin ise gerçel kısımları kullanılacaktır.

$$
\begin{aligned}
{}^1\Theta_S^{(1)}(k) = k\Big[&-{}^1\overline{\alpha}_{1G}\sin\left(\hat{\upsilon}^{(1)}x_2+kx_1\right) + {}^1\overline{\alpha}_{1S}\cos\left(\hat{\upsilon}^{(1)}x_2+kx_1\right) \\
&- {}^1\overline{\alpha}_{2G}\sin\left(\hat{\upsilon}^{(1)}x_2-kx_1\right) - {}^1\overline{\alpha}_{2S}\cos\left(\hat{\upsilon}^{(1)}x_2-kx_1\right)\Big] \\
+\hat{\upsilon}'^{(1)}\Big[&{}^1\overline{\beta}_{1G}\cos\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right) + {}^1\overline{\beta}_{1S}\sin\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right) \\
&+ {}^1\overline{\beta}_{2G}\cos\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right) - {}^1\overline{\beta}_{2S}\sin\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)\Big]
\end{aligned} \tag{E2.109}
$$

$$
\begin{aligned}
{}^1\Omega_G^{(1)}(k) = \hat{\upsilon}^{(1)}\Big[&{}^1\overline{\alpha}_{1G}\sin\left(\hat{\upsilon}^{(1)}x_2+kx_1\right) - {}^1\overline{\alpha}_{1S}\cos\left(\hat{\upsilon}^{(1)}x_2+kx_1\right) \\
&- {}^1\overline{\alpha}_{2G}\sin\left(\hat{\upsilon}^{(1)}x_2-kx_1\right) - {}^1\overline{\alpha}_{2S}\cos\left(\hat{\upsilon}^{(1)}x_2-kx_1\right)\Big] \\
+k\Big[&{}^1\overline{\beta}_{1G}\cos\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right) + {}^1\overline{\beta}_{1S}\sin\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right) \\
&- {}^1\overline{\beta}_{2G}\cos\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right) + {}^1\overline{\beta}_{2S}\sin\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)\Big]
\end{aligned} \tag{E2.110}
$$

$$
\begin{aligned}
{}^1A_G^{(1)}(k) = \hat{\chi}_9\Big[&-{}^1\overline{\alpha}_{1G}\cos\left(\hat{\upsilon}^{(1)}x_2+kx_1\right) - {}^1\overline{\alpha}_{1S}\sin\left(\hat{\upsilon}^{(1)}x_2+kx_1\right) \\
&+ {}^1\overline{\alpha}_{2G}\cos\left(\hat{\upsilon}^{(1)}x_2-kx_1\right) - {}^1\overline{\alpha}_{2S}\sin\left(\hat{\upsilon}^{(1)}x_2-kx_1\right)\Big] \\
-\hat{\chi}'_2\Big[&-{}^1\overline{\beta}_{1G}\sin\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right) + {}^1\overline{\beta}_{1S}\cos\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right) \\
&+ {}^1\overline{\beta}_{2G}\sin\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right) + {}^1\overline{\beta}_{2S}\cos\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)\Big]
\end{aligned} \tag{E2.111}
$$

$$
\begin{aligned}
{}^1B_G^{(1)}(k) = \hat{\chi}_1\Big[&-{}^1\overline{\alpha}_{1G}\cos\left(\hat{\upsilon}^{(1)}x_2+kx_1\right) - {}^1\overline{\alpha}_{1S}\sin\left(\hat{\upsilon}^{(1)}x_2+kx_1\right) \\
&+ {}^1\overline{\alpha}_{2G}\cos\left(\hat{\upsilon}^{(1)}x_2-kx_1\right) - {}^1\overline{\alpha}_{2S}\sin\left(\hat{\upsilon}^{(1)}x_2-kx_1\right)\Big] \\
+\hat{\chi}'_2\Big[&-{}^1\overline{\beta}_{1G}\sin\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right) + {}^1\overline{\beta}_{1S}\cos\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right) \\
&+ {}^1\overline{\beta}_{2G}\sin\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right) + {}^1\overline{\beta}_{2S}\cos\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)\Big]
\end{aligned} \tag{E2.112}
$$

$$
\begin{aligned}
{}^1C_S^{(1)}(k) = -\hat{\chi}'_3\Big[&{}^1\overline{\alpha}_{1G}\cos\left(\hat{\upsilon}^{(1)}x_2+kx_1\right) + {}^1\overline{\alpha}_{1S}\sin\left(\hat{\upsilon}^{(1)}x_2+kx_1\right) \\
&+ {}^1\overline{\alpha}_{2G}\cos\left(\hat{\upsilon}^{(1)}x_2-kx_1\right) - {}^1\overline{\alpha}_{2S}\sin\left(\hat{\upsilon}^{(1)}x_2-kx_1\right)\Big] \\
+\chi_4\Big[&{}^1\overline{\beta}_{1G}\sin\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right) + {}^1\overline{\beta}_{1S}\cos\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right) \\
&+ {}^1\overline{\beta}_{2G}\sin\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right) + {}^1\overline{\beta}_{2S}\cos\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)\Big]
\end{aligned} \tag{E2.113}
$$

$$
\begin{aligned}
{}^1\Theta_S^{(2)}(k) = k\Big[&{}^1\overline{\alpha}_{3G}\sin\left(\hat{\upsilon}^{(2)}x_2+kx_1\right) - {}^1\overline{\alpha}_{3S}\cos\left(\hat{\upsilon}^{(2)}x_2+kx_1\right)\Big] \\
-\hat{\upsilon}'^{(2)}\Big[&{}^1\overline{\beta}_{3G}\cos\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right) + {}^1\overline{\beta}_{3S}\sin\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)\Big]
\end{aligned} \tag{E2.114}
$$

$$
\begin{aligned}
{}^1\Omega_G^{(2)}(k) = \hat{\upsilon}^{(2)}\Big[&-{}^1\overline{\alpha}_{3G}\sin\left(\hat{\upsilon}^{(2)}x_2+kx_1\right) + {}^1\overline{\alpha}_{3S}\cos\left(\hat{\upsilon}^{(2)}x_2+kx_1\right)\Big] \\
-k\Big[&{}^1\overline{\beta}_{3G}\cos\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right) + {}^1\overline{\beta}_{3S}\sin\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)\Big]
\end{aligned} \tag{E2.115}
$$

$$
\begin{aligned}
{}^{1}A_{G}^{(2)}(k) &= \hat{\chi}_{10}\left[{}^{1}\overline{\alpha}_{3G}\cos\left(\hat{\upsilon}^{(2)}x_2+kx_1\right)+{}^{1}\overline{\alpha}_{3S}\sin\left(\hat{\upsilon}^{(2)}x_2+kx_1\right)\right] \\
&\quad +\hat{\chi}_{6}'\left[-{}^{1}\overline{\beta}_{3G}\sin\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)+{}^{1}\overline{\beta}_{3S}\cos\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)\right]
\end{aligned}
\tag{E2.116}
$$

$$
\begin{aligned}
{}^{1}B_{G}^{(2)}(k) &= \hat{\chi}_{5}\left[{}^{1}\overline{\alpha}_{3G}\cos\left(\hat{\upsilon}^{(2)}x_2+kx_1\right)+{}^{1}\overline{\alpha}_{3S}\sin\left(\hat{\upsilon}^{(2)}x_2+kx_1\right)\right] \\
&\quad +\hat{\chi}_{6}'\left[{}^{1}\overline{\beta}_{3G}\sin\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)-{}^{1}\overline{\beta}_{3S}\cos\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)\right]
\end{aligned}
\tag{E2.117}
$$

$$
\begin{aligned}
{}^{1}C_{S}^{(2)}(k) &= \hat{\chi}_{7}'\left[{}^{1}\overline{\alpha}_{3G}\cos\left(\hat{\upsilon}^{(2)}x_2+kx_1\right)+{}^{1}\overline{\alpha}_{3S}\sin\left(\hat{\upsilon}^{(2)}x_2+kx_1\right)\right] \\
&\quad +\chi_{8}\left[-{}^{1}\overline{\beta}_{3G}\sin\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)+{}^{1}\overline{\beta}_{3S}\cos\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)\right]
\end{aligned}
\tag{E2.118}
$$

olup bu eşitliklerin tek kısımları atıldıktan sonra kalan,

$$
\begin{aligned}
{}^{1}\Theta_{SÇ}^{(1)}(k) &= k\left[-({}^{1}\overline{\alpha}_{1G}-{}^{1}\overline{\alpha}_{2G})\cos\left(\hat{\upsilon}^{(1)}x_2\right)\right. \\
&\qquad \left.-\left({}^{1}\overline{\alpha}_{1S}+{}^{1}\overline{\alpha}_{2S}\right)\sin\left(\hat{\upsilon}^{(1)}x_2\right)\right]\sin(kx_1) \\
&\quad +\hat{\upsilon}'^{(1)}\left[-\left({}^{1}\overline{\beta}_{1G}-{}^{1}\overline{\beta}_{2G}\right)\sin\left(\hat{\upsilon}'^{(1)}x_2\right)\right. \\
&\qquad \left.+\left({}^{1}\overline{\beta}_{1S}+{}^{1}\overline{\beta}_{2S}\right)\cos\left(\hat{\upsilon}'^{(1)}x_2\right)\right]\sin(kx_1)
\end{aligned}
\tag{E2.119}
$$

$$
\begin{aligned}
{}^{1}\Omega_{GÇ}^{(1)}(k) &= \hat{\upsilon}^{(1)}\left[({}^{1}\overline{\alpha}_{1G}-{}^{1}\overline{\alpha}_{2G})\sin\left(\hat{\upsilon}^{(1)}x_2\right)\right. \\
&\qquad \left.-\left({}^{1}\overline{\alpha}_{1S}+{}^{1}\overline{\alpha}_{2S}\right)\cos\left(\hat{\upsilon}^{(1)}x_2\right)\right]\cos(kx_1) \\
&\quad +k\left[\left({}^{1}\overline{\beta}_{1G}-{}^{1}\overline{\beta}_{2G}\right)\cos\left(\hat{\upsilon}'^{(1)}x_2\right)\right. \\
&\qquad \left.+\left({}^{1}\overline{\beta}_{1S}+{}^{1}\overline{\beta}_{2S}\right)\sin\left(\hat{\upsilon}'^{(1)}x_2\right)\right]\cos(kx_1)
\end{aligned}
\tag{E2.120}
$$

$$
\begin{aligned}
{}^{1}A_{GÇ}^{(1)}(k) &= \hat{\chi}_{9}\left[-({}^{1}\overline{\alpha}_{1G}-{}^{1}\overline{\alpha}_{2G})\cos\left(\hat{\upsilon}^{(1)}x_2\right)\right. \\
&\qquad \left.-\left({}^{1}\overline{\alpha}_{1S}+{}^{1}\overline{\alpha}_{2S}\right)\sin\left(\hat{\upsilon}^{(1)}x_2\right)\right]\cos(kx_1) \\
&\quad +\hat{\chi}_{2}'\left[\left({}^{1}\overline{\beta}_{1G}-{}^{1}\overline{\beta}_{2G}\right)\sin\left(\hat{\upsilon}'^{(1)}x_2\right)\right. \\
&\qquad \left.-\left({}^{1}\overline{\beta}_{1S}+{}^{1}\overline{\beta}_{2S}\right)\cos\left(\hat{\upsilon}'^{(1)}x_2\right)\right]\cos(kx_1)
\end{aligned}
\tag{E2.121}
$$

$$
\begin{aligned}
{}^{1}B_{GÇ}^{(1)}(k) &= \hat{\chi}_{1}\left[-({}^{1}\overline{\alpha}_{1G}-{}^{1}\overline{\alpha}_{2G})\cos\left(\hat{\upsilon}^{(1)}x_2\right)\right. \\
&\qquad \left.-\left({}^{1}\overline{\alpha}_{1S}+{}^{1}\overline{\alpha}_{2S}\right)\sin\left(\hat{\upsilon}^{(1)}x_2\right)\right]\cos(kx_1) \\
&\quad +\hat{\chi}_{2}'\left[-\left({}^{1}\overline{\beta}_{1G}-{}^{1}\overline{\beta}_{2G}\right)\sin\left(\hat{\upsilon}'^{(1)}x_2\right)\right. \\
&\qquad \left.+\left({}^{1}\overline{\beta}_{1S}+{}^{1}\overline{\beta}_{2S}\right)\cos\left(\hat{\upsilon}'^{(1)}x_2\right)\right]\cos(kx_1)
\end{aligned}
\tag{E2.122}
$$

$$
\begin{aligned}
{}^{1}C_{SÇ}^{(1)}(k) &= \hat{\chi}_{3}'\left[({}^{1}\overline{\alpha}_{1G}-{}^{1}\overline{\alpha}_{2G})\sin\left(\hat{\upsilon}^{(1)}x_2\right)\right. \\
&\qquad \left.-\left({}^{1}\overline{\alpha}_{1S}+{}^{1}\overline{\alpha}_{2S}\right)\cos\left(\hat{\upsilon}^{(1)}x_2\right)\right]\sin(kx_1) \\
&\quad +\chi_{4}\left[\left({}^{1}\overline{\beta}_{1G}-{}^{1}\overline{\beta}_{2G}\right)\cos\left(\hat{\upsilon}'^{(1)}x_2\right)\right. \\
&\qquad \left.+\left({}^{1}\overline{\beta}_{1S}+{}^{1}\overline{\beta}_{2S}\right)\sin\left(\hat{\upsilon}'^{(1)}x_2\right)\right]\sin(kx_1)
\end{aligned}
\tag{E2.123}
$$

$$
\begin{aligned}
{}^{1}\Theta_{SÇ}^{(2)}(k) &= k\left\{\left[\cos(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\overline{\alpha}}_{3G}-\sin(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\overline{\alpha}}_{3S}\right]\cos\left(\hat{\upsilon}^{(2)}x_2\right)\right. \\
&\qquad \left.+\left[\cos(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\overline{\alpha}}_{3S}+\sin(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\overline{\alpha}}_{3G}\right]\sin\left(\hat{\upsilon}^{(2)}x_2\right)\right\}\sin(kx_1) \\
&\quad +\hat{\upsilon}'^{(2)}\left\{\left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\overline{\beta}}_{3G}-\sin(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\overline{\beta}}_{3S}\right]\sin\left(\hat{\upsilon}'^{(2)}x_2\right)\right. \\
&\qquad \left.-\left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\overline{\beta}}_{3S}+\sin(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\overline{\beta}}_{3G}\right]\cos\left(\hat{\upsilon}'^{(2)}x_2\right)\right\}\sin(kx_1)
\end{aligned}
\tag{E2.124}
$$

$$
\begin{aligned}
{}^{1}\Omega_{GÇ}^{(2)}(k) &= \hat{\upsilon}^{(2)}\left\{-\left[\cos(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\overline{\alpha}}_{3G}-\sin(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\overline{\alpha}}_{3S}\right]\sin\left(\hat{\upsilon}^{(2)}x_2\right)\right. \\
&\qquad \left.+\left[\cos(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\overline{\alpha}}_{3S}+\sin(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\overline{\alpha}}_{3G}\right]\cos\left(\hat{\upsilon}^{(2)}x_2\right)\right\}\cos(kx_1) \\
&\quad -k\left\{\left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\overline{\beta}}_{3G}-\sin(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\overline{\beta}}_{3S}\right]\cos\left(\hat{\upsilon}'^{(2)}x_2\right)\right. \\
&\qquad \left.+\left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\overline{\beta}}_{3S}+\sin(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\overline{\beta}}_{3G}\right]\sin\left(\hat{\upsilon}'^{(2)}x_2\right)\right\}\cos(kx_1)
\end{aligned}
\tag{E2.125}
$$

$$\begin{aligned}
{}^{1}A^{(2)}_{G\c{C}}(k) = \hat{\chi}_{10}\Big\{&\Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\tilde{\alpha}}_{3G} - \sin(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\tilde{\alpha}}_{3S}\Big]\cos\big(\hat{\upsilon}^{(2)}x_2\big) \\
&+\Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\tilde{\alpha}}_{3S} + \sin(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\tilde{\alpha}}_{3G}\Big]\sin\big(\hat{\upsilon}^{(2)}x_2\big)\Big\}\cos(kx_1) \\
+\hat{\chi}'_{6}\Big\{&-\Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\tilde{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\tilde{\beta}}_{3S}\Big]\sin\big(\hat{\upsilon}'^{(2)}x_2\big) \\
&+\Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\tilde{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\tilde{\beta}}_{3G}\Big]\cos\big(\hat{\upsilon}'^{(2)}x_2\big)\Big\}\cos(kx_1)
\end{aligned} \tag{E2.126}$$

$$\begin{aligned}
{}^{1}B^{(2)}_{G\c{C}}(k) = \hat{\chi}_{5}\Big\{&\Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\tilde{\alpha}}_{3G} - \sin(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\tilde{\alpha}}_{3S}\Big]\cos\big(\hat{\upsilon}^{(2)}x_2\big) \\
&+\Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\tilde{\alpha}}_{3S} + \sin(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\tilde{\alpha}}_{3G}\Big]\sin\big(\hat{\upsilon}^{(2)}x_2\big)\Big\}\cos(kx_1) \\
+\hat{\chi}'_{6}\Big\{&\Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\tilde{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\tilde{\beta}}_{3S}\Big]\sin\big(\hat{\upsilon}'^{(2)}x_2\big) \\
&-\Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\tilde{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\tilde{\beta}}_{3G}\Big]\cos\big(\hat{\upsilon}'^{(2)}x_2\big)\Big\}\cos(kx_1)
\end{aligned} \tag{E2.127}$$

$$\begin{aligned}
{}^{1}C^{(2)}_{S\c{C}}(k) = \hat{\chi}'_{7}\Big\{&-\Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\tilde{\alpha}}_{3G} - \sin(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\tilde{\alpha}}_{3S}\Big]\sin\big(\hat{\upsilon}^{(2)}x_2\big) \\
&+\Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\tilde{\alpha}}_{3S} + \sin(\hat{\upsilon}^{(2)}h)\ {}^{1}\tilde{\tilde{\alpha}}_{3G}\Big]\cos\big(\hat{\upsilon}^{(2)}x_2\big)\Big\}\sin(kx_1) \\
-\chi_{8}\Big\{&\Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\tilde{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\tilde{\beta}}_{3S}\Big]\cos\big(\hat{\upsilon}'^{(2)}x_2\big) \\
&+\Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\tilde{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^{1}\tilde{\tilde{\beta}}_{3G}\Big]\sin\big(\hat{\upsilon}'^{(2)}x_2\big)\Big\}\sin(kx_1)
\end{aligned} \tag{E2.128}$$

eşitlikleri (3.133)-(3.142) eşitliklerinde kullanılırsa sonuç olarak,

$${}^{1}\overline{u}^{(1)}_{1} = -\frac{P_0}{\pi}\int_0^{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}} {}^{1}\Theta^{(1)}_{S\c{C}}(k)\,dk \tag{E2.129}$$

$${}^{1}\overline{u}^{(1)}_{2} = \frac{P_0}{\pi}\int_0^{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}} {}^{1}\Omega^{(1)}_{G\c{C}}(k)\,dk \tag{E2.130}$$

$${}^{1}\sigma^{(1)}_{11} = \frac{P_0}{\pi}\int_0^{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}} {}^{1}A^{(1)}_{G\c{C}}(k)\,dk \tag{E2.131}$$

$${}^{1}\sigma^{(1)}_{22} = \frac{P_0}{\pi}\int_0^{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}} {}^{1}B^{(1)}_{G\c{C}}(k)\,dk \tag{E2.132}$$

$${}^{1}\sigma^{(1)}_{12} = -\frac{P_0}{\pi}\int_0^{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}} {}^{1}C^{(1)}_{S\c{C}}(k)\,dk \tag{E2.133}$$

$${}^{1}\overline{u}^{(2)}_{1} = -\frac{P_0}{\pi}\int_0^{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}} {}^{1}\Theta^{(2)}_{S\c{C}}(k)\,dk \tag{E2.134}$$

$${}^{1}\overline{u}^{(2)}_{2} = \frac{P_0}{\pi}\int_0^{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}} {}^{1}\Omega^{(2)}_{G\c{C}}(k)\,dk \tag{E2.135}$$

$$
{}^{1}\sigma_{11}^{(2)} = \frac{P_0}{\pi} \int_0^{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}} {}^{1}A_{GÇ}^{(2)}(k)\,dk \qquad \text{(E2.136)}
$$

$$
{}^{1}\sigma_{22}^{(2)} = \frac{P_0}{\pi} \int_0^{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}} {}^{1}B_{GÇ}^{(2)}(k)\,dk \qquad \text{(E2.137)}
$$

$$
{}^{1}\sigma_{12}^{(2)} = -\frac{P_0}{\pi} \int_0^{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}} {}^{1}C_{SÇ}^{(2)}(k)\,dk \qquad \text{(E2.138)}
$$

bulunur.

Ek 3. $k_L^{(1)} / \sqrt{1+\eta_1^{(1)}} < k < k_L^{(2)} \sqrt{1+\eta_1^{(2)}}$ **Durumu.**

Bu durumda (3.52) ve (3.53) uyarınca,

$$\upsilon^{(1)} = \left[k^2(1+\eta_1^{(1)} - k_L^{(1)2}\right]^{1/2} \tag{E3.1}$$
$$\upsilon'^{(1)} = \left[k^2(1+\eta_2^{(1)}) - k_T^{(1)2}\right]^{1/2} = i\left[k_T^{(1)2} - k^2(1+\eta_2^{(1)})\right]^{1/2} = i\,\hat{\upsilon}'^{(1)} \tag{E3.2}$$
$$\upsilon^{(2)} = \left[k^2(1+\eta_1^{(2)}) - k_L^{(2)2}\right]^{1/2} = i\left[k_L^{(2)2} - k^2(1+\eta_1^{(2)})\right]^{1/2} = i\,\hat{\upsilon}^{(2)} \tag{E3.3}$$
$$\upsilon'^{(2)} = \left[k^2(1+\eta_2^{(2)}) - k_T^{(2)2}\right]^{1/2} = i\left[k_T^{(2)2} - k^2(1+\eta_2^{(2)})\right]^{1/2} = i\,\hat{\upsilon}'^{(2)} \tag{E3.4}$$

eşitlikleri kullanılır. Buna göre,

$${}^2\chi_1 = \chi_1 \tag{E3.5}$$
$${}^2\chi_2 = i\hat{\chi}_2' \tag{E3.6}$$
$${}^2\chi_3 = \chi_3 \tag{E3.7}$$
$${}^2\chi_4 = \chi_4 \tag{E3.8}$$
$${}^2\chi_5 = \hat{\chi}_5 \tag{E3.9}$$
$${}^2\chi_6 = i\hat{\chi}_6' \tag{E3.10}$$
$${}^2\chi_7 = i\hat{\chi}_7' \tag{E3.11}$$
$${}^2\chi_8 = \chi_8 \tag{E3.12}$$
$${}^2\chi_9 = \chi_9 \tag{E3.13}$$
$${}^2\chi_{10} = \hat{\chi}_{10} \tag{E3.14}$$

bulunur. Ayrıca,

$${}^2H_1 = H_1 = e^{-\upsilon^{(1)}x_2} \tag{E3.15}$$
$${}^2H_2 = H_2 = e^{\upsilon^{(1)}x_2} \tag{E3.16}$$
$${}^2H_3 = \hat{H}_3 = e^{-i\hat{\upsilon}'^{(1)}x_2} \tag{E3.17}$$
$${}^2H_4 = \hat{H}_4 = e^{i\hat{\upsilon}'^{(1)}x_2} \tag{E3.18}$$
$${}^2H_5 = \hat{H}_5 = e^{-i\hat{\upsilon}^{(2)}x_2} \tag{E3.19}$$
$${}^2H_6 = \hat{H}_6 = e^{-i\hat{\upsilon}'^{(2)}x_2} \tag{E3.20}$$

ve,

$${}^2\overline{H}_1 = \overline{H}_1 = e^{-\upsilon^{(1)}h} \tag{E3.21}$$
$${}^2\overline{H}_2 = \overline{H}_2 = e^{\upsilon^{(1)}h} \tag{E3.22}$$
$${}^2\overline{H}_3 = \hat{\overline{H}}_3 = e^{-i\hat{\upsilon}'^{(1)}h} \tag{E3.23}$$
$${}^2\overline{H}_4 = \hat{\overline{H}}_4 = e^{i\hat{\upsilon}'^{(1)}h} \tag{E3.24}$$
$${}^2\overline{H}_5 = \hat{\overline{H}}_5 = e^{-i\hat{\upsilon}^{(2)}h} \tag{E3.25}$$
$${}^2\overline{H}_6 = \hat{\overline{H}}_6 = e^{-i\hat{\upsilon}'^{(2)}h} \tag{E3.26}$$

olup bütün bu eşitlikler (E1.23)-(E1.36) eşitliklerinde kullanılırsa,

$$\begin{aligned}{}^2\Gamma_{1G} &= k^2(\hat{\chi}_5\chi_8+\chi_1\chi_4-\chi_4\hat{\chi}_5-\chi_1\chi_8-\hat{\chi}'_2\hat{\chi}'_7+\hat{\chi}'_6\hat{\chi}'_7)\\ &\quad +k\hat{\upsilon}^{(2)}(\hat{\chi}'_2\chi_8-\chi_4\hat{\chi}'_6)+k\chi_1\hat{\chi}'_7(\hat{\upsilon}'^{(1)}-\hat{\upsilon}'^{(2)})+\hat{\upsilon}^{(2)}\chi_1(\hat{\upsilon}'^{(2)}\chi_4-\hat{\upsilon}'^{(1)}\chi_8)\end{aligned} \quad \text{(E3.27)}$$

$${}^2\Gamma_{2G}=0 \quad \text{(E3.28)}$$

$$\begin{aligned}{}^2\Gamma_{3G} &= k^2(\hat{\chi}_5\chi_8+\chi_1\chi_4-\chi_4\hat{\chi}_5-\chi_1\chi_8+\hat{\chi}'_2\hat{\chi}'_7+\hat{\chi}'_6\hat{\chi}'_7)\\ &\quad -k\hat{\upsilon}^{(2)}(\hat{\chi}'_2\chi_8+\chi_4\hat{\chi}'_6)-k\chi_1\hat{\chi}'_7(\hat{\upsilon}'^{(1)}+\hat{\upsilon}'^{(2)})+\hat{\upsilon}^{(2)}\chi_1(\hat{\upsilon}'^{(2)}\chi_4+\hat{\upsilon}'^{(1)}\chi_8)\end{aligned} \quad \text{(E3.39)}$$

$${}^2\Gamma_{4G}={}^2\Gamma_{3G} \quad \text{(E3.30)}$$

$${}^2\Gamma_{5G}={}^2\Gamma_{1G} \quad \text{(E3.31)}$$

$$\begin{aligned}{}^2\Gamma_{6G} &= -2k\upsilon^{(1)}(\hat{\chi}'_6\hat{\chi}'_7+\hat{\chi}_5\chi_8)+2k\chi_3(k\hat{\chi}_5+\hat{\upsilon}^{(2)}\hat{\chi}'_6)\\ &\quad +2k\chi_1(\upsilon^{(1)}\chi_8-k\chi_3)-2\hat{\upsilon}'^{(2)}\chi_1(\hat{\upsilon}^{(2)}\chi_3-\upsilon^{(1)}\hat{\chi}'_7)\end{aligned} \quad \text{(E3.32)}$$

$${}^2\Gamma_{7G}=2(\upsilon^{(1)}\chi_4-k\chi_3)(\hat{\upsilon}'^{(1)}\hat{\chi}'_6-\hat{\upsilon}'^{(2)}\hat{\chi}'_2) \quad \text{(E3.33)}$$

$${}^2\Gamma_{8G}=-\,{}^2\Gamma_{7G} \quad \text{(E3.34)}$$

$${}^2\Gamma_{9G}=0 \quad \text{(E3.35)}$$

$${}^2\Gamma_{10G}=0 \quad \text{(E3.36)}$$

$${}^2\Gamma_{11G}=2(\hat{\upsilon}'^{(1)}\chi_1-k\hat{\chi}'_2)(k\hat{\chi}'_7-\hat{\upsilon}^{(2)}\chi_4) \quad \text{(E3.37)}$$

$${}^2\Gamma_{12G}={}^2\Gamma_{11G} \quad \text{(E3.38)}$$

$$\begin{aligned}{}^2\Gamma_{13G} &= -2k\upsilon^{(1)}(\chi_4\hat{\chi}_5-\hat{\chi}'_2\hat{\chi}'_7)+2k\chi_3(k\hat{\chi}_5-\hat{\upsilon}^{(2)}\hat{\chi}'_2)\\ &\quad -2\hat{\upsilon}'^{(1)}\chi_1(\upsilon^{(1)}\hat{\chi}'_7-\hat{\upsilon}^{(2)}\chi_3)+2k\chi_1(\upsilon^{(1)}\chi_4-k\chi_3)\end{aligned} \quad \text{(E3.39)}$$

$$\begin{aligned}{}^2\Gamma_{14G} &= -2k\upsilon^{(1)}(\hat{\chi}'_2\hat{\chi}'_7+\chi_4\hat{\chi}_5)+2k\chi_3(k\hat{\chi}_5+\hat{\upsilon}^{(2)}\hat{\chi}'_2)\\ &\quad +2\hat{\upsilon}'^{(1)}\chi_1(\upsilon^{(1)}\hat{\chi}'_7-\hat{\upsilon}^{(2)}\chi_3)+2k\chi_1(\upsilon^{(1)}\chi_4-k\chi_3)\end{aligned} \quad \text{(E3.40)}$$

ve,

$$\begin{aligned}{}^2\Gamma_{1S} &= k^2(\hat{\chi}'_2\chi_3-\chi_3\hat{\chi}'_6)-k\upsilon^{(1)}(\hat{\chi}'_2\chi_8-\chi_4\hat{\chi}'_6)-k\hat{\chi}_5(\hat{\upsilon}'^{(1)}\chi_3-\hat{\upsilon}'^{(2)}\chi_3)\\ &\quad +\hat{\upsilon}'^{(1)}\upsilon^{(1)}(\hat{\chi}_5\chi_8+\hat{\chi}'_6\hat{\chi}'_7)-\hat{\upsilon}'^{(2)}\upsilon^{(1)}(\hat{\chi}'_2\hat{\chi}'_7+\chi_4\hat{\chi}_5)+\hat{\upsilon}^{(2)}\chi_3(\hat{\upsilon}'^{(2)}\hat{\chi}'_2-\hat{\upsilon}'^{(1)}\hat{\chi}'_6)\end{aligned} \quad \text{(E3.41)}$$

$$\begin{aligned}{}^2\Gamma_{2S} &= -2\hat{\upsilon}'^{(1)}k(\hat{\chi}'_6\hat{\chi}'_7+\hat{\chi}_5\chi_8)+2\hat{\upsilon}'^{(1)}\chi_4(k\hat{\chi}_5+\hat{\upsilon}^{(2)}\hat{\chi}'_6)\\ &\quad +2k^2(\hat{\chi}'_2\chi_8-\hat{\chi}'_2\chi_4)-2\hat{\upsilon}'^{(2)}\hat{\chi}'_2(\hat{\upsilon}^{(2)}\chi_4-k\hat{\chi}'_7)\end{aligned} \quad \text{(E3.42)}$$

$$\begin{aligned}{}^2\Gamma_{3S} &= -k^2(\hat{\chi}'_2\chi_3+\chi_3\hat{\chi}'_6)+k\upsilon^{(1)}(\hat{\chi}'_2\chi_8+\chi_4\hat{\chi}'_6)+k\hat{\chi}_5(\hat{\upsilon}'^{(1)}\chi_3+\hat{\upsilon}'^{(2)}\chi_3)\\ &\quad -\hat{\upsilon}'^{(1)}\upsilon^{(1)}(\hat{\chi}_5\chi_8+\hat{\chi}'_6\hat{\chi}'_7)+\hat{\upsilon}'^{(2)}\upsilon^{(1)}(\hat{\chi}'_2\hat{\chi}'_7-\chi_4\hat{\chi}_5)-\hat{\upsilon}^{(2)}\chi_3(\hat{\upsilon}'^{(2)}\hat{\chi}'_2-\hat{\upsilon}'^{(1)}\hat{\chi}'_6)\end{aligned} \quad \text{(E3.43)}$$

$${}^2\Gamma_{4S}=-\,{}^2\Gamma_{3S} \quad \text{(E3.44)}$$

$${}^2\Gamma_{5S}=-\,{}^2\Gamma_{1S} \quad \text{(E3.45)}$$

$${}^2\Gamma_{6S}=0 \quad \text{(E3.46)}$$

$${}^2\Gamma_{7S}=2(k\chi_8-k\chi_4)(\hat{\upsilon}'^{(1)}\chi_1-k\hat{\chi}'_2) \quad \text{(E3.47)}$$

$${}^2\Gamma_{8S}={}^2\Gamma_{7S} \quad \text{(E3.48)}$$

$$\begin{aligned}{}^2\Gamma_{9S} &= -2\upsilon^{(1)}k(\hat{\chi}'_2\chi_8+\chi_4\hat{\chi}'_6)+2\hat{\upsilon}'^{(2)}\chi_1(\upsilon^{(1)}\chi_4-k\chi_3)\\ &\quad +2k^2(\chi_3\hat{\chi}'_2+\chi_3\hat{\chi}'_6)+2\hat{\upsilon}'^{(1)}\chi_1(\upsilon^{(1)}\chi_8-k\chi_3)\end{aligned} \quad \text{(E3.49)}$$

$$
\begin{aligned}
{}^2\Gamma_{10S} = {} & 2\upsilon^{(1)}k(\hat{\chi}_2'\chi_8 - \chi_4\hat{\chi}_6') + 2\hat{\upsilon}'^{(2)}\chi_1(\upsilon^{(1)}\chi_4 - k\chi_3) \\
& - 2k^2(\chi_3\hat{\chi}_2' - \chi_3\hat{\chi}_6') - 2\hat{\upsilon}'^{(1)}\chi_1(\upsilon^{(1)}\chi_8 - k\chi_3)
\end{aligned}
\tag{E3.50}
$$

$$ {}^2\Gamma_{11S} = 2(k\hat{\chi}_2' - \hat{\upsilon}'^{(1)}\hat{\chi}_5)(k\chi_3 - \upsilon^{(1)}\chi_4) \tag{E3.51} $$

$$ {}^2\Gamma_{12S} = -\ {}^2\Gamma_{11S} \tag{E3.52} $$

$$ {}^2\Gamma_{13S} = 0 \tag{E3.53} $$

$$ {}^2\Gamma_{14S} = 0 \tag{E3.54} $$

olmak üzere,

$$ {}^2\Gamma_j = {}^2\Gamma_{jG} + i\ {}^2\Gamma_{jS} \quad ,j=1,14 \tag{E3.55} $$

yazılabilir. Ayrıca (3.85) eşitliği yardımı ile,

$$
\begin{aligned}
{}^2[\Delta] = {} & (i\hat{\chi}_2'\chi_3 + \chi_1\chi_4)\left[\overline{H}_2\hat{\overline{H}}_3({}^2\Gamma_{1G} + i\ {}^2\Gamma_{1S}) + \overline{H}_1\hat{\overline{H}}_4({}^2\Gamma_{4G} + i\ {}^2\Gamma_{4S})\right] \\
& + (i\hat{\chi}_2'\chi_3 - \chi_1\chi_4)\left[\overline{H}_1\hat{\overline{H}}_3({}^2\Gamma_{5G} + i\ {}^2\Gamma_{5S}) + \overline{H}_2\hat{\hat{\overline{H}}}_4({}^2\Gamma_{3G} + i\ {}^2\Gamma_{3S})\right] \\
& + 2\left[\chi_1\chi_3\hat{\overline{H}}_3\hat{\overline{H}}_4({}^2\Gamma_{2G} + i\ {}^2\Gamma_{2S}) + i\hat{\chi}_2'\chi_4\overline{H}_1\overline{H}_2({}^2\Gamma_{6G} + i\ {}^2\Gamma_{6S})\right]
\end{aligned}
\tag{E3.56}
$$

ve k dönüşüm parametresine göre,

${}^2\Gamma_{1G}, {}^2\Gamma_{3G}, {}^2\Gamma_{4G}, {}^2\Gamma_{5G}, {}^2\Gamma_{11G}, {}^2\Gamma_{12G}$ ve ${}^2\Gamma_{1S}, {}^2\Gamma_{3S}, {}^2\Gamma_{4S}, {}^2\Gamma_{5S}, {}^2\Gamma_{9S}, {}^2\Gamma_{10S}, {}^2\Gamma_{11S}, {}^2\Gamma_{12S}$; çift,
${}^2\Gamma_{6G}, {}^2\Gamma_{7G}, {}^2\Gamma_{8G}, {}^2\Gamma_{13G}, {}^2\Gamma_{14G}$ ve ${}^2\Gamma_{2S}, {}^2\Gamma_{7S}, {}^2\Gamma_{8S}$; tek fonksiyonlar ve
${}^2\Gamma_{2G}, {}^2\Gamma_{6S}, {}^2\Gamma_{9G}, {}^2\Gamma_{10G}, {}^2\Gamma_{13S}, {}^2\Gamma_{14S} = 0$ olduğuna göre,

$$
\begin{aligned}
{}^2[\Delta]_G = {} & -\hat{\chi}_2'\chi_3\Big\{e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{1S} + {}^2\Gamma_{3S}) - \sin(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{1G} - {}^2\Gamma_{3G})\right] \\
& \qquad + e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{4S} + {}^2\Gamma_{5S}) + \sin(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{4G} - {}^2\Gamma_{5G})\right]\Big\} \\
& + \chi_1\chi_4\Big\{e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{1G} - {}^2\Gamma_{3G}) + \sin(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{1S} + {}^2\Gamma_{3S})\right] \\
& \qquad + e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{4G} - {}^2\Gamma_{5G}) - \sin(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{4S} + {}^2\Gamma_{5S})\right]\Big\}
\end{aligned}
\tag{E3.57}
$$

ve,

$$
\begin{aligned}
{}^2[\Delta]_S = {} & \hat{\chi}_2'\chi_3\Big\{e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{1G} + {}^2\Gamma_{3G}) + \sin(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{1S} - {}^2\Gamma_{3S})\right] \\
& \qquad + e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{4G} + {}^2\Gamma_{5G}) - \sin(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{4S} - {}^2\Gamma_{5S})\right]\Big\} \\
& + \chi_1\chi_4\Big\{e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{1S} - {}^2\Gamma_{3S}) - \sin(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{1G} + {}^2\Gamma_{3G})\right] \\
& \qquad + e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{4S} - {}^2\Gamma_{5S}) + \sin(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{4G} + {}^2\Gamma_{5G})\right]\Big\} \\
& + 2\left(\chi_1\chi_3\ {}^2\Gamma_{2S} + \hat{\chi}_2'\chi_4\ {}^2\Gamma_{6G}\right)
\end{aligned}
\tag{E3.58}
$$

olmak üzere,

$$ {}^2[\Delta] = {}^2[\Delta]_G + i\ {}^2[\Delta]_S \tag{E3.59} $$

yazılabilir. (E3.59) eşitliğinin gerçel ve sanal kısımları çift fonksiyonlar olmaktadır. Buradan (E1.49)-(E1.54) yardımı ile ve (E3.28), (E3.35), (E3.36), (E3.46), (E3.53) ve (E3.54) eşitlikleri de gözönüne alınarak,

$$ {}^2\overline{\alpha}_1 = \frac{\chi_4 e^{\upsilon^{(1)}h}\left[e^{-i\hat{\upsilon}'^{(1)}h}({}^2\Gamma_{1G} + i\ {}^2\Gamma_{1S}) - e^{i\hat{\upsilon}'^{(1)}h}({}^2\Gamma_{3G} + i\ {}^2\Gamma_{3S})\right] + i\chi_3\ {}^2\Gamma_{2S}}{{}^2[\Delta]} \qquad \text{(E3.60)} $$

$$ {}^2\overline{\alpha}_2 = \frac{\chi_4 e^{-\upsilon^{(1)}h}\left[e^{-i\hat{\upsilon}'^{(1)}h}({}^2\Gamma_{5G} + i\ {}^2\Gamma_{5S}) - e^{i\hat{\upsilon}'^{(1)}h}({}^2\Gamma_{4G} + i\ {}^2\Gamma_{4S})\right] - i\chi_3\ {}^2\Gamma_{2S}}{{}^2[\Delta]} \qquad \text{(E3.61)} $$

$$ {}^2\overline{\beta}_1 = \frac{\chi_3 e^{i\hat{\upsilon}'^{(1)}h}\left[e^{\upsilon^{(1)}h}({}^2\Gamma_{3G} + i\ {}^2\Gamma_{3S}) + e^{-\upsilon^{(1)}h}({}^2\Gamma_{4G} + i\ {}^2\Gamma_{4S})\right] + \chi_4\ {}^2\Gamma_{6G}}{{}^2[\Delta]} \qquad \text{(E3.62)} $$

$$ {}^2\overline{\beta}_2 = \frac{\chi_3 e^{-i\hat{\upsilon}'^{(1)}h}\left[e^{\upsilon^{(1)}h}({}^2\Gamma_{1G} + i\ {}^2\Gamma_{1S}) + e^{-\upsilon^{(1)}h}({}^2\Gamma_{5G} + i\ {}^2\Gamma_{5S})\right] + \chi_4\ {}^2\Gamma_{6G}}{{}^2[\Delta]} \qquad \text{(E3.63)} $$

$$ {}^2\overline{\alpha}_3 = \frac{\chi_3\left[e^{\upsilon^{(1)}h}({}^2\Gamma_{7G} + i\ {}^2\Gamma_{7S}) + e^{-\upsilon^{(1)}h}({}^2\Gamma_{8G} + i\ {}^2\Gamma_{8S})\right] + i\chi_4\left(e^{-i\hat{\upsilon}'^{(1)}h}\ {}^2\Gamma_{10S} - e^{\hat{\upsilon}'^{(1)}h}\ {}^2\Gamma_{9S}\right)}{e^{-i\hat{\upsilon}^{(2)}h}\ {}^2[\Delta]} \qquad \text{(E3.64)} $$

$$ {}^2\overline{\beta}_3 = \frac{\chi_3\left[e^{\upsilon^{(1)}h}({}^2\Gamma_{11G} + i\ {}^2\Gamma_{11S}) + e^{-\upsilon^{(1)}h}({}^2\Gamma_{12G} + i\ {}^2\Gamma_{12S})\right] + \chi_4\left(e^{i\hat{\upsilon}'^{(1)}h}\,{}^2\Gamma_{13G} - e^{-\hat{\upsilon}'^{(1)}h}\,{}^2\Gamma_{14G}\right)}{e^{-i\hat{\upsilon}'^{(2)}h}\ {}^2[\Delta]} \qquad \text{(E3.65)} $$

eşitlikleri yazılabilir. (E3.60)-(E3.65) eşitliklerini gerçel ve sanal kısımlarına ayırmak için,

$$ {}^2\overline{\alpha}_{1PG} = \chi_4 e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)\,({}^2\Gamma_{1G} - {}^2\Gamma_{3G}) + \sin(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{1S} + {}^2\Gamma_{3S})\right] \qquad \text{(E3.66)} $$

$$ {}^2\overline{\alpha}_{1PS} = \chi_4 e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)\,({}^2\Gamma_{1S} - {}^2\Gamma_{3S}) - \sin(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{1G} + {}^2\Gamma_{3G})\right] + \chi_3\ {}^2\Gamma_{2S} \qquad \text{(E3.67)} $$

$$ {}^2\overline{\alpha}_{2PG} = \chi_4 e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)\,({}^2\Gamma_{5G} - {}^2\Gamma_{4G}) + \sin(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{5S} + {}^2\Gamma_{4S})\right] \qquad \text{(E3.68)} $$

$$ {}^2\overline{\alpha}_{2PS} = \chi_4 e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)\,({}^2\Gamma_{5S} - {}^2\Gamma_{4S}) - \sin(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{5G} + {}^2\Gamma_{4G})\right] - \chi_3\ {}^2\Gamma_{2S} \qquad \text{(E3.69)} $$

$$ \begin{aligned} {}^2\overline{\beta}_{1PG} = \chi_3 \Big[&\cos(\hat{\upsilon}'^{(1)}h)(e^{\upsilon^{(1)}h}\ {}^2\Gamma_{3G} + e^{-\upsilon^{(1)}h}\ {}^2\Gamma_{4G}) \\ &- \sin(\hat{\upsilon}'^{(1)}h)(e^{\upsilon^{(1)}h}\ {}^2\Gamma_{3S} + e^{-\upsilon^{(1)}h}\ {}^2\Gamma_{4S})\Big] + \chi_4\ {}^2\Gamma_{6G} \end{aligned} \qquad \text{(E3.70)} $$

$$ {}^2\overline{\beta}_{1PS} = \chi_3\left[\cos(\hat{\upsilon}'^{(1)}h)(e^{\upsilon^{(1)}h}\ {}^2\Gamma_{3S} + e^{-\upsilon^{(1)}h}\ {}^2\Gamma_{4S}) + \sin(\hat{\upsilon}'^{(1)}h)(e^{\upsilon^{(1)}h}\ {}^2\Gamma_{3G} + e^{-\upsilon^{(1)}h}\ {}^2\Gamma_{4G})\right] \qquad \text{(E3.71)} $$

$$ \begin{aligned} {}^2\overline{\beta}_{2PG} = \chi_3 \Big[&\cos(\hat{\upsilon}'^{(1)}h)(e^{\upsilon^{(1)}h}\ {}^2\Gamma_{1G} + e^{-\upsilon^{(1)}h}\ {}^2\Gamma_{5G}) \\ &+ \sin(\hat{\upsilon}'^{(1)}h)(e^{\upsilon^{(1)}h}\ {}^2\Gamma_{1S} + e^{-\upsilon^{(1)}h}\ {}^2\Gamma_{5S})\Big] + \chi_4\ {}^2\Gamma_{6G} \end{aligned} \qquad \text{(E3.72)} $$

$$ {}^2\overline{\beta}_{2PS} = \chi_3\left[\cos(\hat{\upsilon}'^{(1)}h)(e^{\upsilon^{(1)}h}\ {}^2\Gamma_{1S} + e^{-\upsilon^{(1)}h}\ {}^2\Gamma_{5S}) - \sin(\hat{\upsilon}'^{(1)}h)(e^{\upsilon^{(1)}h}\ {}^2\Gamma_{1G} + e^{-\upsilon^{(1)}h}\ {}^2\Gamma_{5G})\right] \qquad \text{(E3.73)} $$

$$ {}^2\overline{\alpha}_{3PG} = \chi_3(e^{\upsilon^{(1)}h}\ {}^2\Gamma_{7G} + e^{-\upsilon^{(1)}h}\ {}^2\Gamma_{8G}) + \chi_4 \sin(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{10S} + {}^2\Gamma_{9S}) \qquad \text{(E3.74)} $$

$$ {}^2\overline{\alpha}_{3PS} = \chi_3(e^{\upsilon^{(1)}h}\ {}^2\Gamma_{7S} + e^{-\upsilon^{(1)}h}\ {}^2\Gamma_{8S}) + \chi_4 \cos(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{10S} - {}^2\Gamma_{9S}) \qquad \text{(E3.75)} $$

$$ {}^2\overline{\beta}_{3PG} = \chi_3(e^{\upsilon^{(1)}h}\ {}^2\Gamma_{11G} + e^{-\upsilon^{(1)}h}\ {}^2\Gamma_{12G}) + \chi_4 \cos(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{13G} - {}^2\Gamma_{14G}) \qquad \text{(E3.76)} $$

$$ {}^2\overline{\beta}_{3PS} = \chi_3(e^{\upsilon^{(1)}h}\ {}^2\Gamma_{11S} + e^{-\upsilon^{(1)}h}\ {}^2\Gamma_{12S}) + \chi_4 \sin(\hat{\upsilon}'^{(1)}h)({}^2\Gamma_{13G} + {}^2\Gamma_{14G}) \qquad \text{(E3.77)} $$

tanımlamaları yapılırsa,

$$ {}^2\overline{\alpha}_{1G} = \frac{{}^2[\Delta]_G\ {}^2\overline{\alpha}_{1PG} + {}^2[\Delta]_S\ {}^2\overline{\alpha}_{1PS}}{{}^2[\Delta]_G^{\,2} + {}^2[\Delta]_S^{\,2}} \qquad \text{(E3.78)} $$

$$ {}^{2}\overline{\alpha}_{1S}=\frac{{}^{2}[\Delta]_{G}\,{}^{2}\overline{\alpha}_{1PS}-{}^{2}[\Delta]_{S}\,{}^{2}\overline{\alpha}_{1PG}}{{}^{2}[\Delta]_{G}^{2}+{}^{2}[\Delta]_{S}^{2}} \qquad \text{(E3.79)}$$

$$ {}^{2}\overline{\alpha}_{2G}=\frac{{}^{2}[\Delta]_{G}\,{}^{2}\overline{\alpha}_{2PG}+{}^{2}[\Delta]_{S}\,{}^{2}\overline{\alpha}_{2PS}}{{}^{2}[\Delta]_{G}^{2}+{}^{2}[\Delta]_{S}^{2}} \qquad \text{(E3.80)}$$

$$ {}^{2}\overline{\alpha}_{2S}=\frac{{}^{2}[\Delta]_{G}\,{}^{2}\overline{\alpha}_{2PS}-{}^{2}[\Delta]_{S}\,{}^{2}\overline{\alpha}_{2PG}}{{}^{2}[\Delta]_{G}^{2}+{}^{2}[\Delta]_{S}^{2}} \qquad \text{(E3.81)}$$

$$ {}^{2}\overline{\beta}_{1G}=\frac{{}^{2}[\Delta]_{G}\,{}^{2}\overline{\beta}_{1PG}+{}^{2}[\Delta]_{S}\,{}^{2}\overline{\beta}_{1PS}}{{}^{2}[\Delta]_{G}^{2}+{}^{2}[\Delta]_{S}^{2}} \qquad \text{(E3.82)}$$

$$ {}^{2}\overline{\beta}_{1S}=\frac{{}^{2}[\Delta]_{G}\,{}^{2}\overline{\beta}_{1PS}-{}^{2}[\Delta]_{S}\,{}^{2}\overline{\beta}_{1PG}}{{}^{2}[\Delta]_{G}^{2}+{}^{2}[\Delta]_{S}^{2}} \qquad \text{(E3.83)}$$

$$ {}^{2}\overline{\beta}_{2G}=\frac{{}^{2}[\Delta]_{G}\,{}^{2}\overline{\beta}_{2PG}+{}^{2}[\Delta]_{S}\,{}^{2}\overline{\beta}_{2PS}}{{}^{2}[\Delta]_{G}^{2}+{}^{2}[\Delta]_{S}^{2}} \qquad \text{(E3.84)}$$

$$ {}^{2}\overline{\beta}_{2S}=\frac{{}^{2}[\Delta]_{G}\,{}^{2}\overline{\beta}_{2PS}-{}^{2}[\Delta]_{S}\,{}^{2}\overline{\beta}_{2PG}}{{}^{2}[\Delta]_{G}^{2}+{}^{2}[\Delta]_{S}^{2}} \qquad \text{(E3.85)}$$

$$ {}^{2}\tilde{\overline{\alpha}}_{3G}=\frac{{}^{2}[\Delta]_{G}\,{}^{2}\overline{\alpha}_{3PG}+{}^{2}[\Delta]_{S}\,{}^{2}\overline{\alpha}_{3PS}}{{}^{2}[\Delta]_{G}^{2}+{}^{2}[\Delta]_{S}^{2}} \qquad \text{(E3.86)}$$

$$ {}^{2}\tilde{\overline{\alpha}}_{3S}=\frac{{}^{2}[\Delta]_{G}\,{}^{2}\overline{\alpha}_{3PS}-{}^{2}[\Delta]_{S}\,{}^{2}\overline{\alpha}_{3PG}}{{}^{2}[\Delta]_{G}^{2}+{}^{2}[\Delta]_{S}^{2}} \qquad \text{(E3.87)}$$

$$ {}^{2}\tilde{\overline{\beta}}_{3G}=\frac{{}^{2}[\Delta]_{G}\,{}^{2}\overline{\beta}_{3PG}+{}^{2}[\Delta]_{S}\,{}^{2}\overline{\beta}_{3PS}}{{}^{2}[\Delta]_{G}^{2}+{}^{2}[\Delta]_{S}^{2}} \qquad \text{(E3.88)}$$

$$ {}^{2}\tilde{\overline{\beta}}_{3S}=\frac{{}^{2}[\Delta]_{G}\,{}^{2}\overline{\beta}_{3PS}-{}^{2}[\Delta]_{S}\,{}^{2}\overline{\beta}_{3PG}}{{}^{2}[\Delta]_{G}^{2}+{}^{2}[\Delta]_{S}^{2}} \qquad \text{(E3.89)}$$

olmak üzere,

$$ {}^{2}\overline{\alpha}_{1}={}^{2}\overline{\alpha}_{1G}+i\;{}^{2}\overline{\alpha}_{1S} \qquad \text{(E3.90)}$$

$$ {}^{2}\overline{\alpha}_{2}={}^{2}\overline{\alpha}_{2G}+i\;{}^{2}\overline{\alpha}_{2S} \qquad \text{(E3.91)}$$

$$ {}^{2}\overline{\beta}_{1}={}^{2}\overline{\beta}_{1G}+i\;{}^{2}\overline{\beta}_{1S} \qquad \text{(E3.92)}$$

$$ {}^{2}\overline{\beta}_{2}={}^{2}\overline{\beta}_{2G}+i\;{}^{2}\overline{\beta}_{2S} \qquad \text{(E3.93)}$$

$$ {}^{2}\overline{\alpha}_{3}=\cos(\hat{\upsilon}^{(2)}h)\;{}^{2}\tilde{\overline{\alpha}}_{3G}-\sin(\hat{\upsilon}^{(2)}h)\;{}^{2}\tilde{\overline{\alpha}}_{3S}+i\left[\cos(\hat{\upsilon}^{(2)}h)\;{}^{2}\tilde{\overline{\alpha}}_{3S}+\sin(\hat{\upsilon}^{(2)}h)\;{}^{2}\tilde{\overline{\alpha}}_{3G}\right] \qquad \text{(E3.94)}$$

$$ {}^{2}\overline{\beta}_{3}=\cos(\hat{\upsilon}'^{(2)}h)\;{}^{2}\tilde{\overline{\beta}}_{3G}-\sin(\hat{\upsilon}'^{(2)}h)\;{}^{2}\tilde{\overline{\beta}}_{3S}+i\left[\cos(\hat{\upsilon}'^{(2)}h)\;{}^{2}\tilde{\overline{\beta}}_{3S}+\sin(\hat{\upsilon}'^{(2)}h)\;{}^{2}\tilde{\overline{\beta}}_{3G}\right] \qquad \text{(E3.95)}$$

yazılabilir. ${}^{2}\overline{\alpha}_{iPG},{}^{2}\overline{\alpha}_{iPS}$ (i=1,3) fonksiyonları k'ya göre çift, ${}^{2}\overline{\beta}_{iPG},{}^{2}\overline{\beta}_{iPS}$ (i=1,3) ise tek olup, ${}^{2}[\Delta]_{G}$ ve ${}^{2}[\Delta]_{S}$ fonksiyonları da çift olduklarına göre sonuç olarak ${}^{2}\overline{\alpha}_{iG},{}^{2}\overline{\alpha}_{iS}$ (i=1,3) fonksiyonlarının çift, ${}^{2}\overline{\beta}_{iG},{}^{2}\overline{\beta}_{iS}$ (i=1,3) fonksiyonlarının da tek oldukları görülür. (E3.90)-(E3.95) eşitlikleri (3.122)-(3.131) eşitliklerinde kullanılırsa, (E3.5)-(E3.14) ve (E3.15)-(E3.20) eşitlikleri yardımı ile,

$$
\begin{aligned}
{}^2\Theta^{(1)}(k) = k\Big[&\left({}^2\overline{\alpha}_{1G} + i\ {}^2\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2}e^{-ikx_1} - \left({}^2\overline{\alpha}_{2G} + i\ {}^2\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}e^{-ikx_1}\Big] \\
&+ i\hat{\upsilon}'^{(1)}\left[\left({}^2\overline{\beta}_{1G} + i\ {}^2\overline{\beta}_{1S}\right)e^{-i\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right)} + \left({}^2\overline{\beta}_{2G} + i\ {}^2\overline{\beta}_{2S}\right)e^{i\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right)}\right]
\end{aligned} \tag{E3.96}
$$

$$
\begin{aligned}
{}^2\Omega^{(1)}(k) = \upsilon^{(1)}\Big[&\left({}^2\overline{\alpha}_{1G} + i\ {}^2\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2}e^{-ikx_1} + \left({}^2\overline{\alpha}_{2G} + i\ {}^2\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}e^{-ikx_1}\Big] \\
&+ k\left[\left({}^2\overline{\beta}_{1G} + i\ {}^2\overline{\beta}_{1S}\right)e^{-i\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right)} - \left({}^2\overline{\beta}_{2G} + i\ {}^2\overline{\beta}_{2S}\right)e^{i\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right)}\right]
\end{aligned} \tag{E3.97}
$$

$$
\begin{aligned}
{}^2A^{(1)}(k) = \chi_9\Big[&-\left({}^2\overline{\alpha}_{1G} + i\ {}^2\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2}e^{-ikx_1} + \left({}^2\overline{\alpha}_{2G} + i\ {}^2\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}e^{-ikx_1}\Big] \\
&+ i\hat{\chi}'_2\left[\left({}^2\overline{\beta}_{1G} + i\ {}^2\overline{\beta}_{1S}\right)e^{-i\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right)} + \left({}^2\overline{\beta}_{2G} + i\ {}^2\overline{\beta}_{2S}\right)e^{i\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right)}\right]
\end{aligned} \tag{E3.98}
$$

$$
\begin{aligned}
{}^2B^{(1)}(k) = \chi_1\Big[&-\left({}^2\overline{\alpha}_{1G} + i\ {}^2\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2}e^{-ikx_1} + \left({}^2\overline{\alpha}_{2G} + i\ {}^2\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}e^{-ikx_1}\Big] \\
&- i\hat{\chi}'_2\left[\left({}^2\overline{\beta}_{1G} + i\ {}^2\overline{\beta}_{1S}\right)e^{-i\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right)} + \left({}^2\overline{\beta}_{2G} + i\ {}^2\overline{\beta}_{2S}\right)e^{i\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right)}\right]
\end{aligned} \tag{E3.99}
$$

$$
\begin{aligned}
{}^2C^{(1)}(k) = -\chi_3\Big[&\left({}^2\overline{\alpha}_{1G} + i\ {}^2\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2}e^{-ikx_1} + \left({}^2\overline{\alpha}_{2G} + i\ {}^2\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}e^{-ikx_1}\Big] \\
&+ \chi_4\left[-\left({}^2\overline{\beta}_{1G} + i\ {}^2\overline{\beta}_{1S}\right)e^{-i\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right)} + \left({}^2\overline{\beta}_{2G} + i\ {}^2\overline{\beta}_{2S}\right)e^{i\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right)}\right]
\end{aligned} \tag{E3.100}
$$

$$
{}^2\Theta^{(2)}(k) = -k\left({}^2\overline{\alpha}_{3G} + i\ {}^2\overline{\alpha}_{3S}\right)e^{-i\left(\hat{\upsilon}^{(2)}x_2 + kx_1\right)} - i\hat{\upsilon}'^{(2)}\left({}^2\overline{\beta}_{3G} + i\ {}^2\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)} \tag{E3.101}
$$

$$
{}^2\Omega^{(2)}(k) = -i\hat{\upsilon}^{(2)}\left({}^2\overline{\alpha}_{3G} + i\ {}^2\overline{\alpha}_{3S}\right)e^{-i\left(\hat{\upsilon}^{(2)}x_2 + kx_1\right)} - k\left({}^2\overline{\beta}_{3G} + i\ {}^2\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)} \tag{E3.102}
$$

$$
{}^2A^{(2)}(k) = \hat{\chi}_{10}\left({}^2\overline{\alpha}_{3G} + i\ {}^2\overline{\alpha}_{3S}\right)e^{-i\left(\hat{\upsilon}^{(2)}x_2 + kx_1\right)} - i\hat{\chi}'_6\left({}^2\overline{\beta}_{3G} + i\ {}^2\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)} \tag{E3.103}
$$

$$
{}^2B^{(2)}(k) = \hat{\chi}_5\left({}^2\overline{\alpha}_{3G} + i\ {}^2\overline{\alpha}_{3S}\right)e^{-i\left(\hat{\upsilon}^{(2)}x_2 + kx_1\right)} + i\hat{\chi}'_6\left({}^2\overline{\beta}_{3G} + i\ {}^2\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)} \tag{E3.104}
$$

$$
{}^2C^{(2)}(k) = i\hat{\chi}'_7\left({}^2\overline{\alpha}_{3G} + i\ {}^2\overline{\alpha}_{3S}\right)e^{-i\left(\hat{\upsilon}^{(2)}x_2 + kx_1\right)} + \chi_8\left({}^2\overline{\beta}_{3G} + i\ {}^2\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)} \tag{E3.105}
$$

bulunur. (3.133)-(3.142) eşitlikleri uyarınca (E3.96), (E3.100), (E3.101) ve (E3.105) eşitliklerinin sanal, (E3.97), (E3.98), (E3.99), (E3.102), (E3.103) ve (E3.104) eşitliklerinin de gerçel kısımları kullanılacaktır. Buna göre,

$$
\begin{aligned}
{}^2\Theta_S^{(1)}(k) = k\Big\{&e^{-\upsilon^{(1)}x_2}\left[-\ {}^2\overline{\alpha}_{1G}\sin(kx_1) + {}^2\overline{\alpha}_{1S}\cos(kx_1)\right] \\
&- e^{\upsilon^{(1)}x_2}\left[-\ {}^2\overline{\alpha}_{2G}\sin(kx_1) + {}^2\overline{\alpha}_{2S}\cos(kx_1)\right]\Big\} \\
&+ \hat{\upsilon}'^{(1)}\Big[{}^2\overline{\beta}_{1G}\cos\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right) + {}^2\overline{\beta}_{1S}\sin\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right) \\
&\quad + {}^2\overline{\beta}_{2G}\cos\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right) - {}^2\overline{\beta}_{2S}\sin\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right)\Big]
\end{aligned} \tag{E3.106}
$$

$$
\begin{aligned}
{}^2\Omega_G^{(1)}(k) = \upsilon^{(1)}\Big\{&e^{-\upsilon^{(1)}x_2}\left[{}^2\overline{\alpha}_{1G}\cos(kx_1) + {}^2\overline{\alpha}_{1S}\sin(kx_1)\right] \\
&+ e^{\upsilon^{(1)}x_2}\left[{}^2\overline{\alpha}_{2G}\cos(kx_1) + {}^2\overline{\alpha}_{2S}\sin(kx_1)\right]\Big\} \\
&+ k\Big[{}^2\overline{\beta}_{1G}\cos\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right) + {}^2\overline{\beta}_{1S}\sin\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right) \\
&\quad - {}^2\overline{\beta}_{2G}\cos\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right) + {}^2\overline{\beta}_{2S}\sin\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right)\Big]
\end{aligned} \tag{E3.107}
$$

$$
\begin{aligned}
{}^2A_G^{(1)}(k) = \chi_9\Big\{&e^{-\upsilon^{(1)}x_2}\left[-{}^2\overline{\alpha}_{1G}\cos(kx_1) - {}^2\overline{\alpha}_{1S}\sin(kx_1)\right] \\
&+ e^{\upsilon^{(1)}x_2}\left[{}^2\overline{\alpha}_{2G}\cos(kx_1) + {}^2\overline{\alpha}_{2S}\sin(kx_1)\right]\Big\} \\
&- \hat{\chi}'_2\Big[-{}^2\overline{\beta}_{1G}\sin\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right) + {}^2\overline{\beta}_{1S}\cos\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right) \\
&\quad + {}^2\overline{\beta}_{2G}\sin\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right) + {}^2\overline{\beta}_{2S}\cos\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right)\Big]
\end{aligned} \tag{E3.108}
$$

$$
\begin{aligned}
{}^2B_G^{(1)}(k) = \chi_1 \Big\{ & e^{-\upsilon^{(1)}x_2}\left[-{}^2\overline{\alpha}_{1G}\cos(kx_1) - {}^2\overline{\alpha}_{1S}\sin(kx_1)\right] \\
& + e^{\upsilon^{(1)}x_2}\left[{}^2\overline{\alpha}_{2G}\cos(kx_1) + {}^2\overline{\alpha}_{2S}\sin(kx_1)\right]\Big\} \\
& + \hat{\chi}_2'\left[-{}^2\overline{\beta}_{1G}\sin\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right) + {}^2\overline{\beta}_{1S}\cos\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right)\right. \\
& \left. + {}^2\overline{\beta}_{2G}\sin\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right) + {}^2\overline{\beta}_{2S}\cos\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right)\right]
\end{aligned}
\tag{E3.109}
$$

$$
\begin{aligned}
{}^2C_S^{(1)}(k) = -\chi_3 \Big\{ & e^{-\upsilon^{(1)}x_2}\left[-{}^2\overline{\alpha}_{1G}\sin(kx_1) + {}^2\overline{\alpha}_{1S}\cos(kx_1)\right] \\
& + e^{\upsilon^{(1)}x_2}\left[-{}^2\overline{\alpha}_{2G}\sin(kx_1) + {}^2\overline{\alpha}_{2S}\cos(kx_1)\right]\Big\} \\
& + \chi_4\left[{}^2\overline{\beta}_{1G}\sin\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right) - {}^2\overline{\beta}_{1S}\cos\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right)\right. \\
& \left. + {}^2\overline{\beta}_{2G}\sin\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right) + {}^2\overline{\beta}_{2S}\cos\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right)\right]
\end{aligned}
\tag{E3.110}
$$

$$
\begin{aligned}
{}^2\Theta_S^{(2)}(k) = & k\left[{}^2\overline{\alpha}_{3G}\sin\left(\hat{\upsilon}^{(2)}x_2 + kx_1\right) - {}^2\overline{\alpha}_{3S}\cos\left(\hat{\upsilon}^{(2)}x_2 + kx_1\right)\right] \\
& - \hat{\upsilon}'^{(2)}\left[{}^2\overline{\beta}_{3G}\cos\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right) + {}^2\overline{\beta}_{3S}\sin\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)\right]
\end{aligned}
\tag{E3.111}
$$

$$
\begin{aligned}
{}^2\Omega_G^{(2)}(k) = & \hat{\upsilon}^{(2)}\left[-{}^2\overline{\alpha}_{3G}\sin\left(\hat{\upsilon}^{(2)}x_2 + kx_1\right) + {}^2\overline{\alpha}_{3S}\cos\left(\hat{\upsilon}^{(2)}x_2 + kx_1\right)\right] \\
& - k\left[{}^2\overline{\beta}_{3G}\cos\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right) + {}^2\overline{\beta}_{3S}\sin\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)\right]
\end{aligned}
\tag{E3.112}
$$

$$
\begin{aligned}
{}^2A_G^{(2)}(k) = & \hat{\chi}_{10}\left[{}^2\overline{\alpha}_{3G}\cos\left(\hat{\upsilon}^{(2)}x_2 + kx_1\right) + {}^2\overline{\alpha}_{3S}\sin\left(\hat{\upsilon}^{(2)}x_2 + kx_1\right)\right] \\
& + \hat{\chi}_6'\left[-{}^2\overline{\beta}_{3G}\sin\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right) + {}^2\overline{\beta}_{3S}\cos\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)\right]
\end{aligned}
\tag{E3.113}
$$

$$
\begin{aligned}
{}^2B_G^{(2)}(k) = & \hat{\chi}_5\left[{}^2\overline{\alpha}_{3G}\cos\left(\hat{\upsilon}^{(2)}x_2 + kx_1\right) + {}^2\overline{\alpha}_{3S}\sin\left(\hat{\upsilon}^{(2)}x_2 + kx_1\right)\right] \\
& + \hat{\chi}_6'\left[{}^2\overline{\beta}_{3G}\sin\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right) - {}^2\overline{\beta}_{3S}\cos\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)\right]
\end{aligned}
\tag{E3.114}
$$

$$
\begin{aligned}
{}^2C_S^{(2)}(k) = & \hat{\chi}_7'\left[{}^2\overline{\alpha}_{3G}\cos\left(\hat{\upsilon}^{(2)}x_2 + kx_1\right) + {}^2\overline{\alpha}_{3S}\sin\left(\hat{\upsilon}^{(2)}x_2 + kx_1\right)\right] \\
& + \chi_8\left[-{}^2\overline{\beta}_{3G}\sin\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right) + {}^2\overline{\beta}_{3S}\cos\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)\right]
\end{aligned}
\tag{E3.115}
$$

yazılıp bu eşitliklerin tek kısımları atıldıktan sonra kalan,

$$
\begin{aligned}
{}^2\Theta_{SÇ}^{(1)}(k) = \Big\{ & k\left(-e^{-\upsilon^{(1)}x_2}\,{}^2\overline{\alpha}_{1G} + e^{\upsilon^{(1)}x_2}\,{}^2\overline{\alpha}_{2G}\right) \\
& + \hat{\upsilon}'^{(1)}\left[-\left({}^2\overline{\beta}_{1G} - {}^2\overline{\beta}_{2G}\right)\sin\left(\hat{\upsilon}'^{(1)}x_2\right) + \left({}^2\overline{\beta}_{1S} + {}^2\overline{\beta}_{2S}\right)\cos\left(\hat{\upsilon}'^{(1)}x_2\right)\right]\Big\}\sin(kx_1)
\end{aligned}
\tag{E3.116}
$$

$$
\begin{aligned}
{}^2\Omega_{GÇ}^{(1)}(k) = \Big\{ & \upsilon^{(1)}\left(e^{-\upsilon^{(1)}x_2}\,{}^2\overline{\alpha}_{1G} + e^{\upsilon^{(1)}x_2}\,{}^2\overline{\alpha}_{2G}\right) \\
& + k\left[\left({}^2\overline{\beta}_{1G} - {}^2\overline{\beta}_{2G}\right)\cos\left(\hat{\upsilon}'^{(1)}x_2\right) + \left({}^2\overline{\beta}_{1S} + {}^2\overline{\beta}_{2S}\right)\sin\left(\hat{\upsilon}'^{(1)}x_2\right)\right]\Big\}\cos(kx_1)
\end{aligned}
\tag{E3.117}
$$

$$
\begin{aligned}
{}^2A_{GÇ}^{(1)}(k) = \Big\{ & \chi_9\left(-e^{-\upsilon^{(1)}x_2}\,{}^2\overline{\alpha}_{1G} + e^{\upsilon^{(1)}x_2}\,{}^2\overline{\alpha}_{2G}\right) \\
& + \hat{\chi}_2'\left[\left({}^2\overline{\beta}_{1G} - {}^2\overline{\beta}_{2G}\right)\sin\left(\hat{\upsilon}'^{(1)}x_2\right) - \left({}^2\overline{\beta}_{1S} + {}^2\overline{\beta}_{2S}\right)\cos\left(\hat{\upsilon}'^{(1)}x_2\right)\right]\Big\}\cos(kx_1)
\end{aligned}
\tag{E3.118}
$$

$$
\begin{aligned}
{}^2B_{GÇ}^{(1)}(k) = \Big\{ & \chi_1\left(-e^{-\upsilon^{(1)}x_2}\,{}^2\overline{\alpha}_{1G} + e^{\upsilon^{(1)}x_2}\,{}^2\overline{\alpha}_{2G}\right) \\
& + \hat{\chi}_2'\left[-\left({}^2\overline{\beta}_{1G} - {}^2\overline{\beta}_{2G}\right)\sin\left(\hat{\upsilon}'^{(1)}x_2\right) + \left({}^2\overline{\beta}_{1S} + {}^2\overline{\beta}_{2S}\right)\cos\left(\hat{\upsilon}'^{(1)}x_2\right)\right]\Big\}\cos(kx_1)
\end{aligned}
\tag{E3.119}
$$

$$
\begin{aligned}
{}^2C_{SÇ}^{(1)}(k) = \Big\{ & \chi_3\left(e^{-\upsilon^{(1)}x_2}\,{}^2\overline{\alpha}_{1G} + e^{\upsilon^{(1)}x_2}\,{}^2\overline{\alpha}_{2G}\right) \\
& + \chi_4\left[\left({}^2\overline{\beta}_{1G} - {}^2\overline{\beta}_{2G}\right)\cos\left(\hat{\upsilon}'^{(1)}x_2\right) + \left({}^2\overline{\beta}_{1S} + {}^2\overline{\beta}_{2S}\right)\sin\left(\hat{\upsilon}'^{(1)}x_2\right)\right]\Big\}\sin(kx_1)
\end{aligned}
\tag{E3.120}
$$

$$
\begin{aligned}
{}^2\Theta_{SÇ}^{(2)}(k) = k\Big\{ & \left[\cos(\hat{\upsilon}^{(2)}h)\,{}^2\tilde{\overline{\alpha}}_{3G} - \sin(\hat{\upsilon}^{(2)}h)\,{}^2\tilde{\overline{\alpha}}_{3S}\right]\cos\left(\hat{\upsilon}^{(2)}x_2\right) \\
& + \left[\cos(\hat{\upsilon}^{(2)}h)\,{}^2\tilde{\overline{\alpha}}_{3S} + \sin(\hat{\upsilon}^{(2)}h)\,{}^2\tilde{\overline{\alpha}}_{3G}\right]\sin\left(\hat{\upsilon}^{(2)}x_2\right)\Big\}\sin(kx_1) \\
+ \hat{\upsilon}'^{(2)}\Big\{ & \left[\cos(\hat{\upsilon}'^{(2)}h)\,{}^2\tilde{\overline{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\,{}^2\tilde{\overline{\beta}}_{3S}\right]\sin\left(\hat{\upsilon}'^{(2)}x_2\right) \\
& - \left[\cos(\hat{\upsilon}'^{(2)}h)\,{}^2\tilde{\overline{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\,{}^2\tilde{\overline{\beta}}_{3G}\right]\cos\left(\hat{\upsilon}'^{(2)}x_2\right)\Big\}\sin(kx_1)
\end{aligned}
\tag{E3.121}
$$

$$
\begin{aligned}
{}^{2}\Omega_{G\c{C}}^{(2)}(k) = \hat{\upsilon}^{(2)}\Big\{ & -\Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3G} - \sin(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3S}\Big]\sin\big(\hat{\upsilon}^{(2)}x_2\big) \\
& + \Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3S} + \sin(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3G}\Big]\cos\big(\hat{\upsilon}^{(2)}x_2\big)\Big\}\cos(kx_1) \\
- k\Big\{ & \Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3S}\Big]\cos\big(\hat{\upsilon}'^{(2)}x_2\big) \\
& + \Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3G}\Big]\sin\big(\hat{\upsilon}'^{(2)}x_2\big)\Big\}\cos(kx_1)
\end{aligned} \tag{E3.122}
$$

$$
\begin{aligned}
{}^{2}A_{G\c{C}}^{(2)}(k) = \hat{\chi}_{10}\Big\{ & \Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3G} - \sin(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3S}\Big]\cos\big(\hat{\upsilon}^{(2)}x_2\big) \\
& + \Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3S} + \sin(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3G}\Big]\sin\big(\hat{\upsilon}^{(2)}x_2\big)\Big\}\cos(kx_1) \\
+ \hat{\chi}'_{6}\Big\{ & -\Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3S}\Big]\sin\big(\hat{\upsilon}'^{(2)}x_2\big) \\
& + \Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3G}\Big]\cos\big(\hat{\upsilon}'^{(2)}x_2\big)\Big\}\cos(kx_1)
\end{aligned} \tag{E3.123}
$$

$$
\begin{aligned}
{}^{2}B_{G\c{C}}^{(2)}(k) = \hat{\chi}_{5}\Big\{ & \Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3G} - \sin(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3S}\Big]\cos\big(\hat{\upsilon}^{(2)}x_2\big) \\
& + \Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3S} + \sin(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3G}\Big]\sin\big(\hat{\upsilon}^{(2)}x_2\big)\Big\}\cos(kx_1) \\
+ \hat{\chi}'_{6}\Big\{ & \Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3S}\Big]\sin\big(\hat{\upsilon}'^{(2)}x_2\big) \\
& - \Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3G}\Big]\cos\big(\hat{\upsilon}'^{(2)}x_2\big)\Big\}\cos(kx_1)
\end{aligned} \tag{E3.124}
$$

$$
\begin{aligned}
{}^{2}C_{S\c{C}}^{(2)}(k) = \hat{\chi}'_{7}\Big\{ & -\Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3G} - \sin(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3S}\Big]\sin\big(\hat{\upsilon}^{(2)}x_2\big) \\
& + \Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3S} + \sin(\hat{\upsilon}^{(2)}h)\ {}^{2}\tilde{\tilde{\alpha}}_{3G}\Big]\cos\big(\hat{\upsilon}^{(2)}x_2\big)\Big\}\sin(kx_1) \\
- \chi_{8}\Big\{ & \Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3S}\Big]\cos\big(\hat{\upsilon}'^{(2)}x_2\big) \\
& + \Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^{2}\tilde{\tilde{\beta}}_{3G}\Big]\sin\big(\hat{\upsilon}'^{(2)}x_2\big)\Big\}\sin(kx_1)
\end{aligned} \tag{E3.125}
$$

eşitlikleri (3.132)-(3.141) eşitliklerinde kullanılırsa sonuç olarak,

$$
{}^{2}\bar{u}_{1}^{(1)} = -\frac{P_0}{\pi}\int_{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{2}\Theta_{S\c{C}}^{(1)}(k)\,dk \tag{E3.126}
$$

$$
{}^{2}\bar{u}_{2}^{(1)} = \frac{P_0}{\pi}\int_{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{2}\Omega_{G\c{C}}^{(1)}(k)\,dk \tag{E3.127}
$$

$$
{}^{2}\sigma_{11}^{(1)} = \frac{P_0}{\pi}\int_{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{2}A_{G\c{C}}^{(1)}(k)\,dk \tag{E3.128}
$$

$$
{}^{2}\sigma_{22}^{(1)} = \frac{P_0}{\pi}\int_{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{2}B_{G\c{C}}^{(1)}(k)\,dk \tag{E3.129}
$$

$$ {}^{2}\sigma_{12}^{(1)} = -\frac{P_0}{\pi} \int_{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{2}C_{SÇ}^{(1)}(k)\,dk \qquad (E3.130) $$

$$ {}^{2}\overline{u}_{1}^{(2)} = -\frac{P_0}{\pi} \int_{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{2}\Theta_{SÇ}^{(2)}(k)\,dk \qquad (E3.131) $$

$$ {}^{2}\overline{u}_{2}^{(2)} = \frac{P_0}{\pi} \int_{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{2}\Omega_{GÇ}^{(2)}(k)\,dk \qquad (E3.132) $$

$$ {}^{2}\sigma_{11}^{(2)} = \frac{P_0}{\pi} \int_{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{2}A_{GÇ}^{(2)}(k)\,dk \qquad (E3.133) $$

$$ {}^{2}\sigma_{22}^{(2)} = \frac{P_0}{\pi} \int_{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{2}B_{GÇ}^{(2)}(k)\,dk \qquad (E3.134) $$

$$ {}^{2}\sigma_{12}^{(2)} = -\frac{P_0}{\pi} \int_{\frac{k_L^{(1)}}{\sqrt{1+\eta_1^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{2}C_{SÇ}^{(2)}(k)\,dk \qquad (E3.135) $$

bulunur.

EK 4. $k_L^{(2)}/\sqrt{1+\eta_1^{(2)}} < k < k_T^{(1)}/\sqrt{1+\eta_2^{(1)}}$ **Durumu.**

Bu durumda (3.52) VE (3.53) eşitliklerinden hareketle,

$$\upsilon^{(1)} = \left[k^2(1+\eta_1^{(1)}) - k_L^{(1)2}\right]^{1/2} \quad \text{(E4.1)}$$
$$\upsilon'^{(1)} = \left[k^2(1+\eta_2^{(1)}) - k_T^{(1)2}\right]^{1/2} = i\left[k_T^{(1)2} - k^2(1+\eta_2^{(1)})\right]^{1/2} = i\,\hat{\upsilon}'^{(1)} \quad \text{(E4.2)}$$
$$\upsilon^{(2)} = \left[k^2(1+\eta_1^{(2)}) - k_L^{(2)2}\right]^{1/2} \quad \text{(E4.3)}$$
$$\upsilon'^{(2)} = \left[k^2(1+\eta_2^{(2)}) - k_T^{(2)2}\right]^{1/2} = i\left[k_T^{(2)2} - k^2(1+\eta_2^{(2)})\right]^{1/2} = i\,\hat{\upsilon}'^{(2)} \quad \text{(E4.4)}$$

eşitlikleri kullanılır. Buna göre,

$$^3\chi_1 = \chi_1 \quad \text{(E4.5)}$$
$$^3\chi_2 = i\hat{\chi}'_2 \quad \text{(E4.6)}$$
$$^3\chi_3 = \chi_3 \quad \text{(E4.7)}$$
$$^3\chi_4 = \chi_4 \quad \text{(E4.8)}$$
$$^3\chi_5 = \chi_5 \quad \text{(E4.9)}$$
$$^3\chi_6 = i\hat{\chi}'_6 \quad \text{(E4.10)}$$
$$^3\chi_7 = \chi_7 \quad \text{(E4.11)}$$
$$^3\chi_8 = \chi_8 \quad \text{(E4.12)}$$
$$^3\chi_9 = \chi_9 \quad \text{(E4.13)}$$
$$^3\chi_{10} = \chi_{10} \quad \text{(E4.14)}$$

bulunur. Ayrıca,

$$^3H_1 = H_1 = e^{-\upsilon^{(1)}x_2} \quad \text{(E4.15)}$$
$$^3H_2 = H_2 = e^{\upsilon^{(1)}x_2} \quad \text{(E4.16)}$$
$$^3H_3 = \hat{H}_3 = e^{-i\hat{\upsilon}'^{(1)}x_2} \quad \text{(E4.17)}$$
$$^3H_4 = \hat{H}_4 = e^{i\hat{\upsilon}'^{(1)}x_2} \quad \text{(E4.18)}$$
$$^3H_5 = H_5 = e^{-\upsilon^{(2)}x_2} \quad \text{(E4.19)}$$
$$^3H_6 = \hat{H}_6 = e^{-i\hat{\upsilon}'^{(2)}x_2} \quad \text{(E4.20)}$$

ve,

$$^3\overline{H}_1 = \overline{H}_1 = e^{-\upsilon^{(1)}h} \quad \text{(E4.21)}$$
$$^3\overline{H}_2 = \overline{H}_2 = e^{\upsilon^{(1)}h} \quad \text{(E4.22)}$$
$$^3\overline{H}_3 = \hat{\overline{H}}_3 = e^{-i\hat{\upsilon}'^{(1)}h} \quad \text{(E4.23)}$$
$$^3\overline{H}_4 = \hat{\overline{H}}_4 = e^{i\hat{\upsilon}'^{(1)}h} \quad \text{(E4.24)}$$
$$^3\overline{H}_5 = \overline{H}_5 = e^{-\upsilon^{(2)}h} \quad \text{(E4.25)}$$
$$^3\overline{H}_6 = \hat{\overline{H}}_6 = e^{-i\hat{\upsilon}'^{(2)}h} \quad \text{(E4.26)}$$

olup bütün bu eşitlikler (E1.23)-(E1.36) bağıntılarında kullanılırsa,

$$^{3}\Gamma_{1G}=k^{2}(\chi_4-\chi_8)(\chi_1-\chi_5)+(\upsilon^{(1)}\chi_7-\upsilon^{(2)}\chi_3)(\hat{\upsilon}'^{(1)}\hat{\chi}'_6-\hat{\upsilon}'^{(2)}\hat{\chi}'_2) \quad \text{(E4.27)}$$

$$^{3}\Gamma_{2G}=2(\hat{\upsilon}'^{(1)}\hat{\chi}'_6-\hat{\upsilon}'^{(2)}\hat{\chi}'_2)(\upsilon^{(2)}\chi_4-k\chi_7) \quad \text{(E4.28)}$$

$$^{3}\Gamma_{3G}=k^{2}(\chi_4-\chi_8)(\chi_1-\chi_5)-(\upsilon^{(1)}\chi_7-\upsilon^{(2)}\chi_3)(\hat{\upsilon}'^{(1)}\hat{\chi}'_6-\hat{\upsilon}'^{(2)}\hat{\chi}'_2) \quad \text{(E4.29)}$$

$$^{3}\Gamma_{4G}={}^{3}\Gamma_{1G} \quad \text{(E4.30)}$$

$$^{3}\Gamma_{5G}={}^{3}\Gamma_{3G} \quad \text{(E4.31)}$$

$$^{3}\Gamma_{6G}=2(\upsilon^{(1)}\chi_8-k\chi_3)(k\chi_1-k\chi_5) \quad \text{(E4.32)}$$

$$^{3}\Gamma_{7G}=2(\upsilon^{(1)}\chi_4-k\chi_3)(\hat{\upsilon}'^{(1)}\hat{\chi}'_6-\hat{\upsilon}'^{(2)}\hat{\chi}'_2) \quad \text{(E4.33)}$$

$$^{3}\Gamma_{8G}=-{}^{3}\Gamma_{7G} \quad \text{(E4.34)}$$

$$^{3}\Gamma_{9G}=0 \quad \text{(E4.35)}$$

$$^{3}\Gamma_{10G}=0 \quad \text{(E4.36)}$$

$$^{3}\Gamma_{11G}=0 \quad \text{(E4.37)}$$

$$^{3}\Gamma_{12G}=0 \quad \text{(E4.38)}$$

$$^{3}\Gamma_{13G}=2(\upsilon^{(1)}\chi_4-k\chi_3)(k\chi_1-k\chi_5) \quad \text{(E4.39)}$$

$$^{3}\Gamma_{14G}={}^{3}\Gamma_{13G} \quad \text{(E4.40)}$$

ve,

$$\begin{aligned}^{3}\Gamma_{1S}=&\,k^{2}(\hat{\chi}'_2-\hat{\chi}'_6)(\chi_3+\chi_7)+k(\chi_4\hat{\chi}'_6-\hat{\chi}'_2\chi_8)(\upsilon^{(1)}+\upsilon^{(2)})\\&-k(\chi_1\chi_7+\chi_3\chi_5)(\hat{\upsilon}'^{(1)}-\hat{\upsilon}'^{(2)})+(\upsilon^{(1)}\chi_5+\upsilon^{(2)}\chi_1)(\hat{\upsilon}'^{(1)}\chi_8-\hat{\upsilon}'^{(2)}\chi_4)\end{aligned} \quad \text{(E4.41)}$$

$$^{3}\Gamma_{2S}=2(k\chi_8-k\chi_4)(k\hat{\chi}'_2-\hat{\upsilon}'^{(1)}\chi_5) \quad \text{(E4.42)}$$

$$\begin{aligned}^{3}\Gamma_{3S}=&-k^{2}(\hat{\chi}'_2+\hat{\chi}'_6)(\chi_3+\chi_7)+k(\chi_4\hat{\chi}'_6+\hat{\chi}'_2\chi_8)(\upsilon^{(1)}+\upsilon^{(2)})\\&+k(\chi_1\chi_7+\chi_3\chi_5)(\hat{\upsilon}'^{(1)}+\hat{\upsilon}'^{(2)})-(\upsilon^{(1)}\chi_5+\upsilon^{(2)}\chi_1)(\hat{\upsilon}'^{(1)}\chi_8+\hat{\upsilon}'^{(2)}\chi_4)\end{aligned} \quad \text{(E4.43)}$$

$$\begin{aligned}^{3}\Gamma_{4S}=&\,k^{2}(\hat{\chi}'_2+\hat{\chi}'_6)(\chi_3-\chi_7)+k(\chi_4\hat{\chi}'_6+\hat{\chi}'_2\chi_8)(\upsilon^{(2)}-\upsilon^{(1)})\\&+k(\chi_1\chi_7-\chi_3\chi_5)(\hat{\upsilon}'^{(1)}+\hat{\upsilon}'^{(2)})+(\upsilon^{(1)}\chi_5-\upsilon^{(2)}\chi_1)(\hat{\upsilon}'^{(1)}\chi_8+\hat{\upsilon}'^{(2)}\chi_4)\end{aligned} \quad \text{(E4.44)}$$

$$\begin{aligned}^{3}\Gamma_{5S}=&-k^{2}(\hat{\chi}'_2-\hat{\chi}'_6)(\chi_3-\chi_7)-k(\chi_4\hat{\chi}'_6-\hat{\chi}'_2\chi_8)(\upsilon^{(1)}-\upsilon^{(2)})\\&-k(\chi_1\chi_7-\chi_3\chi_5)(\hat{\upsilon}'^{(1)}-\hat{\upsilon}'^{(2)})-(\upsilon^{(1)}\chi_5-\upsilon^{(2)}\chi_1)(\hat{\upsilon}'^{(1)}\chi_8-\hat{\upsilon}'^{(2)}\chi_4)\end{aligned} \quad \text{(E4.45)}$$

$$^{3}\Gamma_{6S}=2(\upsilon^{(1)}\chi_7-\upsilon^{(2)}\chi_3)(k\hat{\chi}'_6-\hat{\upsilon}'^{(2)}\chi_1) \quad \text{(E4.46)}$$

$$^{3}\Gamma_{7S}=2(k\chi_8-k\chi_4)(\hat{\upsilon}'^{(1)}\chi_1-k\hat{\chi}'_2) \quad \text{(E4.47)}$$

$$^{3}\Gamma_{8S}={}^{3}\Gamma_{7S} \quad \text{(E4.48)}$$

$$\begin{aligned}^{3}\Gamma_{9S}=&-2\upsilon^{(1)}k(\hat{\chi}'_2\chi_8+\chi_4\hat{\chi}'_6)+2\hat{\upsilon}'^{(2)}\chi_1(\upsilon^{(1)}\chi_4-k\chi_3)\\&+2k^{2}(\chi_3\hat{\chi}'_6+\chi_3\hat{\chi}'_2)+2\hat{\upsilon}'^{(1)}\chi_1(\upsilon^{(1)}\chi_8-k\chi_3)\end{aligned} \quad \text{(E4.49)}$$

$$\begin{aligned}^{3}\Gamma_{10S}=&\,2\upsilon^{(1)}k(\hat{\chi}'_2\chi_8-\chi_4\hat{\chi}'_6)+2\hat{\upsilon}'^{(2)}\chi_1(\upsilon^{(1)}\chi_4-k\chi_3)\\&+2k^{2}(\chi_3\hat{\chi}'_6-\chi_3\hat{\chi}'_2)-2\hat{\upsilon}'^{(1)}\chi_1(\upsilon^{(1)}\chi_8-k\chi_3)\end{aligned} \quad \text{(E4.50)}$$

$$\begin{aligned}^{3}\Gamma_{11S}=&-2k\hat{\upsilon}'^{(1)}(\chi_3\chi_5+\chi_1\chi_7)+2k^{2}(\hat{\chi}'_2\chi_7+\hat{\chi}'_2\chi_3)\\&-2k\hat{\chi}'_2(\upsilon^{(1)}\chi_4+\upsilon^{(2)}\chi_4)+2\hat{\upsilon}'^{(1)}\chi_4(\upsilon^{(1)}\chi_5+\upsilon^{(2)}\chi_1)\end{aligned} \quad \text{(E4.51)}$$

$$\begin{aligned}^{3}\Gamma_{12S}=&\,2k\hat{\upsilon}'^{(1)}(\chi_3\chi_5-\chi_1\chi_7)+2k^{2}(\hat{\chi}'_2\chi_7-\hat{\chi}'_2\chi_3)\\&+2k\hat{\chi}'_2(\upsilon^{(1)}\chi_4-\upsilon^{(2)}\chi_4)-2\hat{\upsilon}'^{(1)}\chi_4(\upsilon^{(1)}\chi_5-\upsilon^{(2)}\chi_1)\end{aligned} \quad \text{(E4.52)}$$

$$^{3}\Gamma_{13S}=2(\upsilon^{(1)}\chi_7-\upsilon^{(2)}\chi_3)(\hat{\upsilon}'^{(1)}\chi_1-k\hat{\chi}'_2) \quad \text{(E4.53)}$$

$$^{3}\Gamma_{14S}=-{}^{3}\Gamma_{13S} \quad \text{(E4.54)}$$

olmak üzere,

$$^3\Gamma_j = {}^3\Gamma_{jG} + i\ {}^3\Gamma_{jS} \quad ,j=1,14 \tag{E4.55}$$

yazılabilir. Ayrıca (3.85) eşitliği yardımı ile,

$$\begin{aligned} ^3[\Delta] = &(i\hat{\chi}_2'\chi_3 + \chi_1\chi_4)\left[\overline{H}_2\hat{\overline{H}}_3({}^3\Gamma_{1G} + i\ {}^3\Gamma_{1S}) + \overline{H}_1\hat{\overline{H}}_4({}^3\Gamma_{4G} + i\ {}^3\Gamma_{4S})\right] \\ &+ (i\hat{\chi}_2'\chi_3 - \chi_1\chi_4)\left[\overline{H}_1\hat{\overline{H}}_3({}^3\Gamma_{5G} + i\ {}^3\Gamma_{5S}) + \overline{H}_2\hat{\overline{H}}_4({}^3\Gamma_{3G} + i\ {}^3\Gamma_{3S})\right] \\ &+ 2\left[\chi_1\chi_3\hat{\overline{H}}_3\hat{\overline{H}}_4({}^3\Gamma_{2G} + i\ {}^3\Gamma_{2S}) + i\hat{\chi}_2'\chi_4\overline{H}_1\overline{H}_2({}^3\Gamma_{6G} + i\ {}^3\Gamma_{6S})\right] \end{aligned} \tag{E4.56}$$

ve k dönüşüm parametresine göre,

$^3\Gamma_{1G}, {}^3\Gamma_{3G}, {}^3\Gamma_{4G}, {}^3\Gamma_{5G}$ ve $^3\Gamma_{1S}, {}^3\Gamma_{3S}, {}^3\Gamma_{4S}, {}^3\Gamma_{5S}, {}^3\Gamma_{9S}, {}^3\Gamma_{10S}, {}^3\Gamma_{11S}, {}^2\Gamma_{12S}$; çift,
$^3\Gamma_{2G}, {}^3\Gamma_{6G}, {}^3\Gamma_{7G}, {}^3\Gamma_{8G}, {}^3\Gamma_{13G}, {}^3\Gamma_{14G}$ ve $^3\Gamma_{2S}, {}^3\Gamma_{6S}, {}^3\Gamma_{7S}, {}^3\Gamma_{8S}, {}^3\Gamma_{13S}, {}^3\Gamma_{14S}$; tek fonksiyonlar ve
$^3\Gamma_{9G}, {}^3\Gamma_{10G}, {}^3\Gamma_{11G}, {}^3\Gamma_{12G} = 0$ olduğuna göre,

$$\begin{aligned} ^3[\Delta]_G = &-\hat{\chi}_2'\chi_3\Big\{e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{1S} + {}^3\Gamma_{3S}) - \sin(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{1G} - {}^3\Gamma_{3G})\right] \\ &\qquad + e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{4S} + {}^3\Gamma_{5S}) + \sin(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{4G} - {}^3\Gamma_{5G})\right]\Big\} \\ &+ \chi_1\chi_4\Big\{e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{1G} - {}^3\Gamma_{3G}) + \sin(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{1S} + {}^3\Gamma_{3S})\right] \\ &\qquad + e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{4G} - {}^3\Gamma_{5G}) - \sin(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{4S} + {}^3\Gamma_{5S})\right]\Big\} \\ &+ 2\left(\chi_1\chi_3\ {}^3\Gamma_{2G} - \hat{\chi}_2'\chi_4\ {}^3\Gamma_{6S}\right) \end{aligned} \tag{E4.57}$$

ve,

$$\begin{aligned} ^3[\Delta]_S = &\hat{\chi}_2'\chi_3\Big\{e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{1G} + {}^3\Gamma_{3G}) + \sin(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{1S} - {}^3\Gamma_{3S})\right] \\ &\qquad + e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{4G} + {}^3\Gamma_{5G}) - \sin(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{4S} - {}^3\Gamma_{5S})\right]\Big\} \\ &+ \chi_1\chi_4\Big\{e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{1S} - {}^3\Gamma_{3S}) - \sin(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{1G} + {}^3\Gamma_{3G})\right] \\ &\qquad + e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{4S} - {}^3\Gamma_{5S}) + \sin(\hat{\upsilon}'^{(1)}h)({}^3\Gamma_{4G} + {}^3\Gamma_{5G})\right]\Big\} \\ &+ 2\left(\chi_1\chi_3\ {}^3\Gamma_{2S} + \hat{\chi}_2'\chi_4\ {}^3\Gamma_{6G}\right) \end{aligned} \tag{E4.58}$$

olmak üzere,

$$^3[\Delta] = {}^3[\Delta]_G + i\ {}^3[\Delta]_S \tag{E4.59}$$

yazılabilir. (E4.59) eşitliğinin gerçel ve sanal kısımları çift fonksiyonlar olmaktadır. Buradan (E1.49)-(E1.54) yardımı ile,

$$^3\overline{\alpha}_1 = \frac{\chi_4 e^{\upsilon^{(1)}h}\left[e^{-i\hat{\upsilon}'^{(1)}h}({}^2\Gamma_{1G} + i\ {}^2\Gamma_{1S}) - e^{i\hat{\upsilon}'^{(1)}h}({}^2\Gamma_{3G} + i\ {}^2\Gamma_{3S})\right] + \chi_3({}^2\Gamma_{2G} + i\ {}^2\Gamma_{2S})}{^3[\Delta]} \tag{E4.60}$$

$$^3\overline{\alpha}_2 = \frac{\chi_4 e^{-\upsilon^{(1)}h}\left[e^{-i\hat{\upsilon}'^{(1)}h}({}^2\Gamma_{5G} + i\ {}^2\Gamma_{5S}) - e^{i\hat{\upsilon}'^{(1)}h}({}^2\Gamma_{4G} + i\ {}^2\Gamma_{4S})\right] - \chi_3({}^2\Gamma_{2G} + i\ {}^2\Gamma_{2S})}{^3[\Delta]} \tag{E4.61}$$

$$ {}^{3}\overline{\beta}_{1} = \frac{\chi_3 e^{i\hat{\upsilon}'^{(1)}h}\left|e^{\upsilon^{(1)}h}({}^{2}\Gamma_{3G} + i\ {}^{2}\Gamma_{3S}) + e^{-\upsilon^{(1)}h}({}^{2}\Gamma_{4G} + i\ {}^{2}\Gamma_{4S})\right| + \chi_4({}^{2}\Gamma_{6G} + i\ {}^{2}\Gamma_{6S})}{{}^{3}[\Delta]} \quad \text{(E4.62)} $$

$$ {}^{3}\overline{\beta}_{2} = \frac{\chi_3 e^{-i\hat{\upsilon}'^{(1)}h}\left|e^{\upsilon^{(1)}h}({}^{2}\Gamma_{1G} + i\ {}^{2}\Gamma_{1S}) + e^{-\upsilon^{(1)}h}({}^{2}\Gamma_{5G} + i\ {}^{2}\Gamma_{5S})\right| + \chi_4({}^{2}\Gamma_{6G} + i\ {}^{2}\Gamma_{6S})}{{}^{3}[\Delta]} \quad \text{(E4.63)} $$

$$ {}^{3}\overline{\alpha}_{3} = \frac{\chi_3\left|e^{\upsilon^{(1)}h}({}^{2}\Gamma_{7G} + i\ {}^{2}\Gamma_{7S}) + e^{-\upsilon^{(1)}h}({}^{2}\Gamma_{8G} + i\ {}^{2}\Gamma_{8S})\right| + i\chi_4\left(e^{-i\hat{\upsilon}'^{(1)}h}\ {}^{2}\Gamma_{10S} - e^{-\hat{\upsilon}'^{(1)}h}\ {}^{2}\Gamma_{9S}\right)}{e^{-\upsilon^{(2)}h}\ {}^{3}[\Delta]} \quad \text{(E4.64)} $$

$$ {}^{3}\overline{\beta}_{3} = \frac{i\chi_3\left(e^{\upsilon^{(1)}h}\ {}^{2}\Gamma_{11S} + e^{-\upsilon^{(1)}h}\ {}^{2}\Gamma_{12S}\right) + \chi_4\left|e^{i\hat{\upsilon}'^{(1)}h}({}^{2}\Gamma_{13G} + i\ {}^{2}\Gamma_{13S}) - e^{-\hat{\upsilon}'^{(1)}h}({}^{2}\Gamma_{14G} + i\ {}^{2}\Gamma_{14S})\right|}{e^{-i\hat{\upsilon}'^{(2)}h}\ {}^{3}[\Delta]} \quad \text{(E4.65)} $$

yazılır. (E4.60)-(E4.65) eşitliklerini gerçel ve sanal kısımlarına ayırmak için,

$$ {}^{3}\overline{\alpha}_{1PG} = \chi_4 e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^{3}\Gamma_{1G} - {}^{3}\Gamma_{3G}) + \sin(\hat{\upsilon}'^{(1)}h)({}^{3}\Gamma_{1S} + {}^{3}\Gamma_{3S})\right] + \chi_3\ {}^{3}\Gamma_{2G} \quad \text{(E4.66)} $$

$$ {}^{3}\overline{\alpha}_{1PS} = \chi_4 e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^{3}\Gamma_{1S} - {}^{3}\Gamma_{3S}) - \sin(\hat{\upsilon}'^{(1)}h)({}^{3}\Gamma_{1G} + {}^{3}\Gamma_{3G})\right] + \chi_3\ {}^{3}\Gamma_{2S} \quad \text{(E4.67)} $$

$$ {}^{3}\overline{\alpha}_{2PG} = \chi_4 e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^{3}\Gamma_{5G} - {}^{3}\Gamma_{4G}) + \sin(\hat{\upsilon}'^{(1)}h)({}^{3}\Gamma_{5S} + {}^{3}\Gamma_{4S})\right] - \chi_3\ {}^{3}\Gamma_{2G} \quad \text{(E4.68)} $$

$$ {}^{3}\overline{\alpha}_{2PS} = \chi_4 e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)({}^{3}\Gamma_{5S} - {}^{3}\Gamma_{4S}) - \sin(\hat{\upsilon}'^{(1)}h)({}^{3}\Gamma_{5G} + {}^{3}\Gamma_{4G})\right] - \chi_3\ {}^{3}\Gamma_{2S} \quad \text{(E4.69)} $$

$$ \begin{aligned} {}^{3}\overline{\beta}_{1PG} = \chi_3 \Big\{ & e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)\ {}^{3}\Gamma_{3G} - \sin(\hat{\upsilon}'^{(1)}h)\,{}^{3}\Gamma_{3S}\right] \\ & + e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)\ {}^{3}\Gamma_{4G} - \sin(\hat{\upsilon}'^{(1)}h)\,{}^{3}\Gamma_{4S}\right]\Big\} + \chi_4\ {}^{3}\Gamma_{6G} \end{aligned} \quad \text{(E4.70)} $$

$$ \begin{aligned} {}^{3}\overline{\beta}_{1PS} = \chi_3 \Big\{ & e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)\ {}^{3}\Gamma_{3S} + \sin(\hat{\upsilon}'^{(1)}h)\,{}^{3}\Gamma_{3G}\right] \\ & + e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)\ {}^{3}\Gamma_{4S} + \sin(\hat{\upsilon}'^{(1)}h)\,{}^{3}\Gamma_{4G}\right]\Big\} + \chi_4\ {}^{3}\Gamma_{6S} \end{aligned} \quad \text{(E4.71)} $$

$$ \begin{aligned} {}^{3}\overline{\beta}_{2PG} = \chi_3 \Big\{ & e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)\ {}^{3}\Gamma_{1G} + \sin(\hat{\upsilon}'^{(1)}h)\,{}^{3}\Gamma_{1S}\right] \\ & + e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)\ {}^{3}\Gamma_{5G} + \sin(\hat{\upsilon}'^{(1)}h)\,{}^{3}\Gamma_{5S}\right]\Big\} + \chi_4\ {}^{3}\Gamma_{6G} \end{aligned} \quad \text{(E4.72)} $$

$$ \begin{aligned} {}^{3}\overline{\beta}_{2PS} = \chi_3 \Big\{ & e^{\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)\ {}^{3}\Gamma_{1S} - \sin(\hat{\upsilon}'^{(1)}h)\,{}^{3}\Gamma_{1G}\right] \\ & + e^{-\upsilon^{(1)}h}\left[\cos(\hat{\upsilon}'^{(1)}h)\ {}^{3}\Gamma_{5S} - \sin(\hat{\upsilon}'^{(1)}h)\,{}^{3}\Gamma_{5G}\right]\Big\} + \chi_4\ {}^{3}\Gamma_{6S} \end{aligned} \quad \text{(E4.73)} $$

$$ {}^{3}\overline{\alpha}_{3PG} = \chi_3(e^{\upsilon^{(1)}h}\ {}^{3}\Gamma_{7G} + e^{-\upsilon^{(1)}h}\ {}^{3}\Gamma_{8G}) + \chi_4 \sin(\hat{\upsilon}'^{(1)}h)({}^{3}\Gamma_{10S} + {}^{3}\Gamma_{9S}) \quad \text{(E4.74)} $$

$$ {}^{3}\overline{\alpha}_{3PS} = \chi_3(e^{\upsilon^{(1)}h}\ {}^{3}\Gamma_{7S} + e^{-\upsilon^{(1)}h}\ {}^{3}\Gamma_{8S}) + \chi_4 \cos(\hat{\upsilon}'^{(1)}h)({}^{3}\Gamma_{10S} - {}^{3}\Gamma_{9S}) \quad \text{(E4.75)} $$

$$ {}^{3}\overline{\beta}_{3PG} = \chi_4\left[\cos(\hat{\upsilon}'^{(1)}h)({}^{3}\Gamma_{13G} - {}^{3}\Gamma_{14G}) - \sin(\hat{\upsilon}'^{(1)}h)({}^{3}\Gamma_{13S} + {}^{3}\Gamma_{14S})\right] \quad \text{(E4.76)} $$

$$ \begin{aligned} {}^{3}\overline{\beta}_{3PS} = {} & \chi_3(e^{\upsilon^{(1)}h}\ {}^{3}\Gamma_{11S} + e^{-\upsilon^{(1)}h}\ {}^{3}\Gamma_{12S}) \\ & + \chi_4\left[\cos(\hat{\upsilon}'^{(1)}h)({}^{3}\Gamma_{13S} - {}^{3}\Gamma_{14S}) + \sin(\hat{\upsilon}'^{(1)}h)({}^{3}\Gamma_{13G} + {}^{3}\Gamma_{14G})\right] \end{aligned} \quad \text{(E4.77)} $$

tanımlamaları yapılırsa,

$$ {}^{3}\overline{\alpha}_{1G} = \frac{{}^{3}[\Delta]_G\ {}^{3}\overline{\alpha}_{1PG} + {}^{3}[\Delta]_S\ {}^{3}\overline{\alpha}_{1PS}}{{}^{3}[\Delta]_G^{\,2} + {}^{3}[\Delta]_S^{\,2}} \quad \text{(E4.78)} $$

$$ {}^{3}\overline{\alpha}_{1S} = \frac{{}^{3}[\Delta]_G\ {}^{3}\overline{\alpha}_{1PS} - {}^{3}[\Delta]_S\ {}^{3}\overline{\alpha}_{1PG}}{{}^{3}[\Delta]_G^{\,2} + {}^{3}[\Delta]_S^{\,2}} \quad \text{(E4.79)} $$

$$ {}^{3}\overline{\alpha}_{2G} = \frac{{}^{3}[\Delta]_G\ {}^{3}\overline{\alpha}_{2PG} + {}^{3}[\Delta]_S\ {}^{3}\overline{\alpha}_{2PS}}{{}^{3}[\Delta]_G^{\,2} + {}^{3}[\Delta]_S^{\,2}} \quad \text{(E4.80)} $$

$$ {}^3\overline{\alpha}_{2S} = \frac{{}^3[\Delta]_G\ {}^3\overline{\alpha}_{2PS} - {}^3[\Delta]_S\ {}^3\overline{\alpha}_{2PG}}{{}^3[\Delta]_G{}^2 + {}^3[\Delta]_S{}^2} \tag{E4.81} $$

$$ {}^3\overline{\beta}_{1G} = \frac{{}^3[\Delta]_G\ {}^3\overline{\beta}_{1PG} + {}^3[\Delta]_S\ {}^3\overline{\beta}_{1PS}}{{}^3[\Delta]_G{}^2 + {}^3[\Delta]_S{}^2} \tag{E4.82} $$

$$ {}^3\overline{\beta}_{1S} = \frac{{}^3[\Delta]_G\ {}^3\overline{\beta}_{1PS} - {}^3[\Delta]_S\ {}^3\overline{\beta}_{1PG}}{{}^3[\Delta]_G{}^2 + {}^3[\Delta]_S{}^2} \tag{E4.83} $$

$$ {}^3\overline{\beta}_{2G} = \frac{{}^3[\Delta]_G\ {}^3\overline{\beta}_{2PG} + {}^3[\Delta]_S\ {}^3\overline{\beta}_{2PS}}{{}^3[\Delta]_G{}^2 + {}^3[\Delta]_S{}^2} \tag{E4.84} $$

$$ {}^3\overline{\beta}_{2S} = \frac{{}^3[\Delta]_G\ {}^3\overline{\beta}_{2PS} - {}^3[\Delta]_S\ {}^3\overline{\beta}_{2PG}}{{}^3[\Delta]_G{}^2 + {}^3[\Delta]_S{}^2} \tag{E4.85} $$

$$ {}^3\widetilde{\overline{\alpha}}_{3G} = \frac{{}^3[\Delta]_G\ {}^3\overline{\alpha}_{3PG} + {}^3[\Delta]_S\ {}^3\overline{\alpha}_{3PS}}{{}^3[\Delta]_G{}^2 + {}^3[\Delta]_S{}^2} \tag{E4.86} $$

$$ {}^3\widetilde{\overline{\alpha}}_{3S} = \frac{{}^3[\Delta]_G\ {}^3\overline{\alpha}_{3PS} - {}^3[\Delta]_S\ {}^3\overline{\alpha}_{3PG}}{{}^3[\Delta]_G{}^2 + {}^3[\Delta]_S{}^2} \tag{E4.87} $$

$$ {}^3\widetilde{\overline{\beta}}_{3G} = \frac{{}^3[\Delta]_G\ {}^3\overline{\beta}_{3PG} + {}^3[\Delta]_S\ {}^3\overline{\beta}_{3PS}}{{}^3[\Delta]_G{}^2 + {}^3[\Delta]_S{}^2} \tag{E4.88} $$

$$ {}^3\widetilde{\overline{\beta}}_{3S} = \frac{{}^3[\Delta]_G\ {}^3\overline{\beta}_{3PS} - {}^3[\Delta]_S\ {}^3\overline{\beta}_{3PG}}{{}^3[\Delta]_G{}^2 + {}^3[\Delta]_S{}^2} \tag{E4.89} $$

olmak üzere,

$$ {}^3\overline{\alpha}_1 = {}^3\overline{\alpha}_{1G} + i\ {}^3\overline{\alpha}_{1S} \tag{E4.90} $$

$$ {}^3\overline{\alpha}_2 = {}^3\overline{\alpha}_{2G} + i\ {}^3\overline{\alpha}_{2S} \tag{E4.91} $$

$$ {}^3\overline{\beta}_1 = {}^3\overline{\beta}_{1G} + i\ {}^3\overline{\beta}_{1S} \tag{E4.92} $$

$$ {}^3\overline{\beta}_2 = {}^3\overline{\beta}_{2G} + i\ {}^3\overline{\beta}_{2S} \tag{E4.93} $$

$$ {}^3\overline{\alpha}_3 = e^{\upsilon^{(2)}h}\left({}^3\overline{\widetilde{\overline{\alpha}}}_{3G} + i\ {}^3\overline{\widetilde{\overline{\alpha}}}_{3S}\right) \tag{E4.94} $$

$$ {}^3\overline{\beta}_3 = \cos(\hat{\upsilon}'^{(2)}h)\ {}^3\overline{\widetilde{\overline{\beta}}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^3\overline{\widetilde{\overline{\beta}}}_{3S} + i\left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^3\overline{\widetilde{\overline{\beta}}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^3\overline{\widetilde{\overline{\beta}}}_{3G}\right] \tag{E4.95} $$

yazılabilir. ${}^3\overline{\alpha}_{iPG}, {}^3\overline{\alpha}_{iPS}$ (i=1,3) fonksiyonları k'ya göre çift, ${}^3\overline{\beta}_{iPG}, {}^3\overline{\beta}_{iPS}$ (i=1,3) ise tek olup, ${}^3[\Delta]_G$ ve ${}^3[\Delta]_S$ fonksiyonları da çift olduklarına göre sonuç olarak ${}^3\overline{\alpha}_{iG}, {}^3\overline{\alpha}_{iS}$ (i=1,3) fonksiyonlarının çift, ${}^3\overline{\beta}_{iG}, {}^3\overline{\beta}_{iS}$ (i=1,3) fonksiyonlarının da tek oldukları görülür. (E4.90)-(E4.95) eşitlikleri (3.123)-(3.132)′de kullanılırsa, (E4.1)-(E4.20) eşitlikleri yardımı ile,

$$ \begin{aligned} {}^3\Theta^{(1)}(k) = k\Big[&\left({}^3\overline{\alpha}_{1G} + i\ {}^3\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2}e^{-ikx_1} - \left({}^3\overline{\alpha}_{2G} + i\ {}^3\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}e^{-ikx_1}\Big] \\ &+ i\hat{\upsilon}'^{(1)}\Big[\left({}^3\overline{\beta}_{1G} + i\ {}^3\overline{\beta}_{1S}\right)e^{-i\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right)} + \left({}^3\overline{\beta}_{2G} + i\ {}^3\overline{\beta}_{2S}\right)e^{i\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right)}\Big] \end{aligned} \tag{E4.96} $$

$$ \begin{aligned} {}^3\Omega^{(1)}(k) = \upsilon^{(1)}\Big[&\left({}^3\overline{\alpha}_{1G} + i\ {}^3\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2}e^{-ikx_1} + \left({}^3\overline{\alpha}_{2G} + i\ {}^3\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}e^{-ikx_1}\Big] \\ &+ k\Big[\left({}^3\overline{\beta}_{1G} + i\ {}^3\overline{\beta}_{1S}\right)e^{-i\left(\hat{\upsilon}'^{(1)}x_2 + kx_1\right)} - \left({}^3\overline{\beta}_{2G} + i\ {}^3\overline{\beta}_{2S}\right)e^{i\left(\hat{\upsilon}'^{(1)}x_2 - kx_1\right)}\Big] \end{aligned} \tag{E4.97} $$

$$^3A^{(1)}(k)=\chi_9\Big[-\left(^3\overline{\alpha}_{1G}+i\,^3\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2}e^{-ikx_1}+\left(^3\overline{\alpha}_{2G}+i\,^3\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}e^{-ikx_1}$$
$$+i\hat{\chi}_2'\Big[\left(^3\overline{\beta}_{1G}+i\,^3\overline{\beta}_{1S}\right)e^{-i\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right)}+\left(^3\overline{\beta}_{2G}+i\,^3\overline{\beta}_{2S}\right)e^{i\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)}\Big]\Big] \qquad \text{(E4.98)}$$

$$^3B^{(1)}(k)=\chi_1\Big[-\left(^3\overline{\alpha}_{1G}+i\,^3\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2}e^{-ikx_1}+\left(^3\overline{\alpha}_{2G}+i\,^3\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}e^{-ikx_1}$$
$$-i\hat{\chi}_2'\Big[\left(^3\overline{\beta}_{1G}+i\,^3\overline{\beta}_{1S}\right)e^{-i\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right)}+\left(^3\overline{\beta}_{2G}+i\,^3\overline{\beta}_{2S}\right)e^{i\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)}\Big]\Big] \qquad \text{(E4.99)}$$

$$^3C^{(1)}(k)=-\chi_3\Big[\left(^3\overline{\alpha}_{1G}+i\,^3\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2}e^{-ikx_1}+\left(^3\overline{\alpha}_{2G}+i\,^3\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}e^{-ikx_1}$$
$$+\chi_4\Big[-\left(^3\overline{\beta}_{1G}+i\,^3\overline{\beta}_{1S}\right)e^{-i\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right)}+\left(^3\overline{\beta}_{2G}+i\,^3\overline{\beta}_{2S}\right)e^{i\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)}\Big]\Big] \qquad \text{(E4.100)}$$

$$^3\Theta^{(2)}(k)=-k\left(^3\overline{\alpha}_{3G}+i\,^3\overline{\alpha}_{3S}\right)e^{-\upsilon^{(2)}x_2}e^{-ikx_1}-i\hat{\upsilon}'^{(2)}\left(^3\overline{\beta}_{3G}+i\,^3\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)} \qquad \text{(E4.101)}$$

$$^3\Omega^{(2)}(k)=-\upsilon^{(2)}\left(^3\overline{\alpha}_{3G}+i\,^3\overline{\alpha}_{3S}\right)e^{-\upsilon^{(2)}x_2}e^{-ikx_1}-k\left(^3\overline{\beta}_{3G}+i\,^3\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)} \qquad \text{(E.4102)}$$

$$^3A^{(2)}(k)=\chi_{10}\left(^3\overline{\alpha}_{3G}+i\,^3\overline{\alpha}_{3S}\right)e^{-\upsilon^{(2)}x_2}e^{-ikx_1}-i\hat{\chi}_6'\left(^3\overline{\beta}_{3G}+i\,^3\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)} \qquad \text{(E4.103)}$$

$$^3B^{(2)}(k)=\chi_5\left(^3\overline{\alpha}_{3G}+i\,^3\overline{\alpha}_{3S}\right)e^{-\upsilon^{(2)}x_2}e^{-ikx_1}+i\hat{\chi}_6'\left(^3\overline{\beta}_{3G}+i\,^3\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)} \qquad \text{(E4.104)}$$

$$^3C^{(2)}(k)=\chi_7\left(^3\overline{\alpha}_{3G}+i\,^3\overline{\alpha}_{3S}\right)e^{-\upsilon^{(2)}x_2}e^{-ikx_1}+\chi_8\left(^3\overline{\beta}_{3G}+i\,^3\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)} \qquad \text{(E4.105)}$$

bulunur. (3.133)-(3.142) uyarınca (E4.96), (E4.100), (E4.101) ve (E4.105) eşitliklerinin sanal, (E4.97), (E4.98), (E4.99), (E4.102), (E4.103) ve (E4.104) eşitliklerinin de gerçel kısımları kullanılacaktır. Buna göre,

$$^3\Theta_S^{(1)}(k)=k\Big\{e^{-\upsilon^{(1)}x_2}\left[-\,^3\overline{\alpha}_{1G}\sin(kx_1)+{}^3\overline{\alpha}_{1S}\cos(kx_1)\right]$$
$$-e^{\upsilon^{(1)}x_2}\left[-\,^3\overline{\alpha}_{2G}\sin(kx_1)+{}^3\overline{\alpha}_{2S}\cos(kx_1)\right]\Big\}$$
$$+\hat{\upsilon}'^{(1)}\Big[{}^3\overline{\beta}_{1G}\cos\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right)+{}^3\overline{\beta}_{1S}\sin\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right)$$
$$+\,^3\overline{\beta}_{2G}\cos\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)-{}^3\overline{\beta}_{2S}\sin\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)\Big] \qquad \text{(E4.106)}$$

$$^3\Omega_G^{(1)}(k)=\upsilon^{(1)}\Big\{e^{-\upsilon^{(1)}x_2}\left[\,^3\overline{\alpha}_{1G}\cos(kx_1)+{}^3\overline{\alpha}_{1S}\sin(kx_1)\right]$$
$$+e^{\upsilon^{(1)}x_2}\left[\,^3\overline{\alpha}_{2G}\cos(kx_1)+{}^3\overline{\alpha}_{2S}\sin(kx_1)\right]\Big\}$$
$$+k\Big[{}^3\overline{\beta}_{1G}\cos\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right)+{}^3\overline{\beta}_{1S}\sin\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right)$$
$$-\,^3\overline{\beta}_{2G}\cos\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)+{}^3\overline{\beta}_{2S}\sin\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)\Big] \qquad \text{(E4.107)}$$

$$^3A_G^{(1)}(k)=\chi_9\Big\{e^{-\upsilon^{(1)}x_2}\left[-{}^3\overline{\alpha}_{1G}\cos(kx_1)-{}^3\overline{\alpha}_{1S}\sin(kx_1)\right]$$
$$+e^{\upsilon^{(1)}x_2}\left[\,^3\overline{\alpha}_{2G}\cos(kx_1)+{}^3\overline{\alpha}_{2S}\sin(kx_1)\right]\Big\}$$
$$-\hat{\chi}_2'\Big[-{}^3\overline{\beta}_{1G}\sin\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right)+{}^3\overline{\beta}_{1S}\cos\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right)$$
$$+{}^3\overline{\beta}_{2G}\sin\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)+{}^3\overline{\beta}_{2S}\cos\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)\Big] \qquad \text{(E4.108)}$$

$$^3B_G^{(1)}(k)=\chi_1\Big\{e^{-\upsilon^{(1)}x_2}\left[-{}^3\overline{\alpha}_{1G}\cos(kx_1)-{}^3\overline{\alpha}_{1S}\sin(kx_1)\right]$$
$$+e^{\upsilon^{(1)}x_2}\left[\,^3\overline{\alpha}_{2G}\cos(kx_1)+{}^3\overline{\alpha}_{2S}\sin(kx_1)\right]\Big\}$$
$$+\hat{\chi}_2'\Big[-{}^3\overline{\beta}_{1G}\sin\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right)+{}^3\overline{\beta}_{1S}\cos\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right)$$
$$+{}^3\overline{\beta}_{2G}\sin\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)+{}^3\overline{\beta}_{2S}\cos\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)\Big] \qquad \text{(E4.109)}$$

$$^3C_S^{(1)}(k)=-\chi_3\Big\{e^{-\upsilon^{(1)}x_2}\left[-\,^3\overline{\alpha}_{1G}\sin(kx_1)+{}^3\overline{\alpha}_{1S}\cos(kx_1)\right]$$
$$+e^{\upsilon^{(1)}x_2}\left[-\,^3\overline{\alpha}_{2G}\sin(kx_1)+{}^3\overline{\alpha}_{2S}\cos(kx_1)\right]\Big\}$$
$$+\chi_4\Big[{}^3\overline{\beta}_{1G}\sin\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right)-{}^3\overline{\beta}_{1S}\cos\left(\hat{\upsilon}'^{(1)}x_2+kx_1\right)$$
$$+{}^3\overline{\beta}_{2G}\sin\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)+{}^3\overline{\beta}_{2S}\cos\left(\hat{\upsilon}'^{(1)}x_2-kx_1\right)\Big] \qquad \text{(E4.110)}$$

$$
\begin{aligned}
{}^3\Theta_S^{(2)}(k) &= k e^{-\upsilon^{(2)}x_2}\left[{}^3\bar{\alpha}_{3G}\sin(kx_1) - {}^3\bar{\alpha}_{3S}\cos(kx_1)\right] \\
&\quad - \hat{\upsilon}'^{(2)}\left[{}^3\bar{\beta}_{3G}\cos\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right) + {}^3\bar{\beta}_{3S}\sin\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)\right]
\end{aligned}
\tag{E4.111}
$$

$$
\begin{aligned}
{}^3\Omega_G^{(2)}(k) &= -\upsilon^{(2)} e^{-\upsilon^{(2)}x_2}\left[{}^3\bar{\alpha}_{3G}\cos(kx_1) + {}^3\bar{\alpha}_{3S}\sin(kx_1)\right] \\
&\quad - k\left[{}^3\bar{\beta}_{3G}\cos\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right) + {}^3\bar{\beta}_{3S}\sin\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)\right]
\end{aligned}
\tag{E4.112}
$$

$$
\begin{aligned}
{}^3A_G^{(2)}(k) &= \chi_{10} e^{-\upsilon^{(2)}x_2}\left[{}^3\bar{\alpha}_{3G}\cos(kx_1) + {}^3\bar{\alpha}_{3S}\sin(kx_1)\right] \\
&\quad + \hat{\chi}'_6\left[-{}^3\bar{\beta}_{3G}\sin\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right) + {}^3\bar{\beta}_{3S}\cos\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)\right]
\end{aligned}
\tag{E4.113}
$$

$$
\begin{aligned}
{}^3B_G^{(2)}(k) &= \chi_5 e^{-\upsilon^{(2)}x_2}\left[{}^3\bar{\alpha}_{3G}\cos(kx_1) + {}^3\bar{\alpha}_{3S}\sin(kx_1)\right] \\
&\quad + \hat{\chi}'_6\left[{}^3\bar{\beta}_{3G}\sin\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right) - {}^3\bar{\beta}_{3S}\cos\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)\right]
\end{aligned}
\tag{E4.114}
$$

$$
\begin{aligned}
{}^3C_S^{(2)}(k) &= \chi_7 e^{-\upsilon^{(2)}x_2}\left[-{}^3\bar{\alpha}_{3G}\sin(kx_1) + {}^3\bar{\alpha}_{3S}\cos(kx_1)\right] \\
&\quad + \chi_8\left[-{}^3\bar{\beta}_{3G}\sin\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right) + {}^3\bar{\beta}_{3S}\cos\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)\right]
\end{aligned}
\tag{E4.115}
$$

yazılıp bu ifadelerin tek kısımları atıldıktan sonra kalan,

$$
\begin{aligned}
{}^3\Theta_{SÇ}^{(1)}(k) &= \Big\{k\left(-e^{-\upsilon^{(1)}x_2}\;{}^3\bar{\alpha}_{1G} + e^{\upsilon^{(1)}x_2}\;{}^3\bar{\alpha}_{2G}\right) \\
&\quad + \hat{\upsilon}'^{(1)}\left[-\left({}^3\bar{\beta}_{1G} - {}^3\bar{\beta}_{2G}\right)\sin\left(\hat{\upsilon}'^{(1)}x_2\right) + \left({}^3\bar{\beta}_{1S} + {}^3\bar{\beta}_{2S}\right)\cos\left(\hat{\upsilon}'^{(1)}x_2\right)\right]\Big\}\sin(kx_1)
\end{aligned}
\tag{E4.116}
$$

$$
\begin{aligned}
{}^3\Omega_{GÇ}^{(1)}(k) &= \Big\{\upsilon^{(1)}\left(e^{-\upsilon^{(1)}x_2}\;{}^3\bar{\alpha}_{1G} + e^{\upsilon^{(1)}x_2}\;{}^3\bar{\alpha}_{2G}\right) \\
&\quad + k\left[\left({}^3\bar{\beta}_{1G} - {}^3\bar{\beta}_{2G}\right)\cos\left(\hat{\upsilon}'^{(1)}x_2\right) + \left({}^3\bar{\beta}_{1S} + {}^3\bar{\beta}_{2S}\right)\sin\left(\hat{\upsilon}'^{(1)}x_2\right)\right]\Big\}\cos(kx_1)
\end{aligned}
\tag{E4.117}
$$

$$
\begin{aligned}
{}^3A_{GÇ}^{(1)}(k) &= \Big\{\chi_9\left(-e^{-\upsilon^{(1)}x_2}\;{}^3\bar{\alpha}_{1G} + e^{\upsilon^{(1)}x_2}\;{}^3\bar{\alpha}_{2G}\right) \\
&\quad + \hat{\chi}'_2\left[\left({}^3\bar{\beta}_{1G} - {}^3\bar{\beta}_{2G}\right)\sin\left(\hat{\upsilon}'^{(1)}x_2\right) - \left({}^3\bar{\beta}_{1S} + {}^3\bar{\beta}_{2S}\right)\cos\left(\hat{\upsilon}'^{(1)}x_2\right)\right]\Big\}\cos(kx_1)
\end{aligned}
\tag{E4.118}
$$

$$
\begin{aligned}
{}^3B_{GÇ}^{(1)}(k) &= \Big\{\chi_1\left(-e^{-\upsilon^{(1)}x_2}\;{}^3\bar{\alpha}_{1G} + e^{\upsilon^{(1)}x_2}\;{}^3\bar{\alpha}_{2G}\right) \\
&\quad + \hat{\chi}'_2\left[-\left({}^3\bar{\beta}_{1G} - {}^3\bar{\beta}_{2G}\right)\sin\left(\hat{\upsilon}'^{(1)}x_2\right) + \left({}^3\bar{\beta}_{1S} + {}^3\bar{\beta}_{2S}\right)\cos\left(\hat{\upsilon}'^{(1)}x_2\right)\right]\Big\}\cos(kx_1)
\end{aligned}
\tag{E4.119}
$$

$$
\begin{aligned}
{}^3C_{SÇ}^{(1)}(k) &= \Big\{\chi_3\left(e^{-\upsilon^{(1)}x_2}\;{}^3\bar{\alpha}_{1G} + e^{\upsilon^{(1)}x_2}\;{}^3\bar{\alpha}_{2G}\right) \\
&\quad + \chi_4\left[\left({}^3\bar{\beta}_{1G} - {}^3\bar{\beta}_{2G}\right)\cos\left(\hat{\upsilon}'^{(1)}x_2\right) + \left({}^3\bar{\beta}_{1S} + {}^3\bar{\beta}_{2S}\right)\sin\left(\hat{\upsilon}'^{(1)}x_2\right)\right]\Big\}\sin(kx_1)
\end{aligned}
\tag{E4.120}
$$

$$
\begin{aligned}
{}^3\Theta_{SÇ}^{(2)}(k) &= k\, e^{\upsilon^{(2)}(h-x_2)}\;{}^3\tilde{\bar{\alpha}}_{3G}\sin(kx_1) \\
&\quad + \hat{\upsilon}'^{(2)}\Big\{\left[\cos(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3S}\right]\sin\left(\hat{\upsilon}'^{(2)}x_2\right) \\
&\qquad - \left[\cos(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3G}\right]\cos\left(\hat{\upsilon}'^{(2)}x_2\right)\Big\}\sin(kx_1)
\end{aligned}
\tag{E4.121}
$$

$$
\begin{aligned}
{}^3\Omega_{GÇ}^{(2)}(k) &= -\upsilon^{(2)} e^{\upsilon^{(2)}(h-x_2)}\;{}^3\tilde{\bar{\alpha}}_{3G}\cos(kx_1) \\
&\quad - k\Big\{\left[\cos(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3S}\right]\cos\left(\hat{\upsilon}'^{(2)}x_2\right) \\
&\qquad + \left[\cos(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3G}\right]\sin\left(\hat{\upsilon}'^{(2)}x_2\right)\Big\}\cos(kx_1)
\end{aligned}
\tag{E4.122}
$$

$$
\begin{aligned}
{}^3A_{GÇ}^{(2)}(k) &= \chi_{10} e^{\upsilon^{(2)}(h-x_2)}\;{}^3\tilde{\bar{\alpha}}_{3G}\cos(kx_1) \\
&\quad + \hat{\chi}'_6\Big\{-\left[\cos(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3S}\right]\sin\left(\hat{\upsilon}'^{(2)}x_2\right) \\
&\qquad + \left[\cos(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3G}\right]\cos\left(\hat{\upsilon}'^{(2)}x_2\right)\Big\}\cos(kx_1)
\end{aligned}
\tag{E4.123}
$$

$$
\begin{aligned}
{}^3B_{GÇ}^{(2)}(k) &= \chi_5\, e^{\upsilon^{(2)}(h-x_2)}\;{}^3\tilde{\bar{\alpha}}_{3G}\cos(kx_1) \\
&\quad + \hat{\chi}'_6\Big\{\left[\cos(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3S}\right]\sin\left(\hat{\upsilon}'^{(2)}x_2\right) \\
&\qquad - \left[\cos(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\;{}^3\tilde{\bar{\beta}}_{3G}\right]\cos\left(\hat{\upsilon}'^{(2)}x_2\right)\Big\}\cos(kx_1)
\end{aligned}
\tag{E4.124}
$$

$$
\begin{aligned}
{}^{3}C_{S\c{C}}^{(2)}(k) = & -\chi_7\, e^{\upsilon^{(2)}(h-x_2)}\, {}^{3}\tilde{\tilde{\alpha}}_{3G} \sin(kx_1) \\
& -\chi_8 \Big\{ \Big[\cos(\hat{\upsilon}'^{(2)}h)\, {}^{3}\tilde{\tilde{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\, {}^{3}\tilde{\tilde{\beta}}_{3S} \Big] \cos\big(\hat{\upsilon}'^{(2)}x_2\big) \\
& \quad + \Big[\cos(\hat{\upsilon}'^{(2)}h)\, {}^{3}\tilde{\tilde{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\, {}^{3}\tilde{\tilde{\beta}}_{3G} \Big] \sin\big(\hat{\upsilon}'^{(2)}x_2\big) \Big\} \sin(kx_1)
\end{aligned}
\tag{E4.125}
$$

eşitlikleri (3.133)-(3.142) de kullanılırsa sonuç olarak,

$$
{}^{3}\bar{u}_1^{(1)} = -\frac{P_0}{\pi} \int_{\frac{k_L^2}{\sqrt{1+\eta_1^{(2)}}}}^{\frac{k_T^1}{\sqrt{1+\eta_2^{(1)}}}} {}^{3}\Theta_{S\c{C}}^{(1)}(k)\, dk \tag{E4.126}
$$

$$
{}^{3}\bar{u}_2^{(1)} = \frac{P_0}{\pi} \int_{\frac{k_L^2}{\sqrt{1+\eta_1^{(2)}}}}^{\frac{k_T^1}{\sqrt{1+\eta_2^{(1)}}}} {}^{3}\Omega_{G\c{C}}^{(1)}(k)\, dk \tag{E4.127}
$$

$$
{}^{3}\sigma_{11}^{(1)} = \frac{P_0}{\pi} \int_{\frac{k_L^2}{\sqrt{1+\eta_1^{(2)}}}}^{\frac{k_T^1}{\sqrt{1+\eta_2^{(1)}}}} {}^{3}A_{G\c{C}}^{(1)}(k)\, dk \tag{E4.128}
$$

$$
{}^{3}\sigma_{22}^{(1)} = \frac{P_0}{\pi} \int_{\frac{k_L^2}{\sqrt{1+\eta_1^{(2)}}}}^{\frac{k_T^1}{\sqrt{1+\eta_2^{(1)}}}} {}^{3}B_{G\c{C}}^{(1)}(k)\, dk \tag{E4.129}
$$

$$
{}^{3}\sigma_{12}^{(1)} = -\frac{P_0}{\pi} \int_{\frac{k_L^2}{\sqrt{1+\eta_1^{(2)}}}}^{\frac{k_T^1}{\sqrt{1+\eta_2^{(1)}}}} {}^{3}C_{S\c{C}}^{(1)}(k)\, dk \tag{E4.130}
$$

$$
{}^{3}\bar{u}_1^{(2)} = -\frac{P_0}{\pi} \int_{\frac{k_L^2}{\sqrt{1+\eta_1^{(2)}}}}^{\frac{k_T^1}{\sqrt{1+\eta_2^{(1)}}}} {}^{3}\Theta_{S\c{C}}^{(2)}(k)\, dk \tag{E4.131}
$$

$$
{}^{3}\bar{u}_2^{(2)} = \frac{P_0}{\pi} \int_{\frac{k_L^2}{\sqrt{1+\eta_1^{(2)}}}}^{\frac{k_T^1}{\sqrt{1+\eta_2^{(1)}}}} {}^{3}\Omega_{G\c{C}}^{(2)}(k)\, dk \tag{E4.132}
$$

$$
{}^{3}\sigma_{11}^{(2)} = \frac{P_0}{\pi} \int_{\frac{k_L^2}{\sqrt{1+\eta_1^{(2)}}}}^{\frac{k_T^1}{\sqrt{1+\eta_2^{(1)}}}} {}^{3}A_{G\c{C}}^{(2)}(k)\, dk \tag{E4.133}
$$

$$ {}^{3}\sigma_{22}^{(2)} = \frac{P_0}{\pi} \int_{\frac{k_L^2}{\sqrt{1+\eta_1^{(2)}}}}^{\frac{k_T^1}{\sqrt{1+\eta_2^{(1)}}}} {}^{3}B_{GÇ}^{(2)}(k)\,dk \tag{E4.134} $$

$$ {}^{3}\sigma_{12}^{(2)} = -\frac{P_0}{\pi} \int_{\frac{k_L^2}{\sqrt{1+\eta_1^{(2)}}}}^{\frac{k_T^1}{\sqrt{1+\eta_2^{(1)}}}} {}^{3}C_{SÇ}^{(2)}(k)\,dk \tag{E4.135} $$

bulunur.

EK 5. $k_T^{(1)}/\sqrt{1+\eta_2^{(1)}} < k < k_T^{(2)}/\sqrt{1+\eta_2^{(2)}}$ **Durumu.**

Bu durumda (3.52) ve (3.53) eşitliklerinden hareketle,

$$\upsilon^{(1)} = \left[k^2(1+\eta_1^{(1)}) - k_L^{(1)2}\right]^{1/2} \quad \text{(E5.1)}$$
$$\upsilon'^{(1)} = \left[k^2(1+\eta_2^{(1)}) - k_T^{(1)2}\right]^{1/2} \quad \text{(E5.2)}$$
$$\upsilon^{(2)} = \left[k^2(1+\eta_1^{(2)}) - k_L^{(2)2}\right]^{1/2} \quad \text{(E5.3)}$$
$$\upsilon'^{(2)} = \left[k^2(1+\eta_2^{(2)}) - k_T^{(2)2}\right]^{1/2} = i\left[k_T^{(2)2} - k^2(1+\eta_2^{(2)})\right]^{1/2} = i\,\hat{\upsilon}'^{(2)} \quad \text{(E5.4)}$$

eşitlikleri kullanılır. Buna göre,

$$^4\chi_1 = \chi_1 \quad \text{(E5.5)}$$
$$^4\chi_2 = \chi_2 \quad \text{(E5.6)}$$
$$^4\chi_3 = \chi_3 \quad \text{(E5.7)}$$
$$^4\chi_4 = \chi_4 \quad \text{(E5.8)}$$
$$^4\chi_5 = \chi_5 \quad \text{(E5.9)}$$
$$^4\chi_6 = i\hat{\chi}'_6 \quad \text{(E5.10)}$$
$$^4\chi_7 = \chi_7 \quad \text{(E5.11)}$$
$$^4\chi_8 = \chi_8 \quad \text{(E5.12)}$$
$$^4\chi_9 = \chi_9 \quad \text{(E5.13)}$$
$$^4\chi_{10} = \chi_{10} \quad \text{(E5.14)}$$

bulunur. Ayrıca,

$$^4H_1 = H_1 = e^{-\upsilon^{(1)}x_2} \quad \text{(E5.15)}$$
$$^4H_2 = H_2 = e^{\upsilon^{(1)}x_2} \quad \text{(E5.16)}$$
$$^4H_3 = H_3 = e^{-\upsilon'^{(1)}x_2} \quad \text{(E5.17)}$$
$$^4H_4 = H_4 = e^{\upsilon'^{(1)}x_2} \quad \text{(E5.18)}$$
$$^4H_5 = H_5 = e^{-\upsilon^{(2)}x_2} \quad \text{(E5.19)}$$
$$^4H_6 = \hat{H}_6 = e^{-i\hat{\upsilon}'^{(2)}x_2} \quad \text{(E5.20)}$$

ve,

$$^4\overline{H}_1 = \overline{H}_1 = e^{-\upsilon^{(1)}h} \quad \text{(E5.21)}$$
$$^4\overline{H}_2 = \overline{H}_2 = e^{\upsilon^{(1)}h} \quad \text{(E5.22)}$$
$$^4\overline{H}_3 = \overline{H}_3 = e^{-\upsilon'^{(1)}h} \quad \text{(E5.23)}$$
$$^4\overline{H}_4 = \overline{H}_4 = e^{\upsilon'^{(1)}h} \quad \text{(E5.24)}$$
$$^4\overline{H}_5 = \overline{H}_5 = e^{-\upsilon^{(2)}h} \quad \text{(E5.25)}$$
$$^4\overline{H}_6 = \hat{\overline{H}}_6 = e^{-i\hat{\upsilon}'^{(2)}h} \quad \text{(E5.26)}$$

olup bütün bu eşitlikler (E1.23)-(E1.36) da kullanılırsa,

$$
\begin{aligned}
{}^4\Gamma_{1G} &= k^2(\chi_5-\chi_1)(\chi_8-\chi_4)+k^2(\chi_2\chi_7+\chi_2\chi_3)-k\chi_2\chi_8(\upsilon^{(1)}+\upsilon^{(2)})\\
&\quad -k(\chi_1\chi_7+\chi_3\chi_5)\upsilon'^{(1)}+\upsilon'^{(1)}\chi_8(\upsilon^{(1)}\chi_5+\upsilon^{(2)}\chi_1)
\end{aligned} \tag{E5.27}
$$

$$ {}^4\Gamma_{2G} = 2(k\chi_4-k\chi_8)(\upsilon'^{(1)}\chi_5-k\chi_2) \tag{E5.28} $$

$$
\begin{aligned}
{}^4\Gamma_{3G} &= k^2(\chi_5-\chi_1)(\chi_8-\chi_4)-k^2(\chi_2\chi_7+\chi_2\chi_3)+k\chi_2\chi_8(\upsilon^{(1)}+\upsilon^{(2)})\\
&\quad +k(\chi_1\chi_7+\chi_3\chi_5)\upsilon'^{(1)}-\upsilon'^{(1)}\chi_8(\upsilon^{(1)}\chi_5+\upsilon^{(2)}\chi_1)
\end{aligned} \tag{E5.29}
$$

$$
\begin{aligned}
{}^4\Gamma_{4G} &= k^2(\chi_5-\chi_1)(\chi_8-\chi_4)+k^2(\chi_2\chi_3-\chi_2\chi_7)+k\chi_2\chi_8(\upsilon^{(2)}-\upsilon^{(1)})\\
&\quad +k(\chi_1\chi_7-\chi_3\chi_5)\upsilon'^{(1)}+\upsilon'^{(1)}\chi_8(\upsilon^{(1)}\chi_5-\upsilon^{(2)}\chi_1)
\end{aligned} \tag{E5.30}
$$

$$
\begin{aligned}
{}^4\Gamma_{5G} &= k^2(\chi_5-\chi_1)(\chi_8-\chi_4)-k^2(\chi_2\chi_3-\chi_2\chi_7)-k\chi_2\chi_8(\upsilon^{(2)}-\upsilon^{(1)})\\
&\quad -k(\chi_1\chi_7-\chi_3\chi_5)\upsilon'^{(1)}-\upsilon'^{(1)}\chi_8(\upsilon^{(1)}\chi_5-\upsilon^{(2)}\chi_1)
\end{aligned} \tag{E5.31}
$$

$$ {}^4\Gamma_{6G} = 2(k\chi_5-k\chi_1)(k\chi_3-\upsilon^{(1)}\chi_8) \tag{E5.32} $$

$$ {}^4\Gamma_{7G} = 2(k\chi_8-k\chi_4)(\upsilon'^{(1)}\chi_1-k\chi_2) \tag{E5.33} $$

$$ {}^4\Gamma_{8G} = {}^4\Gamma_{7G} \tag{E5.34} $$

$$ {}^4\Gamma_{9G} = 2(\upsilon^{(1)}\chi_8-k\chi_3)(\upsilon'^{(1)}\chi_1-k\chi_2) \tag{E5.35} $$

$$ {}^4\Gamma_{10G} = -{}^4\Gamma_{9G} \tag{E5.36} $$

$$
\begin{aligned}
{}^4\Gamma_{11G} &= -2k\upsilon'^{(1)}(\chi_3\chi_5+\chi_1\chi_7)+2k^2(\chi_2\chi_7+\chi_2\chi_3)\\
&\quad +2\upsilon'^{(1)}\chi_4(\upsilon^{(2)}\chi_1+\upsilon^{(1)}\chi_5)-2k\chi_2(\upsilon^{(1)}\chi_4+\upsilon^{(2)}\chi_4)
\end{aligned} \tag{E5.37}
$$

$$
\begin{aligned}
{}^4\Gamma_{12G} &= 2k\upsilon'^{(1)}(\chi_3\chi_5-\chi_1\chi_7)+2k^2(\chi_2\chi_7-\chi_2\chi_3)\\
&\quad +2\upsilon'^{(1)}\chi_4(\upsilon^{(2)}\chi_1-\upsilon^{(1)}\chi_5)+2k\chi_2(\upsilon^{(1)}\chi_4-\upsilon^{(2)}\chi_4)
\end{aligned} \tag{E5.38}
$$

$$
\begin{aligned}
{}^4\Gamma_{13G} &= -2k\upsilon^{(1)}(\chi_4\chi_5+\chi_2\chi_7)+2k\chi_3(k\chi_5+\upsilon^{(2)}\chi_2)\\
&\quad +2\upsilon'^{(1)}\chi_1(\upsilon^{(1)}\chi_7-\upsilon^{(2)}\chi_3)+2k\chi_1(\upsilon^{(1)}\chi_4-k\chi_3)
\end{aligned} \tag{E5.39}
$$

$$
\begin{aligned}
{}^4\Gamma_{14G} &= -2k\upsilon^{(1)}(\chi_4\chi_5-\chi_2\chi_7)+2k\chi_3(k\chi_5-\upsilon^{(2)}\chi_2)\\
&\quad -2\upsilon'^{(1)}\chi_1(\upsilon^{(1)}\chi_7-\upsilon^{(2)}\chi_3)+2k\chi_1(\upsilon^{(1)}\chi_4-k\chi_3)
\end{aligned} \tag{E5.40}
$$

ve,

$$
\begin{aligned}
{}^4\Gamma_{1S} &= -k^2(\hat\chi_6'\chi_7+\chi_3\hat\chi_6')+k\chi_4\hat\chi_6'(\upsilon^{(1)}+\upsilon^{(2)})+k\hat\upsilon'^{(2)}(\chi_1\chi_7+\chi_3\chi_5)\\
&\quad +\upsilon'^{(1)}\hat\chi_6'(\upsilon^{(2)}\chi_3-\upsilon^{(1)}\chi_7)+\hat\upsilon'^{(2)}\upsilon^{(1)}(\chi_2\chi_7-\chi_4\chi_5)-\hat\upsilon'^{(2)}\upsilon^{(2)}(\chi_1\chi_4+\chi_2\chi_3)
\end{aligned} \tag{E5.41}
$$

$$ {}^4\Gamma_{2S} = 2(k\chi_7-\upsilon^{(2)}\chi_4)(\upsilon'^{(1)}\hat\chi_6'-\hat\upsilon'^{(2)}\chi_2) \tag{E5.42} $$

$$
\begin{aligned}
{}^4\Gamma_{3S} &= -k^2(\hat\chi_6'\chi_7+\chi_3\hat\chi_6')+k\chi_4\hat\chi_6'(\upsilon^{(1)}+\upsilon^{(2)})+k\hat\upsilon'^{(2)}(\chi_1\chi_7+\chi_3\chi_5)\\
&\quad -\upsilon'^{(1)}\hat\chi_6'(\upsilon^{(2)}\chi_3-\upsilon^{(1)}\chi_7)-\hat\upsilon'^{(2)}\upsilon^{(1)}(\chi_2\chi_7+\chi_4\chi_5)-\hat\upsilon'^{(2)}\upsilon^{(2)}(\chi_1\chi_4-\chi_2\chi_3)
\end{aligned} \tag{E5.43}
$$

$$
\begin{aligned}
{}^4\Gamma_{4S} &= -k^2(\hat\chi_6'\chi_7-\chi_3\hat\chi_6')+k\chi_4\hat\chi_6'(\upsilon^{(2)}-\upsilon^{(1)})+k\hat\upsilon'^{(2)}(\chi_1\chi_7-\chi_3\chi_5)\\
&\quad +\upsilon'^{(1)}\hat\chi_6'(\upsilon^{(2)}\chi_3-\upsilon^{(1)}\chi_7)+\hat\upsilon'^{(2)}\upsilon^{(1)}(\chi_2\chi_7+\chi_4\chi_5)-\hat\upsilon'^{(2)}\upsilon^{(2)}(\chi_1\chi_4+\chi_2\chi_3)
\end{aligned} \tag{E5.44}
$$

$$
\begin{aligned}
{}^4\Gamma_{5S} &= -k^2(\hat\chi_6'\chi_7-\chi_3\hat\chi_6')+k\chi_4\hat\chi_6'(\upsilon^{(2)}-\upsilon^{(1)})+k\hat\upsilon'^{(2)}(\chi_1\chi_7-\chi_3\chi_5)\\
&\quad -\upsilon'^{(1)}\hat\chi_6'(\upsilon^{(2)}\chi_3-\upsilon^{(1)}\chi_7)+\hat\upsilon'^{(2)}\upsilon^{(1)}(\chi_4\chi_5-\chi_2\chi_7)+\hat\upsilon'^{(2)}\upsilon^{(2)}(\chi_2\chi_3-\chi_1\chi_4)
\end{aligned} \tag{E5.45}
$$

$$ {}^4\Gamma_{6S} = 2(\upsilon^{(1)}\chi_7-\upsilon^{(2)}\chi_3)(k\hat\chi_6'-\hat\upsilon'^{(2)}\chi_1) \tag{E5.46} $$

$$ {}^4\Gamma_{7S} = 2(k\chi_3-\upsilon^{(1)}\chi_4)(\upsilon'^{(1)}\hat\chi_6'-\hat\upsilon'^{(2)}\chi_2) \tag{E5.47} $$

$$ {}^4\Gamma_{8S} = -{}^4\Gamma_{7S} \tag{E5.48} $$

$$ {}^4\Gamma_{9S} = 2(k\chi_3-\upsilon^{(1)}\chi_4)(k\hat\chi_6'-\hat\upsilon'^{(2)}\chi_1) \tag{E5.49} $$

$$ {}^4\Gamma_{10S} = {}^4\Gamma_{9S} \tag{E5.50} $$

$$^4\Gamma_{11S}=0 \tag{E5.51}$$
$$^4\Gamma_{12S}=0 \tag{E5.52}$$
$$^4\Gamma_{13S}=0 \tag{E5.53}$$
$$^4\Gamma_{14S}=0 \tag{E5.54}$$

olmak üzere,

$$^4\Gamma_j={}^4\Gamma_{jG}+i\ {}^4\Gamma_{jS}\ \ ,j=1,14 \tag{E5.55}$$

yazılabilir. Ayrıca (3.85) eşitliği yardımı ile,

$$\begin{aligned}^4[\Delta]=&(\chi_2\chi_3+\chi_1\chi_4)\left[\overline{H}_2\overline{H}_3({}^4\Gamma_{1G}+i\ {}^4\Gamma_{1S})+\overline{H}_1\overline{H}_4({}^4\Gamma_{4G}+i\ {}^4\Gamma_{4S})\right]\\&+(\chi_2\chi_3-\chi_1\chi_4)\left[\overline{H}_1\overline{H}_3({}^4\Gamma_{5G}+i\ {}^4\Gamma_{5S})+\overline{H}_2\overline{H}_4({}^4\Gamma_{3G}+i\ {}^4\Gamma_{3S})\right]\\&+2\left[\chi_1\chi_3\overline{H}_3\overline{H}_4({}^4\Gamma_{2G}+i\ {}^4\Gamma_{2S})+\chi_2\chi_4\overline{H}_1\overline{H}_2({}^4\Gamma_{6G}+i\ {}^4\Gamma_{6S})\right]\end{aligned} \tag{E5.56}$$

ve k dönüşüm parametresine göre,

$^4\Gamma_{1G},{}^4\Gamma_{3G},{}^4\Gamma_{4G},{}^4\Gamma_{5G},{}^4\Gamma_{9G},{}^4\Gamma_{10G},{}^4\Gamma_{11G},{}^4\Gamma_{12G}$ ve $^4\Gamma_{1S},{}^4\Gamma_{3S},{}^4\Gamma_{4S},{}^4\Gamma_{5S},{}^4\Gamma_{9S},{}^4\Gamma_{10S}$; çift, $^4\Gamma_{2G},{}^4\Gamma_{6G},{}^4\Gamma_{7G},{}^4\Gamma_{8G},{}^4\Gamma_{13G},{}^4\Gamma_{14G}$ ve $^4\Gamma_{2S},{}^4\Gamma_{6S},{}^4\Gamma_{7S},{}^4\Gamma_{8S}$; tek fonksiyonlar ve $^4\Gamma_{11S},{}^4\Gamma_{12S},{}^4\Gamma_{13S},{}^4\Gamma_{14S}=0$ olduğuna göre,

$$\begin{aligned}^4[\Delta]_G=&(\chi_2\chi_3+\chi_1\chi_4)\left(e^{(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^4\Gamma_{1G}+e^{-(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^4\Gamma_{4G}\right)\\&+(\chi_2\chi_3-\chi_1\chi_4)\left(e^{-(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^4\Gamma_{5G}+e^{(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^4\Gamma_{3G}\right)\\&+2\left(\chi_1\chi_3\ {}^4\Gamma_{2G}+\chi_2\chi_4\ {}^4\Gamma_{6G}\right)\end{aligned} \tag{E5.57}$$

ve,

$$\begin{aligned}^4[\Delta]_S=&(\chi_2\chi_3+\chi_1\chi_4)\left(e^{(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^4\Gamma_{1S}+e^{-(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^4\Gamma_{4S}\right)\\&+(\chi_2\chi_3-\chi_1\chi_4)\left(e^{-(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^4\Gamma_{5S}+e^{(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^4\Gamma_{3S}\right)\\&+2\left(\chi_1\chi_3\ {}^4\Gamma_{2S}+\chi_2\chi_4\ {}^4\Gamma_{6S}\right)\end{aligned} \tag{E5.58}$$

olmak üzere,

$$^4[\Delta]={}^4[\Delta]_G+i\ {}^4[\Delta]_S \tag{E5.59}$$

yazılabilir. (E5.59) eşitliğinin gerçel ve sanal kısımları çift fonksiyonlar olmaktadır. Buradan (E1.49)-(E1.54) yardımı ile,

$$^4\overline{\alpha}_1=\frac{\chi_4e^{\upsilon^{(1)}h}\left|e^{-\upsilon'^{(1)}h}({}^4\Gamma_{1G}+i\ {}^4\Gamma_{1S})-e^{\upsilon'^{(1)}h}({}^4\Gamma_{3G}+i\ {}^4\Gamma_{3S})\right|+\chi_3({}^4\Gamma_{2G}+i\ {}^4\Gamma_{2S})}{^4[\Delta]} \tag{E5.60}$$

$$^4\overline{\alpha}_2=\frac{\chi_4e^{-\upsilon^{(1)}h}\left|e^{-\upsilon'^{(1)}h}({}^4\Gamma_{5G}+i\ {}^4\Gamma_{5S})-e^{\upsilon'^{(1)}h}({}^4\Gamma_{4G}+i\ {}^4\Gamma_{4S})\right|-\chi_3({}^4\Gamma_{2G}+i\ {}^4\Gamma_{2S})}{^4[\Delta]} \tag{E5.61}$$

$$^4\overline{\beta}_1=\frac{\chi_3e^{\upsilon'^{(1)}h}\left|e^{\upsilon^{(1)}h}({}^4\Gamma_{3G}+i\ {}^4\Gamma_{3S})+e^{-\upsilon^{(1)}h}({}^4\Gamma_{4G}+i\ {}^4\Gamma_{4S})\right|+\chi_4({}^4\Gamma_{6G}+i\ {}^4\Gamma_{6S})}{^4[\Delta]} \tag{E5.62}$$

$$ {}^{4}\overline{\beta}_2 = \frac{\chi_3 e^{-\upsilon'^{(1)}h}\left[e^{\upsilon^{(1)}h}({}^{4}\Gamma_{1G} + i\ {}^{4}\Gamma_{1S}) + e^{-\upsilon^{(1)}h}({}^{4}\Gamma_{5G} + i\ {}^{4}\Gamma_{5S})\right] + \chi_4({}^{4}\Gamma_{6G} + i\ {}^{4}\Gamma_{6S})}{{}^{4}[\Delta]} \qquad \text{(E5.63)} $$

$$ {}^{4}\overline{\alpha}_3 = \frac{\chi_3\left[e^{\upsilon^{(1)}h}({}^{4}\Gamma_{7G} + i\ {}^{4}\Gamma_{7S}) + e^{-\upsilon^{(1)}h}({}^{4}\Gamma_{8G} + i\ {}^{4}\Gamma_{8S})\right]}{e^{-\upsilon^{(2)}h}\ {}^{4}[\Delta]} + \frac{\chi_4\left[e^{-\upsilon'^{(1)}h}({}^{4}\Gamma_{10G} + i\ {}^{4}\Gamma_{10S}) - e^{\upsilon'^{(1)}h}({}^{4}\Gamma_{9G} + i\ {}^{4}\Gamma_{9S})\right]}{e^{-\upsilon^{(2)}h}\ {}^{4}[\Delta]} \qquad \text{(E5.64)} $$

$$ {}^{4}\overline{\beta}_3 = \frac{\chi_3\left(e^{\upsilon^{(1)}h}\ {}^{4}\Gamma_{11G} + e^{-\upsilon^{(1)}h}\ {}^{4}\Gamma_{12G}\right) + \chi_4\left(e^{\upsilon'^{(1)}h}\ {}^{2}\Gamma_{13G} - e^{-\upsilon'^{(1)}h}\ {}^{4}\Gamma_{14G}\right)}{e^{-i\hat{\upsilon}'^{(2)}h}\ {}^{4}[\Delta]} \qquad \text{(E5.65)} $$

yazılır. (E5.60)-(E5.65) eşitliklerini gerçel ve sanal kısımlarına ayırmak için,

$$ {}^{4}\overline{\alpha}_{1PG} = \chi_4\left(e^{(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^{4}\Gamma_{1G} - e^{(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^{4}\Gamma_{3G}\right) + \chi_3\,{}^{4}\Gamma_{2G} \qquad \text{(E5.66)} $$

$$ {}^{4}\overline{\alpha}_{1PS} = \chi_4\left(e^{(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^{4}\Gamma_{1S} - e^{(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^{4}\Gamma_{3S}\right) + \chi_3\,{}^{4}\Gamma_{2S} \qquad \text{(E5.67)} $$

$$ {}^{4}\overline{\alpha}_{2PG} = \chi_4\left(e^{-(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^{4}\Gamma_{5G} - e^{-(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^{4}\Gamma_{4G}\right) - \chi_3\,{}^{4}\Gamma_{2G} \qquad \text{(E5.68)} $$

$$ {}^{4}\overline{\alpha}_{2PS} = \chi_4\left(e^{-(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^{4}\Gamma_{5S} - e^{-(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^{4}\Gamma_{4S}\right) - \chi_3\,{}^{4}\Gamma_{2S} \qquad \text{(E5.69)} $$

$$ {}^{4}\overline{\beta}_{1PG} = \chi_3\left(e^{(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^{4}\Gamma_{3G} + e^{-(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^{4}\Gamma_{4G}\right) + \chi_4\,{}^{4}\Gamma_{6G} \qquad \text{(E5.70)} $$

$$ {}^{4}\overline{\beta}_{1PS} = \chi_3\left(e^{(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^{4}\Gamma_{3S} + e^{-(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^{4}\Gamma_{4S}\right) + \chi_4\,{}^{4}\Gamma_{6S} \qquad \text{(E5.71)} $$

$$ {}^{4}\overline{\beta}_{2PG} = \chi_3\left(e^{(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^{4}\Gamma_{1G} + e^{-(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^{4}\Gamma_{5G}\right) + \chi_4\,{}^{4}\Gamma_{6G} \qquad \text{(E5.72)} $$

$$ {}^{4}\overline{\beta}_{2PS} = \chi_3\left(e^{(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^{4}\Gamma_{1S} + e^{-(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^{4}\Gamma_{5S}\right) + \chi_4\,{}^{4}\Gamma_{6S} \qquad \text{(E5.73)} $$

$$ {}^{4}\overline{\alpha}_{3PG} = \chi_3\left(e^{\upsilon^{(1)}h}\ {}^{4}\Gamma_{7G} + e^{-\upsilon^{(1)}h}\ {}^{4}\Gamma_{8G}\right) + \chi_4\left(e^{-\upsilon'^{(1)}h}\ {}^{4}\Gamma_{10G} - e^{\upsilon'^{(1)}h}\ {}^{4}\Gamma_{9G}\right) \qquad \text{(E5.74)} $$

$$ {}^{4}\overline{\alpha}_{3PS} = \chi_3\left(e^{\upsilon^{(1)}h}\ {}^{4}\Gamma_{7S} + e^{-\upsilon^{(1)}h}\ {}^{4}\Gamma_{8S}\right) + \chi_4\left(e^{-\upsilon'^{(1)}h}\ {}^{4}\Gamma_{10S} - e^{\upsilon'^{(1)}h}\ {}^{4}\Gamma_{9S}\right) \qquad \text{(E5.75)} $$

$$ {}^{4}\overline{\beta}_{3PG} = \chi_3\left(e^{\upsilon^{(1)}h}\ {}^{4}\Gamma_{11G} + e^{-\upsilon^{(1)}h}\ {}^{4}\Gamma_{12G}\right) + \chi_4\left(e^{\upsilon'^{(1)}h}\ {}^{4}\Gamma_{13G} - e^{-\upsilon'^{(1)}h}\ {}^{4}\Gamma_{14G}\right) \qquad \text{(E5.76)} $$

$$ {}^{4}\overline{\beta}_{3PS} = 0 \qquad \text{(E5.77)} $$

tanımlamaları yapılırsa,

$$ {}^{4}\overline{\alpha}_{1G} = \frac{{}^{4}[\Delta]_G\ {}^{4}\overline{\alpha}_{1PG} + {}^{4}[\Delta]_S\ {}^{4}\overline{\alpha}_{1PS}}{{}^{4}[\Delta]_G^{\,2} + {}^{4}[\Delta]_S^{\,2}} \qquad \text{(E5.78)} $$

$$ {}^{4}\overline{\alpha}_{1S} = \frac{{}^{4}[\Delta]_G\ {}^{4}\overline{\alpha}_{1PS} - {}^{4}[\Delta]_S\ {}^{4}\overline{\alpha}_{1PG}}{{}^{4}[\Delta]_G^{\,2} + {}^{4}[\Delta]_S^{\,2}} \qquad \text{(E5.79)} $$

$$ {}^{4}\overline{\alpha}_{2G} = \frac{{}^{4}[\Delta]_G\ {}^{4}\overline{\alpha}_{2PG} + {}^{4}[\Delta]_S\ {}^{4}\overline{\alpha}_{2PS}}{{}^{4}[\Delta]_G^{\,2} + {}^{4}[\Delta]_S^{\,2}} \qquad \text{(E5.80)} $$

$$ {}^{4}\overline{\alpha}_{2S} = \frac{{}^{4}[\Delta]_G\ {}^{4}\overline{\alpha}_{2PS} - {}^{4}[\Delta]_S\ {}^{4}\overline{\alpha}_{2PG}}{{}^{4}[\Delta]_G^{\,2} + {}^{4}[\Delta]_S^{\,2}} \qquad \text{(E5.81)} $$

$$ {}^{4}\overline{\beta}_{1G} = \frac{{}^{4}[\Delta]_G\ {}^{4}\overline{\beta}_{1PG} + {}^{4}[\Delta]_S\ {}^{4}\overline{\beta}_{1PS}}{{}^{4}[\Delta]_G^{\,2} + {}^{4}[\Delta]_S^{\,2}} \qquad \text{(E5.82)} $$

$$ {}^{4}\overline{\beta}_{1S} = \frac{{}^{4}[\Delta]_G\ {}^{4}\overline{\beta}_{1PS} - {}^{4}[\Delta]_S\ {}^{4}\overline{\beta}_{1PG}}{{}^{4}[\Delta]_G^{\,2} + {}^{4}[\Delta]_S^{\,2}} \qquad \text{(E5.83)} $$

$$^4\overline{\beta}_{2G}=\frac{^4[\Delta]_G\ {}^4\overline{\beta}_{2PG}+{}^4[\Delta]_S\ {}^4\overline{\beta}_{2PS}}{^4[\Delta]_G^{\,2}+{}^4[\Delta]_S^{\,2}} \tag{E5.84}$$

$$^4\overline{\beta}_{2S}=\frac{^4[\Delta]_G\ {}^4\overline{\beta}_{2PS}-{}^4[\Delta]_S\ {}^4\overline{\beta}_{2PG}}{^4[\Delta]_G^{\,2}+{}^4[\Delta]_S^{\,2}} \tag{E5.85}$$

$$^4\tilde{\overline{\alpha}}_{3G}=\frac{^4[\Delta]_G\ {}^4\overline{\alpha}_{3PG}+{}^4[\Delta]_S\ {}^4\overline{\alpha}_{3PS}}{^4[\Delta]_G^{\,2}+{}^4[\Delta]_S^{\,2}} \tag{E5.86}$$

$$^4\tilde{\overline{\alpha}}_{3S}=\frac{^4[\Delta]_G\ {}^4\overline{\alpha}_{3PS}-{}^4[\Delta]_S\ {}^4\overline{\alpha}_{3PG}}{^4[\Delta]_G^{\,2}+{}^4[\Delta]_S^{\,2}} \tag{E5.87}$$

$$^4\tilde{\overline{\beta}}_{3G}=\frac{^4[\Delta]_G\ {}^4\overline{\beta}_{3PG}}{^4[\Delta]_G^{\,2}+{}^4[\Delta]_S^{\,2}} \tag{E5.88}$$

$$^4\tilde{\overline{\beta}}_{3S}=\frac{-\ {}^4[\Delta]_S\ {}^4\overline{\beta}_{3PG}}{^4[\Delta]_G^{\,2}+{}^4[\Delta]_S^{\,2}} \tag{E5.89}$$

olmak üzere,

$$^4\overline{\alpha}_1={}^4\overline{\alpha}_{1G}+i\ {}^4\overline{\alpha}_{1S} \tag{E5.90}$$
$$^4\overline{\alpha}_2={}^4\overline{\alpha}_{2G}+i\ {}^4\overline{\alpha}_{2S} \tag{E5.91}$$
$$^4\overline{\beta}_1={}^4\overline{\beta}_{1G}+i\ {}^4\overline{\beta}_{1S} \tag{E5.92}$$
$$^4\overline{\beta}_2={}^4\overline{\beta}_{2G}+i\ {}^4\overline{\beta}_{2S} \tag{E5.93}$$
$$^4\overline{\alpha}_3=e^{\upsilon^{(2)}h}({}^4\tilde{\overline{\alpha}}_{3G}+i\ {}^4\tilde{\overline{\alpha}}_{3S}) \tag{E5.94}$$
$$^4\overline{\beta}_3=\cos(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3G}-\sin(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3S}+i\left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3S}+\sin(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3G}\right] \tag{E5.95}$$

yazılabilir. ${}^4\overline{\alpha}_{iPG},{}^4\overline{\alpha}_{iPS}$ (i=1,3) fonksiyonları k'ya göre çift, ${}^4\overline{\beta}_{iPG},{}^4\overline{\beta}_{iPS}$ (i=1,3) ise tek olup, ${}^4[\Delta]_G$ ve ${}^4[\Delta]_S$ fonksiyonları da çift olduklarına göre sonuç olarak ${}^4\overline{\alpha}_{iG},{}^4\overline{\alpha}_{iS}$ (i=1,3) fonksiyonlarının çift, ${}^4\overline{\beta}_{iG},{}^4\overline{\beta}_{iS}$ (i=1,3) fonksiyonlarının da tek oldukları görülür. (E5.90)-(E5.95) eşitlikleri (3.123)-(3.132) de kullanılırsa, (E5.1)-(E5.20) yardımı ile,

$$\begin{aligned}{}^4\Theta^{(1)}(k)=\Big\{k\Big[&\left({}^4\overline{\alpha}_{1G}+i\ {}^4\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2}-\left({}^4\overline{\alpha}_{2G}+i\ {}^4\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}\Big]\\ &+\upsilon'^{(1)}\Big[\left({}^4\overline{\beta}_{1G}+i\ {}^4\overline{\beta}_{1S}\right)e^{-\upsilon'^{(1)}x_2}+\left({}^4\overline{\beta}_{2G}+i\ {}^4\overline{\beta}_{2S}\right)e^{\upsilon'^{(1)}x_2}\Big]\Big\}e^{-ikx_1}\end{aligned} \tag{E5.96}$$

$$\begin{aligned}{}^4\Omega^{(1)}(k)=\Big\{\upsilon^{(1)}\Big[&\left({}^4\overline{\alpha}_{1G}+i\ {}^4\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2}+\left({}^4\overline{\alpha}_{2G}+i\ {}^4\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}\Big]\\ &+k\Big[\left({}^4\overline{\beta}_{1G}+i\ {}^4\overline{\beta}_{1S}\right)e^{-\upsilon'^{(1)}x_2}-\left({}^4\overline{\beta}_{2G}+i\ {}^4\overline{\beta}_{2S}\right)e^{\upsilon'^{(1)}x_2}\Big]\Big\}e^{-ikx_1}\end{aligned} \tag{E5.97}$$

$$\begin{aligned}{}^4A^{(1)}(k)=\Big\{\chi_9\Big[&-\left({}^4\overline{\alpha}_{1G}+i\ {}^4\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2}+\left({}^4\overline{\alpha}_{2G}+i\ {}^4\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}\Big]\\ &+\chi_2\Big[\left({}^4\overline{\beta}_{1G}+i\ {}^4\overline{\beta}_{1S}\right)e^{-\upsilon'^{(1)}x_2}+\left({}^4\overline{\beta}_{2G}+i\ {}^4\overline{\beta}_{2S}\right)e^{\upsilon'^{(1)}x_2}\Big]\Big\}e^{-ikx_1}\end{aligned} \tag{E5.98}$$

$$\begin{aligned}{}^4B^{(1)}(k)=\Big\{\chi_1\Big[&-\left({}^4\overline{\alpha}_{1G}+i\ {}^4\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2}+\left({}^4\overline{\alpha}_{2G}+i\ {}^4\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}\Big]\\ &-\chi_2\Big[\left({}^4\overline{\beta}_{1G}+i\ {}^4\overline{\beta}_{1S}\right)e^{-\upsilon'^{(1)}x_2}+\left({}^4\overline{\beta}_{2G}+i\ {}^4\overline{\beta}_{2S}\right)e^{\upsilon'^{(1)}x_2}\Big]\Big\}e^{-ikx_1}\end{aligned} \tag{E5.99}$$

$$\begin{aligned}{}^4C^{(1)}(k)=\Big\{-\chi_3\Big[&\left({}^4\overline{\alpha}_{1G}+i\ {}^4\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2}+\left({}^4\overline{\alpha}_{2G}+i\ {}^4\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}\Big]\\ &+\chi_4\Big[-\left({}^4\overline{\beta}_{1G}+i\ {}^4\overline{\beta}_{1S}\right)e^{-\upsilon'^{(1)}x_2}+\left({}^4\overline{\beta}_{2G}+i\ {}^4\overline{\beta}_{2S}\right)e^{\upsilon'^{(1)}x_2}\Big]\Big\}e^{-ikx_1}\end{aligned} \tag{E5.100}$$

$$^4\Theta^{(2)}(k)=-k\left({}^4\overline{\alpha}_{3G}+i\ {}^4\overline{\alpha}_{3S}\right)e^{-\upsilon^{(2)}x_2}e^{-ikx_1}-i\hat{\upsilon}'^{(2)}\left({}^4\overline{\beta}_{3G}+i\ {}^4\overline{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)} \tag{E5.101}$$

$$ {}^4\Omega^{(2)}(k) = -\upsilon^{(2)}\left({}^4\bar{\alpha}_{3G} + i\,{}^4\bar{\alpha}_{3S}\right)e^{-\upsilon^{(2)}x_2}e^{-ikx_1} - k\left({}^4\bar{\beta}_{3G} + i\,{}^4\bar{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)} \quad \text{(E5.102)} $$

$$ {}^4A^{(2)}(k) = \chi_{10}\left({}^4\bar{\alpha}_{3G} + i\,{}^4\bar{\alpha}_{3S}\right)e^{-\upsilon^{(2)}x_2}e^{-ikx_1} - i\hat{\chi}'_6\left({}^4\bar{\beta}_{3G} + i\,{}^4\bar{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)} \quad \text{(E5.103)} $$

$$ {}^4B^{(2)}(k) = \chi_5\left({}^4\bar{\alpha}_{3G} + i\,{}^4\bar{\alpha}_{3S}\right)e^{-\upsilon^{(2)}x_2}e^{-ikx_1} + i\hat{\chi}'_6\left({}^4\bar{\beta}_{3G} + i\,{}^4\bar{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)} \quad \text{(E5.104)} $$

$$ {}^4C^{(2)}(k) = \chi_7\left({}^4\bar{\alpha}_{3G} + i\,{}^4\bar{\alpha}_{3S}\right)e^{-\upsilon^{(2)}x_2}e^{-ikx_1} + \chi_8\left({}^4\bar{\beta}_{3G} + i\,{}^4\bar{\beta}_{3S}\right)e^{-i\left(\hat{\upsilon}'^{(2)}x_2+kx_1\right)} \quad \text{(E5.105)} $$

bulunur. (3.133)-(3.142) uyarınca (E5.96), (E5.100), (E5.101) ve (E5.105) eşitliklerinin sanal, (E5.97), (E5.98), (E5.99), (E5.102), (E5.103) ve (E5.104) eşitliklerinin de gerçel kısımları kullanılacaktır. Buna göre,

$$ \begin{aligned} {}^4\Theta_S^{(1)}(k) = k\Big\{ & -e^{-\upsilon^{(1)}x_2}\left[{}^4\bar{\alpha}_{1G}\sin(kx_1) - {}^4\bar{\alpha}_{1S}\cos(kx_1)\right] \\ & + e^{\upsilon^{(1)}x_2}\left[{}^4\bar{\alpha}_{2G}\sin(kx_1) - {}^4\bar{\alpha}_{2S}\cos(kx_1)\right]\Big\} \\ - \upsilon'^{(1)}\Big\{ & e^{-\upsilon'^{(1)}x_2}\left[{}^4\bar{\beta}_{1G}\sin(kx_1) - {}^4\bar{\beta}_{1S}\cos(kx_1)\right] \\ & + e^{\upsilon'^{(1)}x_2}\left[{}^4\bar{\beta}_{2G}\sin(kx_1) - {}^4\bar{\beta}_{2S}\cos(kx_1)\right]\Big\} \end{aligned} \quad \text{(E5.106)} $$

$$ \begin{aligned} {}^4\Omega_G^{(1)}(k) = \upsilon^{(1)}\Big\{ & e^{-\upsilon^{(1)}x_2}\left[{}^4\bar{\alpha}_{1G}\cos(kx_1) + {}^4\bar{\alpha}_{1S}\sin(kx_1)\right] \\ & + e^{\upsilon^{(1)}x_2}\left[{}^4\bar{\alpha}_{2G}\cos(kx_1) + {}^4\bar{\alpha}_{2S}\sin(kx_1)\right]\Big\} \\ + k\Big\{ & e^{-\upsilon'^{(1)}x_2}\left[{}^4\bar{\beta}_{1G}\cos(kx_1) + {}^4\bar{\beta}_{1S}\sin(kx_1)\right] \\ & - e^{\upsilon'^{(1)}x_2}\left[{}^4\bar{\beta}_{2G}\cos(kx_1) + {}^4\bar{\beta}_{2S}\sin(kx_1)\right]\Big\} \end{aligned} \quad \text{(E5.107)} $$

$$ \begin{aligned} {}^4A_G^{(1)}(k) = \chi_9\Big\{ & -e^{-\upsilon^{(1)}x_2}\left[{}^4\bar{\alpha}_{1G}\cos(kx_1) + {}^4\bar{\alpha}_{1S}\sin(kx_1)\right] \\ & + e^{\upsilon^{(1)}x_2}\left[{}^4\bar{\alpha}_{2G}\cos(kx_1) + {}^4\bar{\alpha}_{2S}\sin(kx_1)\right]\Big\} \\ + \chi_2\Big\{ & e^{-\upsilon'^{(1)}x_2}\left[{}^4\bar{\beta}_{1G}\cos(kx_1) + {}^4\bar{\beta}_{1S}\sin(kx_1)\right] \\ & + e^{\upsilon'^{(1)}x_2}\left[{}^4\bar{\beta}_{2G}\cos(kx_1) + {}^4\bar{\beta}_{2S}\sin(kx_1)\right]\Big\} \end{aligned} \quad \text{(E5.108)} $$

$$ \begin{aligned} {}^4B_G^{(1)}(k) = \chi_1\Big\{ & -e^{-\upsilon^{(1)}x_2}\left[{}^4\bar{\alpha}_{1G}\cos(kx_1) + {}^4\bar{\alpha}_{1S}\sin(kx_1)\right] \\ & + e^{\upsilon^{(1)}x_2}\left[{}^4\bar{\alpha}_{2G}\cos(kx_1) + {}^4\bar{\alpha}_{2S}\sin(kx_1)\right]\Big\} \\ - \chi_2\Big\{ & e^{-\upsilon'^{(1)}x_2}\left[{}^4\bar{\beta}_{1G}\cos(kx_1) + {}^4\bar{\beta}_{1S}\sin(kx_1)\right] \\ & + e^{\upsilon'^{(1)}x_2}\left[{}^4\bar{\beta}_{2G}\cos(kx_1) + {}^4\bar{\beta}_{2S}\sin(kx_1)\right]\Big\} \end{aligned} \quad \text{(E5.109)} $$

$$ \begin{aligned} {}^4C_S^{(1)}(k) = \chi_3\Big\{ & e^{-\upsilon^{(1)}x_2}\left[{}^4\bar{\alpha}_{1G}\sin(kx_1) - {}^4\bar{\alpha}_{1S}\cos(kx_1)\right] \\ & + e^{\upsilon^{(1)}x_2}\left[{}^4\bar{\alpha}_{2G}\sin(kx_1) - {}^4\bar{\alpha}_{2S}\cos(kx_1)\right]\Big\} \\ + \chi_4\Big\{ & e^{-\upsilon'^{(1)}x_2}\left[{}^4\bar{\beta}_{1G}\sin(kx_1) - {}^4\bar{\beta}_{1S}\cos(kx_1)\right] \\ & - e^{\upsilon'^{(1)}x_2}\left[{}^4\bar{\beta}_{2G}\sin(kx_1) - {}^4\bar{\beta}_{2S}\cos(kx_1)\right]\Big\} \end{aligned} \quad \text{(E5.110)} $$

$$ \begin{aligned} {}^4\Theta_S^{(2)}(k) = {} & ke^{-\upsilon^{(2)}x_2}\left[{}^4\bar{\alpha}_{3G}\sin(kx_1) - {}^4\bar{\alpha}_{3S}\cos(kx_1)\right] \\ & - \hat{\upsilon}'^{(2)}\left[{}^4\bar{\beta}_{3G}\cos\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right) + {}^4\bar{\beta}_{3S}\sin\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)\right] \end{aligned} \quad \text{(E5.111)} $$

$$ \begin{aligned} {}^4\Omega_G^{(2)}(k) = {} & -\upsilon^{(2)}e^{-\upsilon^{(2)}x_2}\left[{}^4\bar{\alpha}_{3G}\cos(kx_1) + {}^4\bar{\alpha}_{3S}\sin(kx_1)\right] \\ & - k\left[{}^4\bar{\beta}_{3G}\cos\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right) + {}^4\bar{\beta}_{3S}\sin\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)\right] \end{aligned} \quad \text{(E5.112)} $$

$$ \begin{aligned} {}^4A_G^{(2)}(k) = {} & \chi_{10}e^{-\upsilon^{(2)}x_2}\left[{}^4\bar{\alpha}_{3G}\cos(kx_1) + {}^4\bar{\alpha}_{3S}\sin(kx_1)\right] \\ & + \hat{\chi}'_6\left[-{}^4\bar{\beta}_{3G}\sin\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right) + {}^4\bar{\beta}_{3S}\cos\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)\right] \end{aligned} \quad \text{(E5.113)} $$

$$ {}^4B_G^{(2)}(k) = \chi_5 e^{-\upsilon^{(2)}x_2}\left[{}^4\overline{\alpha}_{3G}\cos(kx_1) + {}^4\overline{\alpha}_{3S}\sin(kx_1)\right] + \hat{\chi}_6'\left[{}^4\overline{\beta}_{3G}\sin\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right) - {}^4\overline{\beta}_{3S}\cos\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)\right] \tag{E5.114} $$

$$ {}^4C_S^{(2)}(k) = \chi_7 e^{-\upsilon^{(2)}x_2}\left[-{}^4\overline{\alpha}_{3G}\sin(kx_1) + {}^4\overline{\alpha}_{3S}\cos(kx_1)\right] + \chi_8\left[-{}^4\overline{\beta}_{3G}\sin\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right) + {}^4\overline{\beta}_{3S}\cos\left(\hat{\upsilon}'^{(2)}x_2 + kx_1\right)\right] \tag{E5.115} $$

yazılıp bu eşitliklerin tek kısımları atıldıktan sonra kalan,

$$ {}^4\Theta_{SÇ}^{(1)}(k) = \left[k\left(-e^{-\upsilon^{(1)}x_2}\ {}^4\overline{\alpha}_{1G} + e^{\upsilon^{(1)}x_2}\ {}^4\overline{\alpha}_{2G}\right) - \upsilon'^{(1)}\left(e^{-\upsilon'^{(1)}x_2}\ {}^4\overline{\beta}_{1G} + e^{\upsilon'^{(1)}x_2}\ {}^4\overline{\beta}_{2G}\right)\right]\sin(kx_1) \tag{E5.116} $$

$$ {}^4\Omega_{GÇ}^{(1)}(k) = \left[\upsilon^{(1)}\left(e^{-\upsilon^{(1)}x_2}\ {}^4\overline{\alpha}_{1G} + e^{\upsilon^{(1)}x_2}\ {}^4\overline{\alpha}_{2G}\right) + k\left(e^{-\upsilon'^{(1)}x_2}\ {}^4\overline{\beta}_{1G} - e^{\upsilon'^{(1)}x_2}\ {}^4\overline{\beta}_{2G}\right)\right]\cos(kx_1) \tag{E5.117} $$

$$ {}^4A_{GÇ}^{(1)}(k) = \left[\chi_9\left(-e^{-\upsilon^{(1)}x_2}\ {}^4\overline{\alpha}_{1G} + e^{\upsilon^{(1)}x_2}\ {}^4\overline{\alpha}_{2G}\right) + \chi_2\left(e^{-\upsilon'^{(1)}x_2}\ {}^4\overline{\beta}_{1G} + e^{\upsilon'^{(1)}x_2}\ {}^4\overline{\beta}_{2G}\right)\right]\cos(kx_1) \tag{E5.118} $$

$$ {}^4B_{GÇ}^{(1)}(k) = \left[\chi_1\left(-e^{-\upsilon^{(1)}x_2}\ {}^4\overline{\alpha}_{1G} + e^{\upsilon^{(1)}x_2}\ {}^4\overline{\alpha}_{2G}\right) - \chi_2\left(e^{-\upsilon'^{(1)}x_2}\ {}^4\overline{\beta}_{1G} + e^{\upsilon'^{(1)}x_2}\ {}^4\overline{\beta}_{2G}\right)\right]\cos(kx_1) \tag{E5.119} $$

$$ {}^4C_{SÇ}^{(1)}(k) = \left[\chi_3\left(e^{-\upsilon^{(1)}x_2}\ {}^4\overline{\alpha}_{1G} + e^{\upsilon^{(1)}x_2}\ {}^4\overline{\alpha}_{2G}\right) + \chi_4\left(e^{-\upsilon'^{(1)}x_2}\ {}^4\overline{\beta}_{1G} - e^{\upsilon'^{(1)}x_2}\ {}^4\overline{\beta}_{2G}\right)\right]\sin(kx_1) \tag{E5.120} $$

$$ \begin{aligned} {}^4\Theta_{SÇ}^{(2)}(k) = {} & k\, e^{\upsilon^{(2)}(h-x_2)}\ {}^4\tilde{\overline{\alpha}}_{3G}\sin(kx_1) \\ & + \hat{\upsilon}'^{(2)}\Big\{\left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3S}\right]\sin\left(\hat{\upsilon}'^{(2)}x_2\right) \\ & - \left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3G}\right]\cos\left(\hat{\upsilon}'^{(2)}x_2\right)\Big\}\sin(kx_1) \end{aligned} \tag{E5.121} $$

$$ \begin{aligned} {}^4\Omega_{GÇ}^{(2)}(k) = {} & -\upsilon^{(2)}e^{\upsilon^{(2)}(h-x_2)}\ {}^4\tilde{\overline{\alpha}}_{3G}\cos(kx_1) \\ & - k\Big\{\left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3S}\right]\cos\left(\hat{\upsilon}'^{(2)}x_2\right) \\ & + \left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3G}\right]\sin\left(\hat{\upsilon}'^{(2)}x_2\right)\Big\}\cos(kx_1) \end{aligned} \tag{E5.122} $$

$$ \begin{aligned} {}^4A_{GÇ}^{(2)}(k) = {} & \chi_{10}e^{\upsilon^{(2)}(h-x_2)}\ {}^4\tilde{\overline{\alpha}}_{3G}\cos(kx_1) \\ & + \hat{\chi}_6'\Big\{-\left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3S}\right]\sin\left(\hat{\upsilon}'^{(2)}x_2\right) \\ & + \left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3G}\right]\cos\left(\hat{\upsilon}'^{(2)}x_2\right)\Big\}\cos(kx_1) \end{aligned} \tag{E5.123} $$

$$ \begin{aligned} {}^4B_{GÇ}^{(2)}(k) = {} & \chi_5\, e^{\upsilon^{(2)}(h-x_2)}\ {}^4\tilde{\overline{\alpha}}_{3G}\cos(kx_1) \\ & + \hat{\chi}_6'\Big\{\left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3S}\right]\sin\left(\hat{\upsilon}'^{(2)}x_2\right) \\ & - \left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3G}\right]\cos\left(\hat{\upsilon}'^{(2)}x_2\right)\Big\}\cos(kx_1) \end{aligned} \tag{E5.124} $$

$$ \begin{aligned} {}^4C_{SÇ}^{(2)}(k) = {} & -\chi_7\, e^{\upsilon^{(2)}(h-x_2)}\ {}^4\tilde{\overline{\alpha}}_{3G}\sin(kx_1) \\ & - \chi_8\Big\{\left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3S}\right]\cos\left(\hat{\upsilon}'^{(2)}x_2\right) \\ & + \left[\cos(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^4\tilde{\overline{\beta}}_{3G}\right]\sin\left(\hat{\upsilon}'^{(2)}x_2\right)\Big\}\sin(kx_1) \end{aligned} \tag{E5.125} $$

bağıntıları (3.133)-(3.142) eşitliklerinde kullanılırsa sonuç olarak,

$$^{4}\bar{u}_1^{(1)} = -\frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}} {}^{4}\Theta_{SÇ}^{(1)}(k)\,dk \tag{E5.126}$$

$$^{4}\bar{u}_2^{(1)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}} {}^{4}\Omega_{GÇ}^{(1)}(k)\,dk \tag{E5.127}$$

$$^{4}\sigma_{11}^{(1)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}} {}^{4}A_{GÇ}^{(1)}(k)\,dk \tag{E5.128}$$

$$^{4}\sigma_{22}^{(1)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}} {}^{4}B_{GÇ}^{(1)}(k)\,dk \tag{E5.129}$$

$$^{4}\sigma_{12}^{(1)} = -\frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}} {}^{4}C_{SÇ}^{(1)}(k)\,dk \tag{E5.130}$$

$$^{4}\bar{u}_1^{(2)} = -\frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}} {}^{4}\Theta_{SÇ}^{(2)}(k)\,dk \tag{E5.131}$$

$$^{4}\bar{u}_2^{(2)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}} {}^{4}\Omega_{GÇ}^{(2)}(k)\,dk \tag{E5.132}$$

$$^{4}\sigma_{11}^{(2)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}} {}^{4}A_{GÇ}^{(2)}(k)\,dk \tag{E5.133}$$

$$^{4}\sigma_{22}^{(2)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}} {}^{4}B_{GÇ}^{(2)}(k)\,dk \tag{E5.134}$$

$$^{4}\sigma_{12}^{(2)} = -\frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}} {}^{4}C_{SÇ}^{(2)}(k)\,dk \tag{E5.135}$$

bulunur.

EK 6. $k_T^2/\sqrt{1+\eta_2^{(2)}} < k < \infty$ **Durumu.**

Bu durumda (3.52)-(3.53), (E1.1)-(E1.36), (3.85), ve (E1.49)-(E1.54) eşitlikleri geçerli olup buna göre (3.123)-(3.132) yardımı ile,

$$ {}^5\Theta^{(1)}(k) = \left[k\,(\overline{\alpha}_1 H_1 - \overline{\alpha}_2 H_2) + \upsilon'^{(1)}(\overline{\beta}_1 H_3 + \overline{\beta}_2 H_4)\right]e^{-ikx_1} \tag{E6.1} $$

$$ {}^5\Omega^{(1)}(k) = \left[\upsilon^{(1)}(\overline{\alpha}_1 H_1 + \overline{\alpha}_2 H_2) + k(\overline{\beta}_1 H_3 - \overline{\beta}_2 H_4)\right]e^{-ikx_1} \tag{E6.2} $$

$$ {}^5A^{(1)}(k) = \left[\chi_9(-\overline{\alpha}_1 H_1 + \overline{\alpha}_2 H_2) + \chi_2(\overline{\beta}_1 H_3 + \overline{\beta}_2 H_4)\right]e^{-ikx_1} \tag{E6.3} $$

$$ {}^5B^{(1)}(k) = \left[\chi_1(-\overline{\alpha}_1 H_1 + \overline{\alpha}_2 H_2) - \chi_2(\overline{\beta}_1 H_3 + \overline{\beta}_2 H_4)\right]e^{-ikx_1} \tag{E6.4} $$

$$ {}^5C^{(1)}(k) = \left[-\chi_3(\overline{\alpha}_1 H_1 + \overline{\alpha}_2 H_2) + \chi_4(-\overline{\beta}_1 H_3 + \overline{\beta}_2 H_4)\right]e^{-ikx_1} \tag{E6.5} $$

$$ {}^5\Theta^{(2)}(k) = -\left(k\,\overline{\alpha}_3 H_5 + \upsilon'^{(2)}\,\overline{\beta}_3 H_6\right)e^{-ikx_1} \tag{E6.6} $$

$$ {}^5\Omega^{(2)}(k) = -\left(\upsilon^{(2)}\,\overline{\alpha}_3 H_5 + k\,\overline{\beta}_3 H_6\right)e^{-ikx_1} \tag{E6.7} $$

$$ {}^5A^{(2)}(k) = \left(\chi_{10}\,\overline{\alpha}_3 H_5 - \chi_6\,\overline{\beta}_3 H_6\right)e^{-ikx_1} \tag{E6.8} $$

$$ {}^5B^{(2)}(k) = \left(\chi_5\,\overline{\alpha}_3 H_5 + \chi_6\,\overline{\beta}_3 H_6\right)e^{-ikx_1} \tag{E6.9} $$

$$ {}^5C^{(2)}(k) = \left(\chi_7\,\overline{\alpha}_3 H_5 + \chi_8\,\overline{\beta}_3 H_6\right)e^{-ikx_1} \tag{E6.10} $$

bulunur ve (3.133)-(3.142) eşitlikleri yardımı ile,

$$ {}^5\overline{u}_1^{(1)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}}^{\infty} {}^5\Theta_{SÇ}^{(1)}(k)\,dk \tag{E6.11} $$

$$ {}^5\overline{u}_2^{(1)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}}^{\infty} {}^5\Omega_{GÇ}^{(1)}(k)\,dk \tag{E6.12} $$

$$ {}^5\sigma_{11}^{(1)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}}^{\infty} {}^5A_{GÇ}^{(1)}(k)\,dk \tag{E6.13} $$

$$ {}^5\sigma_{22}^{(1)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}}^{\infty} {}^5B_{GÇ}^{(1)}(k)\,dk \tag{E6.14} $$

$$ {}^5\sigma_{12}^{(1)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\infty} {}^5C_{SÇ}^{(1)}(k)\,dk \tag{E6.15} $$

$$ {}^5\overline{u}_1^{(2)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}}^{\infty} {}^5\Theta_{SÇ}^{(2)}(k)\,dk \tag{E6.16} $$

$$ {}^{5}\overline{u}_{2}^{(2)} = \frac{P_0}{\pi} \int\limits_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\infty} {}^{5}\Omega_{GÇ}^{(2)}(k)\,dk \tag{E6.17} $$

$$ {}^{5}\sigma_{11}^{(2)} = \frac{P_0}{\pi} \int\limits_{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}}^{\infty} {}^{5}A_{GÇ}^{(2)}(k)\,dk \tag{E6.18} $$

$$ {}^{5}\sigma_{22}^{(2)} = \frac{P_0}{\pi} \int\limits_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\infty} {}^{5}B_{GÇ}^{(2)}(k)\,dk \tag{E6.19} $$

$$ {}^{5}\sigma_{12}^{(2)} = \frac{P_0}{\pi} \int\limits_{\frac{k_T^{(2)}}{\sqrt{1+\eta_2^{(2)}}}}^{\infty} {}^{5}C_{SÇ}^{(2)}(k)\,dk \tag{E6.20} $$

bulunur.

EK 7. $k_T^{(1)}/\sqrt{1+\eta_2^{(1)}} < k < k_L^{(2)}/\sqrt{1+\eta_1^{(2)}}$ **Durumu.**

Bu durumda (3.52) ve (3.53) eşitliklerinden hareketle,

$$\upsilon^{(1)} = \left[k^2(1+\eta_1^{(1)}) - k_L^{(1)2}\right]^{1/2} \tag{E7.1}$$
$$\upsilon'^{(1)} = \left[k^2(1+\eta_2^{(1)}) - k_T^{(1)2}\right]^{1/2} \tag{E7.2}$$
$$\upsilon^{(2)} = \left[k^2(1+\eta_1^{(2)}) - k_L^{(2)2}\right]^{1/2} = i\left[k_L^{(2)2} - k^2(1+\eta_1^{(2)})\right]^{1/2} = i\,\hat{\upsilon}^{(2)} \tag{E7.3}$$
$$\upsilon'^{(2)} = \left[k^2(1+\eta_2^{(2)}) - k_T^{(2)2}\right]^{1/2} = i\left[k_T^{(2)2} - k^2(1+\eta_2^{(2)})\right]^{1/2} = i\,\hat{\upsilon}'^{(2)} \tag{E7.4}$$

eşitlikleri kullanılır. Buna göre,

$$^6\chi_1 = \chi_1 \tag{E7.5}$$
$$^6\chi_2 = \chi_2 \tag{E7.6}$$
$$^6\chi_3 = \chi_3 \tag{E7.7}$$
$$^6\chi_4 = \chi_4 \tag{E7.8}$$
$$^6\chi_5 = \hat{\chi}_5 \tag{E7.9}$$
$$^6\chi_6 = i\hat{\chi}'_6 \tag{E7.10}$$
$$^6\chi_7 = i\hat{\chi}'_7 \tag{E7.11}$$
$$^6\chi_8 = \chi_8 \tag{E7.12}$$
$$^6\chi_9 = \chi_9 \tag{E7.13}$$
$$^6\chi_{10} = \hat{\chi}_{10} \tag{E7.14}$$

bulunur. Ayrıca,

$$^6H_1 = H_1 = e^{-\upsilon^{(1)}x_2} \tag{E7.15}$$
$$^6H_2 = H_2 = e^{\upsilon^{(1)}x_2} \tag{E7.16}$$
$$^6H_3 = H_3 = e^{-\upsilon'^{(1)}x_2} \tag{E7.17}$$
$$^6H_4 = H_4 = e^{\upsilon'^{(1)}x_2} \tag{E7.18}$$
$$^6H_5 = \hat{H}_5 = e^{-i\hat{\upsilon}^{(2)}x_2} \tag{E7.19}$$
$$^6H_6 = \hat{H}_6 = e^{-i\hat{\upsilon}'^{(2)}x_2} \tag{E7.20}$$

ve,

$$^6\overline{H}_1 = \overline{H}_1 = e^{-\upsilon^{(1)}h} \tag{E7.21}$$
$$^6\overline{H}_2 = \overline{H}_2 = e^{\upsilon^{(1)}h} \tag{E7.22}$$
$$^6\overline{H}_3 = \overline{H}_3 = e^{-\upsilon'^{(1)}h} \tag{E7.23}$$
$$^6\overline{H}_4 = \overline{H}_4 = e^{\upsilon'^{(1)}h} \tag{E7.24}$$
$$^6\overline{H}_5 = \hat{\overline{H}}_5 = e^{-i\hat{\upsilon}^{(2)}h} \tag{E7.25}$$
$$^6\overline{H}_6 = \hat{\overline{H}}_6 = e^{-i\hat{\upsilon}'^{(2)}h} \tag{E7.26}$$

olup bütün bu eşitlikler (E1.23)-(E1.36) eşitliklerinde kullanılırsa,

$$\begin{aligned}{}^{6}\Gamma_{1G} &= k^{2}(\chi_4-\chi_8)(\chi_1-\hat{\chi}_5)+k^{2}(\chi_2\chi_3+\hat{\chi}_6'\hat{\chi}_7')\\ &\quad -k\chi_4\hat{\chi}_6'\hat{\upsilon}^{(2)}-k\chi_2\chi_8\upsilon^{(1)}-k\chi_1\hat{\chi}_7'\hat{\upsilon}'^{(2)}-k\chi_3\hat{\chi}_5\upsilon'^{(1)}\\ &\quad +\upsilon'^{(1)}\upsilon^{(1)}(\hat{\chi}_5\chi_8+\hat{\chi}_6'\hat{\chi}_7')-\upsilon'^{(1)}\hat{\upsilon}^{(2)}\chi_3\hat{\chi}_6'\\ &\quad -\hat{\upsilon}'^{(2)}\upsilon^{(1)}\chi_2\hat{\chi}_7'+\hat{\upsilon}'^{(2)}\hat{\upsilon}^{(2)}(\chi_1\chi_4+\chi_2\chi_3)\end{aligned}\tag{E7.27}$$

$$\begin{aligned}{}^{6}\Gamma_{2G} &= -2\upsilon'^{(1)}k(\hat{\chi}_5\chi_8+\hat{\chi}_6'\hat{\chi}_7')+2\upsilon'^{(1)}\chi_4(k\hat{\chi}_5+\hat{\upsilon}^{(2)}\hat{\chi}_6')\\ &\quad +2k^{2}(\chi_2\chi_8-\chi_2\chi_4)-2\hat{\upsilon}'^{(2)}\chi_2(\hat{\upsilon}^{(2)}\chi_4-k\hat{\chi}_7')\end{aligned}\tag{E7.28}$$

$$\begin{aligned}{}^{6}\Gamma_{3G} &= k^{2}(\chi_4-\chi_8)(\chi_1-\hat{\chi}_5)+k^{2}(-\chi_2\chi_3+\hat{\chi}_6'\hat{\chi}_7')\\ &\quad -k\chi_4\hat{\chi}_6'\hat{\upsilon}^{(2)}+k\chi_2\chi_8\upsilon^{(1)}-k\chi_1\hat{\chi}_7'\hat{\upsilon}'^{(2)}+k\chi_3\hat{\chi}_5\upsilon'^{(1)}\\ &\quad -\upsilon'^{(1)}\upsilon^{(1)}(\hat{\chi}_5\chi_8+\hat{\chi}_6'\hat{\chi}_7')+\upsilon'^{(1)}\hat{\upsilon}^{(2)}\chi_3\hat{\chi}_6'\\ &\quad +\hat{\upsilon}'^{(2)}\upsilon^{(1)}\chi_2\hat{\chi}_7'-\hat{\upsilon}'^{(2)}\hat{\upsilon}^{(2)}(\chi_2\chi_3-\chi_1\chi_4)\end{aligned}\tag{E7.29}$$

$${}^{6}\Gamma_{4G}={}^{6}\Gamma_{1G}\tag{E7.30}$$

$${}^{6}\Gamma_{5G}={}^{6}\Gamma_{3G}\tag{E7.31}$$

$$\begin{aligned}{}^{6}\Gamma_{6G} &= -2\,k\,\upsilon^{(1)}(\hat{\chi}_5\chi_8+\hat{\chi}_6'\hat{\chi}_7')+2\,k\,\chi_3(k\hat{\chi}_5+\hat{\upsilon}^{(2)}\hat{\chi}_6')\\ &\quad +2\,k\,\chi_1(\upsilon^{(1)}\chi_8-k\chi_3)-2\,\hat{\upsilon}'^{(2)}\chi_1(\hat{\upsilon}^{(2)}\chi_3-\upsilon^{(1)}\hat{\chi}_7')\end{aligned}\tag{E7.32}$$

$${}^{6}\Gamma_{7G}=2\,k\,(\chi_8-\chi_4)\,(\upsilon'^{(1)}\chi_1-k\chi_2)\tag{E7.33}$$

$${}^{6}\Gamma_{8G}={}^{6}\Gamma_{7G}\tag{E7.34}$$

$${}^{6}\Gamma_{9G}=2\,(\upsilon^{(1)}\chi_8-k\chi_3)\,(\upsilon'^{(1)}\chi_1-k\chi_2)\tag{E7.35}$$

$${}^{6}\Gamma_{10G}=-{}^{6}\Gamma_{9G}\tag{E7.36}$$

$${}^{6}\Gamma_{11G}=2\,(\upsilon'^{(1)}\hat{\chi}_5-k\chi_2)\,(\upsilon^{(1)}\chi_4-k\chi_3)\tag{E7.37}$$

$${}^{6}\Gamma_{12G}=-{}^{6}\Gamma_{11G}\tag{E7.38}$$

$${}^{6}\Gamma_{13G}=2\,k\,(\chi_1-\hat{\chi}_5)\,(\upsilon^{(1)}\chi_4-k\chi_3)\tag{E7.39}$$

$${}^{6}\Gamma_{14G}={}^{6}\Gamma_{13G}\tag{E7.40}$$

ve,

$$\begin{aligned}{}^{6}\Gamma_{1S} &= k^{2}(\chi_2\hat{\chi}_7'-\chi_3\hat{\chi}_6')+\upsilon^{(1)}\chi_4(k\hat{\chi}_6'-\hat{\upsilon}'^{(2)}\hat{\chi}_5)+\hat{\upsilon}^{(2)}\chi_8\,(\upsilon'^{(1)}\chi_1-k\chi_2)\\ &\quad -k\chi_1\hat{\chi}_7'\upsilon'^{(1)}+k\chi_3\hat{\chi}_5\hat{\upsilon}'^{(2)}\end{aligned}\tag{E7.41}$$

$${}^{6}\Gamma_{2S}=0\tag{E7.42}$$

$$\begin{aligned}{}^{6}\Gamma_{3S} &= -k^{2}(\chi_2\hat{\chi}_7'+\chi_3\hat{\chi}_6')+\upsilon^{(1)}\chi_4(k\hat{\chi}_6'-\hat{\upsilon}'^{(2)}\hat{\chi}_5)-\hat{\upsilon}^{(2)}\chi_8\,(\upsilon'^{(1)}\chi_1-k\chi_2)\\ &\quad +k\chi_1\hat{\chi}_7'\upsilon'^{(1)}+k\chi_3\hat{\chi}_5\hat{\upsilon}'^{(2)}\end{aligned}\tag{E7.43}$$

$${}^{6}\Gamma_{4S}=-{}^{6}\Gamma_{1S}\tag{E7.44}$$

$${}^{6}\Gamma_{5S}=-{}^{6}\Gamma_{3S}\tag{E7.45}$$

$${}^{6}\Gamma_{6S}=0\tag{E7.46}$$

$${}^{6}\Gamma_{7S}=2\,(\upsilon^{(1)}\chi_4-k\chi_3)\,(\hat{\upsilon}'^{(2)}\chi_2-\upsilon'^{(1)}\hat{\chi}_6')\tag{E7.47}$$

$${}^{6}\Gamma_{8S}=-{}^{6}\Gamma_{7S}\tag{E7.48}$$

$${}^{6}\Gamma_{9S}=2\,(\upsilon^{(1)}\chi_4-k\chi_3)\,(\hat{\upsilon}'^{(2)}\chi_1-k\hat{\chi}_6')\tag{E7.49}$$

$${}^{6}\Gamma_{10S}={}^{6}\Gamma_{9S}\tag{E7.50}$$

$${}^{6}\Gamma_{11S}=2\,(\upsilon'^{(1)}\chi_1-k\chi_2)\,(\hat{\upsilon}^{(2)}\chi_4-k\hat{\chi}_7')\tag{E7.51}$$

$$^{6}\Gamma_{12S} = {}^{6}\Gamma_{11S} \tag{E7.52}$$

$$^{6}\Gamma_{13S} = 2(\upsilon^{(1)}\hat{\chi}_7' - \hat{\upsilon}^{(2)}\chi_3)(\upsilon'^{(1)}\chi_1 - k\chi_2) \tag{E7.53}$$

$$^{6}\Gamma_{14S} = -{}^{6}\Gamma_{13S} \tag{E7.54}$$

olmak üzere,

$$^{6}\Gamma_j = {}^{6}\Gamma_{jG} + i\ {}^{6}\Gamma_{jS} \quad ,j=1,14 \tag{E7.55}$$

yazılabilir. Ayrıca (3.85) eşitliği yardımı ile,

$$\begin{aligned} {}^{6}[\Delta] = &(\chi_2\chi_3 + \chi_1\chi_4)\left[\overline{H}_2\overline{H}_3({}^{6}\Gamma_{1G} + i\ {}^{6}\Gamma_{1S}) + \overline{H}_1\overline{H}_4({}^{6}\Gamma_{4G} + i\ {}^{6}\Gamma_{4S})\right] \\ &+ (\chi_2\chi_3 - \chi_1\chi_4)\left[\overline{H}_1\overline{H}_3({}^{6}\Gamma_{5G} + i\ {}^{6}\Gamma_{5S}) + \overline{H}_2\overline{H}_4({}^{6}\Gamma_{3G} + i\ {}^{6}\Gamma_{3S})\right] \\ &+ 2\left(\chi_1\chi_3\,{}^{6}\Gamma_{2G} + \chi_2\chi_4\,{}^{6}\Gamma_{6G}\right) \end{aligned} \tag{E7.56}$$

ve k dönüşüm parametresine göre,

${}^{6}\Gamma_{1G}, {}^{6}\Gamma_{3G}, {}^{6}\Gamma_{4G}, {}^{6}\Gamma_{5G}, {}^{6}\Gamma_{9G}, {}^{6}\Gamma_{10G}, {}^{6}\Gamma_{11G}, {}^{6}\Gamma_{12G}$ ve ${}^{6}\Gamma_{1S}, {}^{6}\Gamma_{3S}, {}^{6}\Gamma_{4S}, {}^{6}\Gamma_{5S}, {}^{6}\Gamma_{9S}, {}^{6}\Gamma_{10S}, {}^{6}\Gamma_{11S}, {}^{6}\Gamma_{12S}$; çift, ${}^{6}\Gamma_{2G}, {}^{6}\Gamma_{6G}, {}^{6}\Gamma_{7G}, {}^{6}\Gamma_{8G}, {}^{6}\Gamma_{13G}, {}^{6}\Gamma_{14G}$ ve ${}^{6}\Gamma_{7S}, {}^{6}\Gamma_{8S}, {}^{6}\Gamma_{13S}, {}^{6}\Gamma_{14S}$; tek fonksiyonlar ve ${}^{6}\Gamma_{2S}, {}^{6}\Gamma_{6S} = 0$ olduğuna göre,

$$\begin{aligned} {}^{6}[\Delta]_G = &(\chi_2\chi_3 + \chi_1\chi_4)\left(e^{(\upsilon^{(1)} - \upsilon'^{(1)})h}\ {}^{6}\Gamma_{1G} + e^{-(\upsilon^{(1)} - \upsilon'^{(1)})h}\ {}^{6}\Gamma_{4G}\right) \\ &+ (\chi_2\chi_3 - \chi_1\chi_4)\left(e^{-(\upsilon^{(1)} + \upsilon'^{(1)})h}\ {}^{6}\Gamma_{5G} + e^{(\upsilon^{(1)} + \upsilon'^{(1)})h}\ {}^{6}\Gamma_{3G}\right) \\ &+ 2\left(\chi_1\chi_3\ {}^{6}\Gamma_{2G} + \chi_2\chi_4\ {}^{6}\Gamma_{6G}\right) \end{aligned} \tag{E7.57}$$

ve,

$$\begin{aligned} {}^{6}[\Delta]_S = &(\chi_2\chi_3 + \chi_1\chi_4)\left(e^{(\upsilon^{(1)} - \upsilon'^{(1)})h}\ {}^{6}\Gamma_{1S} + e^{-(\upsilon^{(1)} - \upsilon'^{(1)})h}\ {}^{6}\Gamma_{4S}\right) \\ &+ (\chi_2\chi_3 - \chi_1\chi_4)\left(e^{-(\upsilon^{(1)} + \upsilon'^{(1)})h}\ {}^{6}\Gamma_{5S} + e^{(\upsilon^{(1)} + \upsilon'^{(1)})h}\ {}^{6}\Gamma_{3S}\right) \end{aligned} \tag{E7.58}$$

olmak üzere,

$${}^{6}[\Delta] = {}^{6}[\Delta]_G + i\ {}^{6}[\Delta]_S \tag{E7.59}$$

yazılabilir. (E7.59) eşitliğinin gerçel ve sanal kısımları çift fonksiyonlar olmaktadır. Buradan (E1.49)-(E1.54) yardımı ile,

$${}^{6}\overline{\alpha}_1 = \frac{\chi_4 e^{\upsilon'^{(1)}h}\left[e^{-\upsilon'^{(1)}h}({}^{6}\Gamma_{1G} + i\ {}^{6}\Gamma_{1S}) - e^{\upsilon'^{(1)}h}({}^{6}\Gamma_{3G} + i\ {}^{6}\Gamma_{3S})\right] + \chi_3\ {}^{6}\Gamma_{2G}}{{}^{6}[\Delta]} \tag{E7.60}$$

$${}^{6}\overline{\alpha}_2 = \frac{\chi_4 e^{-\upsilon^{(1)}h}\left[e^{-\upsilon'^{(1)}h}({}^{6}\Gamma_{5G} + i\ {}^{6}\Gamma_{5S}) - e^{\upsilon'^{(1)}h}({}^{6}\Gamma_{4G} + i\ {}^{6}\Gamma_{4S})\right] - \chi_3\ {}^{6}\Gamma_{2G}}{{}^{6}[\Delta]} \tag{E7.61}$$

$${}^{6}\overline{\beta}_1 = \frac{\chi_3 e^{\upsilon'^{(1)}h}\left[e^{\upsilon^{(1)}h}({}^{6}\Gamma_{3G} + i\ {}^{6}\Gamma_{3S}) + e^{-\upsilon^{(1)}h}({}^{6}\Gamma_{4G} + i\ {}^{6}\Gamma_{4S})\right] + \chi_4\ {}^{6}\Gamma_{6G}}{{}^{6}[\Delta]} \tag{E7.62}$$

$$ {}^6\overline{\beta}_2 = \frac{\chi_3 e^{-\upsilon'^{(1)}h}\left[e^{\upsilon^{(1)}h}({}^6\Gamma_{1G} + i\ {}^6\Gamma_{1S}) + e^{-\upsilon^{(1)}h}({}^6\Gamma_{5G} + i\ {}^6\Gamma_{5S})\right] + \chi_4\ {}^6\Gamma_{6G}}{{}^6[\Delta]} \qquad \text{(E5.63)} $$

$$ {}^6\overline{\alpha}_3 = \frac{\chi_3\left[e^{\upsilon^{(1)}h}({}^6\Gamma_{7G} + i\ {}^6\Gamma_{7S}) + e^{-\upsilon^{(1)}h}({}^6\Gamma_{8G} + i\ {}^6\Gamma_{8S})\right]}{e^{-i\hat{\upsilon}^{(2)}h}\ {}^6[\Delta]} + \frac{\chi_4\left[e^{-\upsilon'^{(1)}h}({}^6\Gamma_{10G} + i\ {}^6\Gamma_{10S}) - e^{\upsilon'^{(1)}h}({}^6\Gamma_{9G} + i\ {}^6\Gamma_{9S})\right]}{e^{-i\hat{\upsilon}^{(2)}h}\ {}^6[\Delta]} \qquad \text{(E5.64)} $$

$$ {}^6\overline{\beta}_3 = \frac{\chi_3\left[e^{\upsilon^{(1)}h}({}^6\Gamma_{11G} + i\ {}^6\Gamma_{11S}) + e^{-\upsilon^{(1)}h}({}^6\Gamma_{12G} + i\ {}^6\Gamma_{12S})\right]}{e^{-i\hat{\upsilon}'^{(2)}h}\ {}^6[\Delta]} + \frac{\chi_4\left[e^{\upsilon'^{(1)}h}({}^6\Gamma_{13G} + i\ {}^6\Gamma_{13S}) - e^{-\upsilon'^{(1)}h}({}^6\Gamma_{14G} + i\ {}^6\Gamma_{14S})\right]}{e^{-i\hat{\upsilon}'^{(2)}h}\ {}^6[\Delta]} \qquad \text{(E5.65)} $$

yazılır. (E7.60)-(E7.65) eşitliklerini gerçel ve sanal kısımlarına ayırmak için,

$$ {}^6\overline{\alpha}_{1PG} = \chi_4\left(e^{(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^6\Gamma_{1G} - e^{(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^6\Gamma_{3G}\right) + \chi_3\,{}^6\Gamma_{2G} \qquad \text{(E7.66)} $$

$$ {}^6\overline{\alpha}_{1PS} = \chi_4\left(e^{(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^6\Gamma_{1S} - e^{(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^6\Gamma_{3S}\right) \qquad \text{(E7.67)} $$

$$ {}^6\overline{\alpha}_{2PG} = \chi_4\left(e^{-(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^6\Gamma_{5G} - e^{-(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^6\Gamma_{4G}\right) - \chi_3\,{}^6\Gamma_{2G} \qquad \text{(E7.68)} $$

$$ {}^6\overline{\alpha}_{2PS} = \chi_4\left(e^{-(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^6\Gamma_{5S} - e^{-(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^6\Gamma_{4S}\right) \qquad \text{(E7.69)} $$

$$ {}^6\overline{\beta}_{1PG} = \chi_3\left(e^{(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^6\Gamma_{3G} + e^{-(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^6\Gamma_{4G}\right) + \chi_4\,{}^6\Gamma_{6G} \qquad \text{(E7.70)} $$

$$ {}^6\overline{\beta}_{1PS} = \chi_3\left(e^{(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^6\Gamma_{3S} + e^{-(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^6\Gamma_{4S}\right) \qquad \text{(E7.71)} $$

$$ {}^6\overline{\beta}_{2PG} = \chi_3\left(e^{(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^6\Gamma_{1G} + e^{-(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^6\Gamma_{5G}\right) + \chi_4\,{}^6\Gamma_{6G} \qquad \text{(E7.72)} $$

$$ {}^6\overline{\beta}_{2PS} = \chi_3\left(e^{(\upsilon^{(1)}-\upsilon'^{(1)})h}\ {}^6\Gamma_{1S} + e^{-(\upsilon^{(1)}+\upsilon'^{(1)})h}\ {}^6\Gamma_{5S}\right) \qquad \text{(E7.73)} $$

$$ {}^6\overline{\alpha}_{3PG} = \chi_3\left(e^{\upsilon^{(1)}h}\ {}^6\Gamma_{7G} + e^{-\upsilon^{(1)}h}\ {}^6\Gamma_{8G}\right) + \chi_4\left(e^{-\upsilon'^{(1)}h}\ {}^6\Gamma_{10G} - e^{\upsilon'^{(1)}h}\ {}^6\Gamma_{9G}\right) \qquad \text{(E7.74)} $$

$$ {}^6\overline{\alpha}_{3PS} = \chi_3\left(e^{\upsilon^{(1)}h}\ {}^6\Gamma_{7S} + e^{-\upsilon^{(1)}h}\ {}^6\Gamma_{8S}\right) + \chi_4\left(e^{-\upsilon'^{(1)}h}\ {}^6\Gamma_{10S} - e^{\upsilon'^{(1)}h}\ {}^6\Gamma_{9S}\right) \qquad \text{(E7.75)} $$

$$ {}^6\overline{\beta}_{3PG} = \chi_3\left(e^{\upsilon^{(1)}h}\ {}^6\Gamma_{11G} + e^{-\upsilon^{(1)}h}\ {}^6\Gamma_{12G}\right) + \chi_4\left(e^{\upsilon'^{(1)}h}\ {}^6\Gamma_{13G} - e^{-\upsilon'^{(1)}h}\ {}^6\Gamma_{14G}\right) \qquad \text{(E7.76)} $$

$$ {}^6\overline{\beta}_{3PS} = \chi_3\left(e^{\upsilon^{(1)}h}\ {}^6\Gamma_{11S} + e^{-\upsilon^{(1)}h}\ {}^6\Gamma_{12S}\right) + \chi_4\left(e^{\upsilon'^{(1)}h}\ {}^6\Gamma_{13S} - e^{-\upsilon'^{(1)}h}\ {}^6\Gamma_{14S}\right) \qquad \text{(E7.77)} $$

tanımlamaları yapılırsa,

$$ {}^6\overline{\alpha}_{1G} = \frac{{}^6[\Delta]_G\ {}^6\overline{\alpha}_{1PG} + {}^6[\Delta]_S\ {}^6\overline{\alpha}_{1PS}}{{}^6[\Delta]_G^{\,2} + {}^6[\Delta]_S^{\,2}} \qquad \text{(E7.78)} $$

$$ {}^6\overline{\alpha}_{1S} = \frac{{}^6[\Delta]_G\ {}^6\overline{\alpha}_{1PS} - {}^6[\Delta]_S\ {}^6\overline{\alpha}_{1PG}}{{}^6[\Delta]_G^{\,2} + {}^6[\Delta]_S^{\,2}} \qquad \text{(E7.79)} $$

$$ {}^6\overline{\alpha}_{2G} = \frac{{}^6[\Delta]_G\ {}^6\overline{\alpha}_{2PG} + {}^6[\Delta]_S\ {}^6\overline{\alpha}_{2PS}}{{}^6[\Delta]_G^{\,2} + {}^6[\Delta]_S^{\,2}} \qquad \text{(E7.80)} $$

$$ {}^6\overline{\alpha}_{2S} = \frac{{}^6[\Delta]_G\ {}^6\overline{\alpha}_{2PS} - {}^6[\Delta]_S\ {}^6\overline{\alpha}_{2PG}}{{}^6[\Delta]_G^{\,2} + {}^6[\Delta]_S^{\,2}} \qquad \text{(E7.81)} $$

$$ {}^6\overline{\beta}_{1G} = \frac{{}^6[\Delta]_G\ {}^6\overline{\beta}_{1PG} + {}^6[\Delta]_S\ {}^6\overline{\beta}_{1PS}}{{}^6[\Delta]_G^{\,2} + {}^6[\Delta]_S^{\,2}} \qquad \text{(E7.82)} $$

$$ {}^6\overline{\beta}_{1S} = \frac{{}^6[\Delta]_G\,{}^6\overline{\beta}_{1PS} - {}^6[\Delta]_S\,{}^6\overline{\beta}_{1PG}}{{}^6[\Delta]_G^{\,2} + {}^6[\Delta]_S^{\,2}} \tag{E7.83} $$

$$ {}^6\overline{\beta}_{2G} = \frac{{}^6[\Delta]_G\,{}^6\overline{\beta}_{2PG} + {}^6[\Delta]_S\,{}^6\overline{\beta}_{2PS}}{{}^6[\Delta]_G^{\,2} + {}^6[\Delta]_S^{\,2}} \tag{E7.84} $$

$$ {}^6\overline{\beta}_{2S} = \frac{{}^6[\Delta]_G\,{}^6\overline{\beta}_{2PS} - {}^6[\Delta]_S\,{}^6\overline{\beta}_{2PG}}{{}^6[\Delta]_G^{\,2} + {}^6[\Delta]_S^{\,2}} \tag{E7.85} $$

$$ {}^6\tilde{\overline{\alpha}}_{3G} = \frac{{}^6[\Delta]_G\,{}^6\overline{\alpha}_{3PG} + {}^6[\Delta]_S\,{}^6\overline{\alpha}_{3PS}}{{}^6[\Delta]_G^{\,2} + {}^6[\Delta]_S^{\,2}} \tag{E7.86} $$

$$ {}^6\tilde{\overline{\alpha}}_{3S} = \frac{{}^6[\Delta]_G\,{}^6\overline{\alpha}_{3PS} - {}^6[\Delta]_S\,{}^6\overline{\alpha}_{3PG}}{{}^6[\Delta]_G^{\,2} + {}^6[\Delta]_S^{\,2}} \tag{E7.87} $$

$$ {}^6\tilde{\overline{\beta}}_{3G} = \frac{{}^6[\Delta]_G\,{}^6\overline{\beta}_{3PG} + {}^6[\Delta]_S\,{}^6\overline{\beta}_{3PS}}{{}^6[\Delta]_G^{\,2} + {}^6[\Delta]_S^{\,2}} \tag{E7.88} $$

$$ {}^6\tilde{\overline{\beta}}_{3S} = \frac{{}^6[\Delta]_G\,{}^6\overline{\beta}_{3PS} - {}^6[\Delta]_S\,{}^6\overline{\beta}_{3PG}}{{}^6[\Delta]_G^{\,2} + {}^6[\Delta]_S^{\,2}} \tag{E7.89} $$

olmak üzere,

$$ {}^6\overline{\alpha}_1 = {}^6\overline{\alpha}_{1G} + i\,{}^6\overline{\alpha}_{1S} \tag{E7.90} $$

$$ {}^6\overline{\alpha}_2 = {}^6\overline{\alpha}_{2G} + i\,{}^6\overline{\alpha}_{2S} \tag{E7.91} $$

$$ {}^6\overline{\beta}_1 = {}^6\overline{\beta}_{1G} + i\,{}^6\overline{\beta}_{1S} \tag{E7.92} $$

$$ {}^6\overline{\beta}_2 = {}^6\overline{\beta}_{2G} + i\,{}^6\overline{\beta}_{2S} \tag{E7.93} $$

$$ {}^6\overline{\alpha}_3 = \cos(\hat{\upsilon}^{(2)}h)\,{}^6\tilde{\overline{\alpha}}_{3G} - \sin(\hat{\upsilon}^{(2)}h)\,{}^6\tilde{\overline{\alpha}}_{3S} + i\left[\cos(\hat{\upsilon}^{(2)}h)\,{}^6\tilde{\overline{\alpha}}_{3S} + \sin(\hat{\upsilon}^{(2)}h)\,{}^6\tilde{\overline{\alpha}}_{3G}\right] \tag{E7.94} $$

$$ {}^6\overline{\beta}_3 = \cos(\hat{\upsilon}'^{(2)}h)\,{}^6\tilde{\overline{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\,{}^6\tilde{\overline{\beta}}_{3S} + i\left[\cos(\hat{\upsilon}'^{(2)}h)\,{}^6\tilde{\overline{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\,{}^6\tilde{\overline{\beta}}_{3G}\right] \tag{E7.95} $$

yazılabilir. ${}^6\overline{\alpha}_{iPG}, {}^6\overline{\alpha}_{iPS}$ (i=1,3) fonksiyonları k'ya göre çift, ${}^6\overline{\beta}_{iPG}, {}^6\overline{\beta}_{iPS}$ (i=1,3) ise tek olup, ${}^6[\Delta]_G$ ve ${}^6[\Delta]_S$ fonksiyonları da çift olduklarına göre sonuç olarak ${}^6\overline{\alpha}_{iG}, {}^6\overline{\alpha}_{iS}$ (i=1,3) fonksiyonlarının çift, ${}^6\overline{\beta}_{iG}, {}^6\overline{\beta}_{iS}$ (i=1,3) fonksiyonlarının da tek oldukları görülür. (E7.90)-(E7.95) eşitlikleri (3.123)-(3.132) de kullanılırsa, (E7.1)-(E7.20) yardımı ile,

$$ \begin{aligned} {}^6\Theta^{(1)}(k) = \Big\{ & k\Big[\left({}^6\overline{\alpha}_{1G} + i\,{}^6\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2} - \left({}^6\overline{\alpha}_{2G} + i\,{}^6\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}\Big] \\ & + \upsilon'^{(1)}\Big[\left({}^6\overline{\beta}_{1G} + i\,{}^6\overline{\beta}_{1S}\right)e^{-\upsilon'^{(1)}x_2} + \left({}^6\overline{\beta}_{2G} + i\,{}^6\overline{\beta}_{2S}\right)e^{\upsilon'^{(1)}x_2}\Big]\Big\}e^{-ikx_1} \end{aligned} \tag{E7.96} $$

$$ \begin{aligned} {}^6\Omega^{(1)}(k) = \Big\{ & \upsilon^{(1)}\Big[\left({}^6\overline{\alpha}_{1G} + i\,{}^6\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2} + \left({}^6\overline{\alpha}_{2G} + i\,{}^6\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}\Big] \\ & + k\Big[\left({}^6\overline{\beta}_{1G} + i\,{}^6\overline{\beta}_{1S}\right)e^{-\upsilon'^{(1)}x_2} - \left({}^6\overline{\beta}_{2G} + i\,{}^6\overline{\beta}_{2S}\right)e^{\upsilon'^{(1)}x_2}\Big]\Big\}e^{-ikx_1} \end{aligned} \tag{E7.97} $$

$$ \begin{aligned} {}^6A^{(1)}(k) = \Big\{ & \chi_9\Big[-\left({}^6\overline{\alpha}_{1G} + i\,{}^6\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2} + \left({}^6\overline{\alpha}_{2G} + i\,{}^6\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}\Big] \\ & + \chi_2\Big[\left({}^6\overline{\beta}_{1G} + i\,{}^6\overline{\beta}_{1S}\right)e^{-\upsilon'^{(1)}x_2} + \left({}^6\overline{\beta}_{2G} + i\,{}^6\overline{\beta}_{2S}\right)e^{\upsilon'^{(1)}x_2}\Big]\Big\}e^{-ikx_1} \end{aligned} \tag{E7.98} $$

$$ \begin{aligned} {}^6B^{(1)}(k) = \Big\{ & \chi_1\Big[-\left({}^6\overline{\alpha}_{1G} + i\,{}^6\overline{\alpha}_{1S}\right)e^{-\upsilon^{(1)}x_2} + \left({}^6\overline{\alpha}_{2G} + i\,{}^6\overline{\alpha}_{2S}\right)e^{\upsilon^{(1)}x_2}\Big] \\ & - \chi_2\Big[\left({}^6\overline{\beta}_{1G} + i\,{}^6\overline{\beta}_{1S}\right)e^{-\upsilon'^{(1)}x_2} + \left({}^6\overline{\beta}_{2G} + i\,{}^6\overline{\beta}_{2S}\right)e^{\upsilon'^{(1)}x_2}\Big]\Big\}e^{-ikx_1} \end{aligned} \tag{E7.99} $$

$$
\begin{aligned}
{}^6C^{(1)}(k) = \Big\{ -\chi_3 \Big[\big({}^6\overline{\alpha}_{1G} + i\,{}^6\overline{\alpha}_{1S}\big) e^{-\upsilon^{(1)}x_2} + \big({}^6\overline{\alpha}_{2G} + i\,{}^6\overline{\alpha}_{2S}\big) e^{\upsilon^{(1)}x_2} \Big] \\
+ \chi_4 \Big[-\big({}^6\overline{\beta}_{1G} + i\,{}^6\overline{\beta}_{1S}\big) e^{-\upsilon'^{(1)}x_2} + \big({}^6\overline{\beta}_{2G} + i\,{}^6\overline{\beta}_{2S}\big) e^{\upsilon'^{(1)}x_2} \Big] \Big\} e^{-ikx_1}
\end{aligned}
\tag{E7100}
$$

$$
{}^6\Theta^{(2)}(k) = -\Big[k\big({}^6\overline{\alpha}_{3G} + i\,{}^6\overline{\alpha}_{3S}\big) e^{-i\hat{\upsilon}^{(2)}x_2} + i\hat{\upsilon}'^{(2)}\big({}^6\overline{\beta}_{3G} + i\,{}^6\overline{\beta}_{3S}\big) e^{-i\hat{\upsilon}'^{(2)}x_2} \Big] e^{-ikx_1}
\tag{E7.101}
$$

$$
{}^6\Omega^{(2)}(k) = -\Big[i\hat{\upsilon}^{(2)}\big({}^6\overline{\alpha}_{3G} + i\,{}^6\overline{\alpha}_{3S}\big) e^{-i\hat{\upsilon}^{(2)}x_2} + k\big({}^6\overline{\beta}_{3G} + i\,{}^6\overline{\beta}_{3S}\big) e^{-i\hat{\upsilon}'^{(2)}x_2} \Big] e^{-ikx_1}
\tag{E7.102}
$$

$$
{}^6A^{(2)}(k) = \Big[\hat{\chi}_{10}\big({}^6\overline{\alpha}_{3G} + i\,{}^6\overline{\alpha}_{3S}\big) e^{-i\hat{\upsilon}^{(2)}x_2} - i\hat{\chi}'_6\big({}^6\overline{\beta}_{3G} + i\,{}^6\overline{\beta}_{3S}\big) e^{-i\hat{\upsilon}'^{(2)}x_2} \Big] e^{-ikx_1}
\tag{E7.103}
$$

$$
{}^6B^{(2)}(k) = \Big[\hat{\chi}_5\big({}^6\overline{\alpha}_{3G} + i\,{}^6\overline{\alpha}_{3S}\big) e^{-i\hat{\upsilon}^{(2)}x_2} + i\hat{\chi}'_6\big({}^6\overline{\beta}_{3G} + i\,{}^6\overline{\beta}_{3S}\big) e^{-i\hat{\upsilon}'^{(2)}x_2} \Big] e^{-ikx_1}
\tag{E7.104}
$$

$$
{}^6C^{(2)}(k) = \Big[i\hat{\chi}'_7\big({}^6\overline{\alpha}_{3G} + i\,{}^6\overline{\alpha}_{3S}\big) e^{-i\hat{\upsilon}^{(2)}x_2} + \chi_8\big({}^6\overline{\beta}_{3G} + i\,{}^6\overline{\beta}_{3S}\big) e^{-i\hat{\upsilon}'^{(2)}x_2} \Big] e^{-ikx_1}
\tag{E7.105}
$$

bulunur. (3.133)-(3.142) uyarınca (E7.96), (E7.100), (E7.101) ve (E7.105) eşitliklerinin sanal, (E7.97), (E7.98), (E7.99), (E7.102), (E7.103) ve (E7.104) eşitliklerinin de gerçel kısımları kullanılacaktır. Buna göre,

$$
\begin{aligned}
{}^6\Theta_S^{(1)}(k) = k \Big\{ & e^{-\upsilon^{(1)}x_2}\big[-{}^6\overline{\alpha}_{1G}\sin(kx_1) + {}^6\overline{\alpha}_{1S}\cos(kx_1) \big] \\
& - e^{\upsilon^{(1)}x_2}\big[-{}^6\overline{\alpha}_{2G}\sin(kx_1) + {}^6\overline{\alpha}_{2S}\cos(kx_1) \big] \Big\} \\
+ \upsilon'^{(1)} \Big\{ & e^{-\upsilon'^{(1)}x_2}\big[-{}^6\overline{\beta}_{1G}\sin(kx_1) + {}^6\overline{\beta}_{1S}\cos(kx_1) \big] \\
& + e^{\upsilon'^{(1)}x_2}\big[-{}^6\overline{\beta}_{2G}\sin(kx_1) + {}^6\overline{\beta}_{2S}\cos(kx_1) \big] \Big\}
\end{aligned}
\tag{E7.106}
$$

$$
\begin{aligned}
{}^6\Omega_G^{(1)}(k) = \upsilon^{(1)} \Big\{ & e^{-\upsilon^{(1)}x_2}\big[{}^6\overline{\alpha}_{1G}\cos(kx_1) + {}^6\overline{\alpha}_{1S}\sin(kx_1) \big] \\
& + e^{\upsilon^{(1)}x_2}\big[{}^6\overline{\alpha}_{2G}\cos(kx_1) + {}^6\overline{\alpha}_{2S}\sin(kx_1) \big] \Big\} \\
+ k \Big\{ & e^{-\upsilon'^{(1)}x_2}\big[{}^6\overline{\beta}_{1G}\cos(kx_1) + {}^6\overline{\beta}_{1S}\sin(kx_1) \big] \\
& - e^{\upsilon'^{(1)}x_2}\big[{}^6\overline{\beta}_{2G}\cos(kx_1) + {}^6\overline{\beta}_{2S}\sin(kx_1) \big] \Big\}
\end{aligned}
\tag{E7.107}
$$

$$
\begin{aligned}
{}^6A_G^{(1)}(k) = \chi_9 \Big\{ & -e^{-\upsilon^{(1)}x_2}\big[{}^6\overline{\alpha}_{1G}\cos(kx_1) + {}^6\overline{\alpha}_{1S}\sin(kx_1) \big] \\
& + e^{\upsilon^{(1)}x_2}\big[{}^6\overline{\alpha}_{2G}\cos(kx_1) + {}^6\overline{\alpha}_{2S}\sin(kx_1) \big] \Big\} \\
+ \chi_2 \Big\{ & e^{-\upsilon'^{(1)}x_2}\big[{}^6\overline{\beta}_{1G}\cos(kx_1) + {}^6\overline{\beta}_{1S}\sin(kx_1) \big] \\
& + e^{\upsilon'^{(1)}x_2}\big[{}^6\overline{\beta}_{2G}\cos(kx_1) + {}^6\overline{\beta}_{2S}\sin(kx_1) \big] \Big\}
\end{aligned}
\tag{E7.108}
$$

$$
\begin{aligned}
{}^6B_G^{(1)}(k) = \chi_1 \Big\{ & -e^{-\upsilon^{(1)}x_2}\big[{}^6\overline{\alpha}_{1G}\cos(kx_1) + {}^6\overline{\alpha}_{1S}\sin(kx_1) \big] \\
& + e^{\upsilon^{(1)}x_2}\big[{}^6\overline{\alpha}_{2G}\cos(kx_1) + {}^6\overline{\alpha}_{2S}\sin(kx_1) \big] \Big\} \\
- \chi_2 \Big\{ & e^{-\upsilon'^{(1)}x_2}\big[{}^6\overline{\beta}_{1G}\cos(kx_1) + {}^6\overline{\beta}_{1S}\sin(kx_1) \big] \\
& + e^{\upsilon'^{(1)}x_2}\big[{}^6\overline{\beta}_{2G}\cos(kx_1) + {}^6\overline{\beta}_{2S}\sin(kx_1) \big] \Big\}
\end{aligned}
\tag{E7.109}
$$

$$
\begin{aligned}
{}^6C_S^{(1)}(k) = -\chi_3 \Big\{ & e^{-\upsilon^{(1)}x_2}\big[-{}^6\overline{\alpha}_{1G}\sin(kx_1) + {}^6\overline{\alpha}_{1S}\cos(kx_1) \big] \\
& + e^{\upsilon^{(1)}x_2}\big[-{}^6\overline{\alpha}_{2G}\sin(kx_1) + {}^6\overline{\alpha}_{2S}\cos(kx_1) \big] \Big\} \\
+ \chi_4 \Big\{ & -e^{-\upsilon'^{(1)}x_2}\big[-{}^6\overline{\beta}_{1G}\sin(kx_1) + {}^6\overline{\beta}_{1S}\cos(kx_1) \big] \\
& + e^{\upsilon'^{(1)}x_2}\big[-{}^6\overline{\beta}_{2G}\sin(kx_1) + {}^6\overline{\beta}_{2S}\cos(kx_1) \big] \Big\}
\end{aligned}
\tag{E7.110}
$$

$$
\begin{aligned}
{}^6\Theta_S^{(2)}(k) = -k & \big[{}^6\overline{\alpha}_{3S}\cos\big(\hat{\upsilon}^{(2)}x_2 + kx_1\big) - {}^6\overline{\alpha}_{3G}\sin\big(\hat{\upsilon}^{(2)}x_2 + kx_1\big) \big] \\
- \hat{\upsilon}'^{(2)} & \big[{}^6\overline{\beta}_{3G}\cos\big(\hat{\upsilon}'^{(2)}x_2 + kx_1\big) + {}^6\overline{\beta}_{3S}\sin\big(\hat{\upsilon}'^{(2)}x_2 + kx_1\big) \big]
\end{aligned}
\tag{E7.111}
$$

$$
\begin{aligned}
{}^{6}\Omega_{G}^{(2)}(k) = \hat{\upsilon}^{(2)}\Big[&{}^{6}\overline{\alpha}_{3S}\cos\big(\hat{\upsilon}^{(2)}x_2+kx_1\big)-{}^{6}\overline{\alpha}_{3G}\sin\big(\hat{\upsilon}^{(2)}x_2+kx_1\big)\Big] \\
&-k\Big[{}^{6}\overline{\beta}_{3G}\cos\big(\hat{\upsilon}'^{(2)}x_2+kx_1\big)+{}^{6}\overline{\beta}_{3S}\sin\big(\hat{\upsilon}'^{(2)}x_2+kx_1\big)\Big]
\end{aligned}
\tag{E7.112}
$$

$$
\begin{aligned}
{}^{6}A_{G}^{(2)}(k) = \hat{\chi}_{10}\Big[&{}^{6}\overline{\alpha}_{3S}\sin\big(\hat{\upsilon}^{(2)}x_2+kx_1\big)+{}^{6}\overline{\alpha}_{3G}\cos\big(\hat{\upsilon}^{(2)}x_2+kx_1\big)\Big] \\
&+\hat{\chi}'_{6}\Big[-{}^{6}\overline{\beta}_{3G}\sin\big(\hat{\upsilon}'^{(2)}x_2+kx_1\big)+{}^{6}\overline{\beta}_{3S}\cos\big(\hat{\upsilon}'^{(2)}x_2+kx_1\big)\Big]
\end{aligned}
\tag{E7.113}
$$

$$
\begin{aligned}
{}^{6}B_{G}^{(2)}(k) = \hat{\chi}_{5}\Big[&{}^{6}\overline{\alpha}_{3S}\sin\big(\hat{\upsilon}^{(2)}x_2+kx_1\big)+{}^{6}\overline{\alpha}_{3G}\cos\big(\hat{\upsilon}^{(2)}x_2+kx_1\big)\Big] \\
&+\hat{\chi}'_{6}\Big[{}^{6}\overline{\beta}_{3G}\sin\big(\hat{\upsilon}'^{(2)}x_2+kx_1\big)-{}^{6}\overline{\beta}_{3S}\cos\big(\hat{\upsilon}'^{(2)}x_2+kx_1\big)\Big]
\end{aligned}
\tag{E7.114}
$$

$$
\begin{aligned}
{}^{6}C_{S}^{(2)}(k) = \hat{\chi}'_{7}\Big[&{}^{6}\overline{\alpha}_{3S}\sin\big(\hat{\upsilon}^{(2)}x_2+kx_1\big)+{}^{6}\overline{\alpha}_{3G}\cos\big(\hat{\upsilon}^{(2)}x_2+kx_1\big)\Big] \\
&+\chi_{8}\Big[-{}^{6}\overline{\beta}_{3G}\sin\big(\hat{\upsilon}'^{(2)}x_2+kx_1\big)+{}^{6}\overline{\beta}_{3S}\cos\big(\hat{\upsilon}'^{(2)}x_2+kx_1\big)\Big]
\end{aligned}
\tag{E7.115}
$$

yazılıp bu eşitliklerin tek kısımları atıldıktan sonra kalan,

$$
\begin{aligned}
{}^{6}\Theta_{SÇ}^{(1)}(k) = \Big[&k\Big(-e^{-\upsilon^{(1)}x_2}\,{}^{6}\overline{\alpha}_{1G}+e^{\upsilon^{(1)}x_2}\,{}^{6}\overline{\alpha}_{2G}\Big) \\
&-\upsilon'^{(1)}\Big(e^{-\upsilon'^{(1)}x_2}\,{}^{6}\overline{\beta}_{1G}+e^{\upsilon'^{(1)}x_2}\,{}^{6}\overline{\beta}_{2G}\Big)\Big]\sin(kx_1)
\end{aligned}
\tag{E7.116}
$$

$$
\begin{aligned}
{}^{6}\Omega_{GÇ}^{(1)}(k) = \Big[&\upsilon^{(1)}\Big(e^{-\upsilon^{(1)}x_2}\,{}^{6}\overline{\alpha}_{1G}+e^{\upsilon^{(1)}x_2}\,{}^{6}\overline{\alpha}_{2G}\Big) \\
&+k\Big(e^{-\upsilon'^{(1)}x_2}\,{}^{6}\overline{\beta}_{1G}-e^{\upsilon'^{(1)}x_2}\,{}^{6}\overline{\beta}_{2G}\Big)\Big]\cos(kx_1)
\end{aligned}
\tag{E7.117}
$$

$$
\begin{aligned}
{}^{6}A_{GÇ}^{(1)}(k) = \Big[&\chi_{9}\Big(-e^{-\upsilon^{(1)}x_2}\,{}^{6}\overline{\alpha}_{1G}+e^{\upsilon^{(1)}x_2}\,{}^{6}\overline{\alpha}_{2G}\Big) \\
&+\chi_{2}\Big(e^{-\upsilon'^{(1)}x_2}\,{}^{6}\overline{\beta}_{1G}+e^{\upsilon'^{(1)}x_2}\,{}^{6}\overline{\beta}_{2G}\Big)\Big]\cos(kx_1)
\end{aligned}
\tag{E7.118}
$$

$$
\begin{aligned}
{}^{6}B_{GÇ}^{(1)}(k) = \Big[&\chi_{1}\Big(-e^{-\upsilon^{(1)}x_2}\,{}^{6}\overline{\alpha}_{1G}+e^{\upsilon^{(1)}x_2}\,{}^{6}\overline{\alpha}_{2G}\Big) \\
&-\chi_{2}\Big(e^{-\upsilon'^{(1)}x_2}\,{}^{6}\overline{\beta}_{1G}+e^{\upsilon'^{(1)}x_2}\,{}^{6}\overline{\beta}_{2G}\Big)\Big]\cos(kx_1)
\end{aligned}
\tag{E7.119}
$$

$$
\begin{aligned}
{}^{6}C_{SÇ}^{(1)}(k) = \Big[&\chi_{3}\Big(e^{-\upsilon^{(1)}x_2}\,{}^{6}\overline{\alpha}_{1G}+e^{\upsilon^{(1)}x_2}\,{}^{6}\overline{\alpha}_{2G}\Big) \\
&+\chi_{4}\Big(e^{-\upsilon'^{(1)}x_2}\,{}^{6}\overline{\beta}_{1G}-e^{\upsilon'^{(1)}x_2}\,{}^{6}\overline{\beta}_{2G}\Big)\Big]\sin(kx_1)
\end{aligned}
\tag{E7.120}
$$

$$
\begin{aligned}
{}^{6}\Theta_{SÇ}^{(2)}(k) = k\Big\{&\Big[\cos(\hat{\upsilon}^{(2)}h)\,{}^{6}\tilde{\overline{\alpha}}_{3S}+\sin(\hat{\upsilon}^{(2)}h)\,{}^{6}\tilde{\overline{\alpha}}_{3G}\Big]\sin\big(\hat{\upsilon}^{(2)}x_2\big) \\
&+\Big[\cos(\hat{\upsilon}^{(2)}h)\,{}^{6}\tilde{\overline{\alpha}}_{3G}-\sin(\hat{\upsilon}^{(2)}h)\,{}^{6}\tilde{\overline{\alpha}}_{3S}\Big]\cos\big(\hat{\upsilon}^{(2)}x_2\big)\Big\}\sin(kx_1) \\
-\hat{\upsilon}'^{(2)}\Big\{&-\Big[\cos(\hat{\upsilon}'^{(2)}h)\,{}^{6}\tilde{\overline{\beta}}_{3G}-\sin(\hat{\upsilon}'^{(2)}h)\,{}^{6}\tilde{\overline{\beta}}_{3S}\Big]\sin\big(\hat{\upsilon}'^{(2)}x_2\big) \\
&+\Big[\cos(\hat{\upsilon}'^{(2)}h)\,{}^{6}\tilde{\overline{\beta}}_{3S}+\sin(\hat{\upsilon}'^{(2)}h)\,{}^{6}\tilde{\overline{\beta}}_{3G}\Big]\cos\big(\hat{\upsilon}'^{(2)}x_2\big)\Big\}\sin(kx_1)
\end{aligned}
\tag{E7.121}
$$

$$
\begin{aligned}
{}^{6}\Omega_{GÇ}^{(2)}(k) = \hat{\upsilon}^{(2)}\Big\{&\Big[\cos(\hat{\upsilon}^{(2)}h)\,{}^{6}\tilde{\overline{\alpha}}_{3S}+\sin(\hat{\upsilon}^{(2)}h)\,{}^{6}\tilde{\overline{\alpha}}_{3G}\Big]\cos\big(\hat{\upsilon}^{(2)}x_2\big) \\
&-\Big[\cos(\hat{\upsilon}^{(2)}h)\,{}^{6}\tilde{\overline{\alpha}}_{3G}-\sin(\hat{\upsilon}^{(2)}h)\,{}^{6}\tilde{\overline{\alpha}}_{3S}\Big]\sin\big(\hat{\upsilon}^{(2)}x_2\big)\Big\}\cos(kx_1) \\
-k\Big\{&\Big[\cos(\hat{\upsilon}'^{(2)}h)\,{}^{6}\tilde{\overline{\beta}}_{3G}-\sin(\hat{\upsilon}'^{(2)}h)\,{}^{6}\tilde{\overline{\beta}}_{3S}\Big]\cos\big(\hat{\upsilon}'^{(2)}x_2\big) \\
&+\Big[\cos(\hat{\upsilon}'^{(2)}h)\,{}^{6}\tilde{\overline{\beta}}_{3S}+\sin(\hat{\upsilon}'^{(2)}h)\,{}^{6}\tilde{\overline{\beta}}_{3G}\Big]\sin\big(\hat{\upsilon}'^{(2)}x_2\big)\Big\}\cos(kx_1)
\end{aligned}
\tag{E7.122}
$$

$$
\begin{aligned}
{}^{6}A_{GÇ}^{(2)}(k) = \hat{\chi}_{10}\Big\{&\Big[\cos(\hat{\upsilon}^{(2)}h)\,{}^{6}\tilde{\overline{\alpha}}_{3G}-\sin(\hat{\upsilon}^{(2)}h)\,{}^{6}\tilde{\overline{\alpha}}_{3S}\Big]\cos\big(\hat{\upsilon}^{(2)}x_2\big) \\
&+\Big[\cos(\hat{\upsilon}^{(2)}h)\,{}^{6}\tilde{\overline{\alpha}}_{3S}+\sin(\hat{\upsilon}^{(2)}h)\,{}^{6}\tilde{\overline{\alpha}}_{3G}\Big]\sin\big(\hat{\upsilon}^{(2)}x_2\big)\Big\}\cos(kx_1) \\
+\hat{\chi}'_{6}\Big\{&\Big[\cos(\hat{\upsilon}'^{(2)}h)\,{}^{6}\tilde{\overline{\beta}}_{3S}+\sin(\hat{\upsilon}'^{(2)}h)\,{}^{6}\tilde{\overline{\beta}}_{3G}\Big]\cos\big(\hat{\upsilon}'^{(2)}x_2\big) \\
&-\Big[\cos(\hat{\upsilon}'^{(2)}h)\,{}^{6}\tilde{\overline{\beta}}_{3G}-\sin(\hat{\upsilon}'^{(2)}h)\,{}^{6}\tilde{\overline{\beta}}_{3S}\Big]\sin\big(\hat{\upsilon}'^{(2)}x_2\big)\Big\}\cos(kx_1)
\end{aligned}
\tag{E7.123}
$$

$$
\begin{aligned}
{}^{6}B_{GÇ}^{(2)}(k) = \hat{\chi}_5 \Big\{ & \Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{6}\tilde{\tilde{\alpha}}_{3G} - \sin(\hat{\upsilon}^{(2)}h)\ {}^{6}\tilde{\tilde{\alpha}}_{3S}\Big]\cos\left(\hat{\upsilon}^{(2)}x_2\right) \\
& + \Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{6}\tilde{\tilde{\alpha}}_{3S} + \sin(\hat{\upsilon}^{(2)}h)\ {}^{6}\tilde{\tilde{\alpha}}_{3G}\Big]\sin\left(\hat{\upsilon}^{(2)}x_2\right)\Big\}\cos(kx_1) \\
& - \hat{\chi}'_6 \Big\{ \Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{6}\tilde{\tilde{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^{6}\tilde{\tilde{\beta}}_{3G}\Big]\cos\left(\hat{\upsilon}'^{(2)}x_2\right) \\
& \quad - \Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{6}\tilde{\tilde{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^{6}\tilde{\tilde{\beta}}_{3S}\Big]\sin\left(\hat{\upsilon}'^{(2)}x_2\right)\Big\}\cos(kx_1)
\end{aligned}
\qquad \text{(E7.124)}
$$

$$
\begin{aligned}
{}^{6}C_{GÇ}^{(2)}(k) = \hat{\chi}'_7 \Big\{ & -\Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{6}\tilde{\tilde{\alpha}}_{3G} - \sin(\hat{\upsilon}^{(2)}h)\ {}^{6}\tilde{\tilde{\alpha}}_{3S}\Big]\sin\left(\hat{\upsilon}^{(2)}x_2\right) \\
& + \Big[\cos(\hat{\upsilon}^{(2)}h)\ {}^{6}\tilde{\tilde{\alpha}}_{3S} + \sin(\hat{\upsilon}^{(2)}h)\ {}^{6}\tilde{\tilde{\alpha}}_{3G}\Big]\cos\left(\hat{\upsilon}^{(2)}x_2\right)\Big\}\sin(kx_1) \\
& - \chi_8 \Big\{ \Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{6}\tilde{\tilde{\beta}}_{3S} + \sin(\hat{\upsilon}'^{(2)}h)\ {}^{6}\tilde{\tilde{\beta}}_{3G}\Big]\sin\left(\hat{\upsilon}'^{(2)}x_2\right) \\
& \quad + \Big[\cos(\hat{\upsilon}'^{(2)}h)\ {}^{6}\tilde{\tilde{\beta}}_{3G} - \sin(\hat{\upsilon}'^{(2)}h)\ {}^{6}\tilde{\tilde{\beta}}_{3S}\Big]\cos\left(\hat{\upsilon}'^{(2)}x_2\right)\Big\}\sin(kx_1)
\end{aligned}
\qquad \text{(E7.125)}
$$

eşitlikleri (3.133)-(3.142) eşitliklerinde kullanılırsa sonuç olarak,

$$
{}^{6}\overline{u}_1^{(1)} = -\frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{6}\Theta_{SÇ}^{(1)}(k)\,dk \qquad \text{(E7.126)}
$$

$$
{}^{6}\overline{u}_2^{(1)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{6}\Omega_{GÇ}^{(1)}(k)\,dk \qquad \text{(E7.127)}
$$

$$
{}^{6}\sigma_{11}^{(1)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{6}A_{GÇ}^{(1)}(k)\,dk \qquad \text{(E7.128)}
$$

$$
{}^{6}\sigma_{22}^{(1)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{6}B_{GÇ}^{(1)}(k)\,dk \qquad \text{(E7.129)}
$$

$$
{}^{6}\sigma_{12}^{(1)} = -\frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{6}C_{SÇ}^{(1)}(k)\,dk \qquad \text{(E7.130)}
$$

$$
{}^{6}\overline{u}_1^{(2)} = -\frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{6}\Theta_{SÇ}^{(2)}(k)\,dk \qquad \text{(E7.131)}
$$

$$
{}^{6}\overline{u}_2^{(2)} = \frac{P_0}{\pi}\int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{6}\Omega_{GÇ}^{(2)}(k)\,dk \qquad \text{(E7.132)}
$$

$$ {}^{6}\sigma_{11}^{(2)} = \frac{P_0}{\pi} \int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{6}A_{GÇ}^{(2)}(k)\, dk \tag{E7.133} $$

$$ {}^{6}\sigma_{22}^{(2)} = \frac{P_0}{\pi} \int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{6}B_{GÇ}^{(2)}(k)\, dk \tag{E7.134} $$

$$ {}^{6}\sigma_{12}^{(2)} = -\frac{P_0}{\pi} \int_{\frac{k_T^{(1)}}{\sqrt{1+\eta_2^{(1)}}}}^{\frac{k_L^{(2)}}{\sqrt{1+\eta_1^{(2)}}}} {}^{6}C_{SÇ}^{(2)}(k)\, dk \tag{E7.135} $$

bulunur.

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